Physics 1401 - Formula List

- **1D Kinematics**

General 1D Motion: x as a function of t

$v_{ave} = \frac{\Delta x}{\Delta t}$. $v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$, $a_{ave} = \frac{\Delta v}{\Delta t}$. $a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$

Constant Vel.: $x(t) = x_0 + v\,t \implies \Delta x = v\,t$

Constant Acc.: $v(t) = v_0 + a\,t$ and $x(t) = x_0 + v_0\,t + \frac{1}{2}\,a\,t^2$

$v = v_0 + a\,t$ $\quad \Delta x = \frac{1}{2}\,(v_0 + v)\,t$

$\Delta x = v_0\,t + \frac{1}{2}\,a\,t^2$ $\quad v^2 = v_0^2 + 2\,a\,\Delta x$

Free Fall : $x \to y$ (y is up) and $a = -g$

- **General Vectors** $\vec{A} = \langle A_x, A_y \rangle = A_x\,\hat{x} + A_y\,\hat{y}$

Mag. & Dir. angle $\implies$ Components $A_x = A\cos\theta$ and $A_y = A\sin\theta$.

Components $\implies$ Mag. & Dir. angle

$$A = \sqrt{A_x^2 + A_y^2} \text{ and } \theta = \begin{cases} \tan^{-1}\left(\frac{A_y}{A_x}\right) & \text{for } A_x > 0 \\ 180° + \tan^{-1}\left(\frac{A_y}{A_x}\right) & \text{for } A_x < 0 \end{cases}$$

- **Relative Motion** $\vec{v}_{AC} = \vec{v}_{AB} + \vec{v}_{BC}$ and $\vec{v}_{BA} = -\vec{v}_{AB}$
- **General 2D Kinematics** $\vec{r}$ as a function of t

$\vec{v}_{ave} = \frac{\Delta\vec{r}}{\Delta t}$, $\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$, $\vec{a}_{ave} = \frac{\Delta\vec{v}}{\Delta t}$, $\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t}$

Ave. Speed = $\frac{\text{Total Distance}}{\text{Total Time}}$

- **Projectiles**

Horizontal: $a_x = 0 \implies v_x$ is const. Vertical: $a_y = -g$

$v_{0x} = v_0\cos\theta$ and $v_{0y} = v_0\sin\theta$.

- **Newton's Laws**

First Law: $\vec{v}$ is const., unless net force.

Second Law: $\vec{F}_{net} = m\,\vec{a}$

Third Law: $\vec{F}_{12} = -\vec{F}_{21}$

Weight ∝ Mass: $W = m\,g$

- **Friction between surfaces** $f_s \le \mu_s\,N$ (static), $f_k = \mu_k\,N$ (kinetic)
- **Springs - Hooke's Law:** $F = -k\,x$
- **Circular Motion**

Uniform Circular Motion: $a_c = \frac{v^2}{r}$, Also $v = \frac{2\pi r}{T} \implies a_c = \left(\frac{2\pi}{T}\right)^2 r$

General Circular Motion: $a_c = \frac{v^2}{r}$, $a_t = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$

- **Work** $W = \sum_i F_{\parallel}^{(i)}\,d^{(i)}$ $\left(\text{small } \vec{d}^{(i)}\right)$

const. force: $W = F\,d\cos\theta = F_{\parallel}\,d = F\,d_{\parallel}$ in 1D: $W = F\,d$

Work done by gravity: $W_{grav} = -m\,g\,\Delta y$

Variable force in 1D: $W = \pm$(area under F vs. x graph)

Work done by spring: $W = -\frac{1}{2}\,k\left(x_f^2 - x_i^2\right)$

- **Work-Energy Theorem** $W_{net} = \Delta K$

W_{net} (net work). $K = \frac{1}{2}\,m\,v^2$ (kinetic energy)

- **Potential Energy and Mechanical Energy**

$\vec{F}$ is conservative $\Longleftrightarrow$ Work is independent of path.

conservative forces $\implies \Delta U = -W$ (U is potential energy)

Gravity: $U = m\,g\,y$ Spring: $U = \frac{1}{2}\,k\,x^2$

W_{nc} is work of all nonconservative forces.

$E = E^{mech} = K_{tot} + U_{tot} \implies E_i + W_{nc} = E_f$, $W_{nc} = 0 \implies E_i = E_f$

Average Power: $\mathcal{P}_{ave} = \frac{W}{\Delta t} = F\,v$

- **Momentum and Impulse-Momentum Theorem**

$\vec{p} = m\,\vec{v}$ (mom.) I = area under F vs. t graph. (impulse)

$\vec{F}_{net,ave}\,\Delta t = \vec{I}_{net} = \Delta\,\vec{p} = m\left(\vec{v}_f - \vec{v}_i\right)$

- **Center of Mass for a System of Particles**

$M = m_1 + m_2 + \ldots$ $\quad \vec{r}_{cm} = \frac{1}{M}\left(m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots\right)$

$\vec{v}_{cm} = \frac{1}{M}\left(m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots\right)$ $\quad \vec{a}_{cm} = \frac{1}{M}\left(m_1\vec{a}_1 + m_2\vec{a}_2 + \ldots\right)$

- **Second Law for a Particle and System**

particle : $\vec{F}_{net} = m\,\vec{a}$ $\quad \vec{F}_{net} = \lim_{\Delta t \to 0} \frac{\Delta\vec{p}}{\Delta t}$ $\quad \vec{F}_{net,ave} = \frac{\Delta\vec{p}}{\Delta t}$

system : $\vec{F}_{net}^{ext} = M\,\vec{a}_{cm}$ $\quad \vec{F}_{net}^{ext} = \lim_{\Delta t \to 0} \frac{\Delta\vec{p}_{tot}}{\Delta t}$ $\quad \vec{F}_{net,ave}^{ext} = \frac{\Delta\vec{p}_{tot}}{\Delta t}$

$\vec{p}_{tot} = m_1\,\vec{v}_1 + m_2\,\vec{v}_2 + \ldots = M\,\vec{v}_{cm}$

- **Conservation of Momentum**

$\vec{F}_{net}^{ext} = 0 \implies \Delta\,\vec{p}_{tot} = 0 \implies \vec{p}_{tot,i} = \vec{p}_{tot,f}$

$F_{net,x}^{ext} = 0 \implies \Delta p_{tot,x} = 0 \implies p_{tot,i,x} = p_{tot,f,x}$

- **Collisions**

$\vec{p}_{tot,i} = \vec{p}_{tot,f} \implies m_1\,\vec{v}_{1i} + m_2\,\vec{v}_{2i} = m_1\,\vec{v}_{1f} + m_2\,\vec{v}_{2f}$

Elastic $\Longleftrightarrow$ $K_{tot,i} = K_{tot,f}$

Totally Inelastic: $\vec{v}_{1\,f} = \vec{v}_{2\,f} = \vec{v}_f \implies m_1\,\vec{v}_{1i} + m_2\,\vec{v}_{2i} = (m_1 + m_2)\,\vec{v}_f$

Elastic Collisions in 1D:

$v_{if} = \frac{m_1 - m_2}{m_1 + m_2}\,v_{1i} + \frac{2\,m_2}{m_1 + m_2}\,v_{2i}$ and $v_{2f} = \frac{2\,m_1}{m_1 + m_2}\,v_{1i} + \frac{m_2 - m}{m_1 + m_2}\,v_{2i}$

- **Rocket Propulsion** thrust = $v_e\,\frac{|\Delta m|}{\Delta t}$
- **General Rotations about fixed axis:**

$\omega_{ave} = \frac{\Delta\theta}{\Delta t}$, $\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$, $\alpha_{ave} = \frac{\Delta\omega}{\Delta t}$, $\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t}$

$\alpha = 0 \Longleftrightarrow \omega = \text{const} = \frac{2\pi}{T}$

- **Constant Angular Acceleration**

$\omega = \omega_0 + \alpha\,t$ $\quad \Delta\theta = \frac{1}{2}\,(\omega + \omega_0)\,t$

$\Delta\theta = \omega_0\,t + \frac{1}{2}\,\alpha\,t^2$ $\quad \omega^2 = \omega_0^2 + 2\,\alpha\,\Delta\theta$

- **Rotational and Linear Quantities**

$v_t = r\,\omega$, $v_c = 0$, $a_t = r\,\alpha$, $a_c = \omega^2\,r$, $a = \sqrt{a_c^2 + a_t^2}$

- **Rotational Energy**

$K = \frac{1}{2}\,I\omega^2$, $U = M g\,y_{cm}$, $K_{tot} = \frac{1}{2}\,M v^2 + \frac{1}{2}\,I\omega^2 = \frac{1}{2}\left(M + \frac{I}{R^2}\right)v^2$

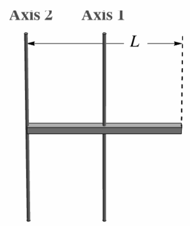
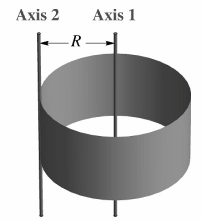
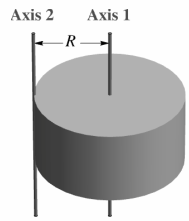
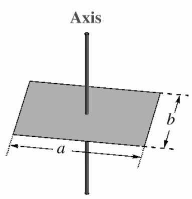
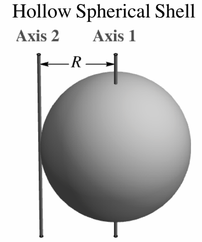
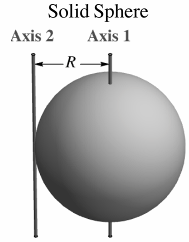
- **Torque** $\tau = r\,F_{\perp} = r_{\perp}\,F = r\,F\sin\theta$ Due to gravity : $\tau_{gravity} = r_{\perp}\,M\,g$
- **Angular Momentum of Particle** $L = r\,p_{\perp} = r_{\perp}\,p = r\,p\sin\theta$
- **General Rigid Body Dynamics**

2nd Law: $\tau_{net} = I\,\alpha$ and $\tau_{net} = \lim_{\Delta t \to 0} \frac{\Delta L}{\Delta t}$ Angular Momentum: $L = I\,\omega$

- **System** $\tau_{\text{net}}^{\text{ext}} = \frac{dL_{\text{tot}}}{dt}$ $\quad \tau_{\text{net}}^{\text{ext}} = 0 \implies \Delta L_{\text{tot}} = 0$ (Conservation)
- **Equilibrium** $\vec{F}_{\text{net}} = \vec{0}$ and $\tau_{\text{net}} = 0$
- **Moment of Inertia** r is $\perp$ dist.from axis

Point Masses: $I = m_1 r_1^2 + m_2 r_2^2 + \ldots$

Moments for uniform bodies:

Thin Rod	Cylindrical Shell or Hoop	Solid Cylinder or Disk
		
$I_2 = \frac{1}{3} ML^2,\ I_1 = \frac{1}{12} ML^2$	$I_2 = 2MR^2,\ I_1 = MR^2$	$I_2 = \frac{3}{2} MR^2,\ I_1 = \frac{1}{2} MR^2$
Rectangular Plate	**Hollow Spherical Shell**	**Solid Sphere**
		
$I = \frac{1}{12} M\left(a^2 + b^2\right)$	$I_2 = \frac{5}{3} MR^2,\ I_1 = \frac{2}{3} MR^2$	$I_2 = \frac{7}{5} MR^2,\ I_1 = \frac{2}{5} MR^2$

- **Newton's Law of Gravity**

Magnitude: $F = G \frac{m_1 m_2}{r^2}$ Direction is attractive.

Discrete Distribution: $\vec{F} = -G\, m \sum \frac{m_i}{r_i^2} \hat{r}_i$

Sph. Shell: $F = G \frac{M m}{r^2}$ $(r > R)$

$g = G \frac{M}{R^2}$ (at surface of spherical planet)

- **Gravitational Potential Energy**

Two masses: $U = -G \frac{M m}{r}$, Several masses: $U = -G \sum_{i<j} \frac{m_i m_j}{r_{ij}}$

Escape speed: $v_{\text{esc}} = \sqrt{\frac{2 G M}{R}}$

- **Circular Orbits** $v^2 = G \frac{M}{r}$ and $T^2 = \frac{4 \pi^2}{G M} r^3$
- **Simple Harmonic Motion** $a = -\omega^2 x$, $\omega = 2 \pi f = \frac{2 \pi}{T}$

$x(t) = A \cos(\omega t + \phi) = x_0 \cos \omega t + \frac{v_0}{\omega} \sin \omega t$

- **Energy** $E = \frac{1}{2} m v^2 + \frac{1}{2} k x^2 = \begin{cases} \frac{1}{2} k A^2 \\ \frac{1}{2} m v_{\max}^2 \end{cases}$ (mass/spring)

$v = \pm \omega \sqrt{A^2 - x^2}$ and $v_{\max} = \omega A$ (in general)

- **Examples of Simple Harmonic Motion**

Mass/Spring: $\omega = \sqrt{k/m}$
Physical Pendulum: $\omega = \sqrt{m g d / I}$
Simple Pendulum: $\omega = \sqrt{g/L}$

- **1D Waves**

General Solution: $u(x, t) = f(x - v t) + g(x + v t)$

- **Sinusoidal Waves** $u(x, t) = A \cos(k x \pm \omega t + \phi)$

$\lambda = \frac{2 \pi}{k}$, $\quad f = \frac{\omega}{2 \pi}$, $\quad v = f \lambda = \frac{\omega}{k}$

- **Waves on a String** $u(x, t) \Rightarrow y(x, t)$

Speed: $v = \sqrt{F_T / \mu}$ where F_T = Tension, Power: $\mathcal{P} = \frac{1}{2} \mu\, \omega^2 A^2 v$

- **Doppler Effect** v_s = source vel. and v_o = observer vel.

$f' = \frac{v + v_o}{v - v_s} f$, $v_o > 0$ and $v_s > 0$ when toward the other.

Mach Angle: $\sin \theta_{\text{Mach}} = v / v_s$ when $v_s > v$

- **Sound Intensity** $I = \frac{\mathcal{P}_{\text{ave}}}{A} = \frac{E_{\text{ave}}}{A \Delta t}$

Sound Level (in dB): $\beta = (10\ \text{dB}) \log(I / I_0)$ where $I_0 = 10^{-12}\ \text{W}/\text{m}^2$

- **Standing Waves**

$A \sin(k x - \omega t) + A \sin(k x + \omega t) = 2 A \sin(k x) \cos(\omega t)$

- **String - Both Ends Fixed or Pipe - Both Ends Open**

L = Length $\quad \lambda_n = \frac{2 L}{n}$, $n = 1, 2, 3, \ldots$

$f_1 = \frac{v}{2 L}$ (fundamental freq.) $\quad f_n = n f_1$ (harmonic freq.)

- **Pipe - One End Opened, Other Closed**

L = Length, $\lambda_n = \frac{4 L}{n}$, $f_1 = \frac{v}{4 L}$, $f_n = n f_1$, $n = 1, 3, 5, \ldots$

- **Fluid Statics** Pressure = Force/Area or $P = F/A$

density = mass/volume or $\rho = M/V$

Variation of P with depth: $P_{\text{bottom}} = P_{\text{top}} + \rho g h$

Buoyant Force: $B = \rho_{\text{fluid}}\, g V$, V is displaced volume

- **Temperature and Thermal Expansion**

$T_F = \frac{9}{5} T_C + 32$, $\Delta T_F = \frac{9}{5} \Delta T_C$, $T_K = T_C + 273$
$\Delta L = \alpha L_0 \Delta T$, $\Delta V = \beta V_0 \Delta T$, $\beta = 3 \alpha$ (for solid)

- **Heat** Q = Heat added to system

$Q = m c \Delta T$ (Temp. change), $Q = \pm m L$ (phase change)

- **Ideal Gas Law** $P V = N k T$ and $P V = n R T$

n = # of moles, $N = N_A n$ = # of molecules
Masses: $m_{\text{tot}} = n\, m_{\text{mol}} = N\, m_{\text{molecule}}$

- **Kinetic Theory** $\frac{1}{2} m v_{\text{rms}}^2 = K_{\text{trans,ave}} = \frac{3}{2} k_B T \Longrightarrow v_{\text{rms}} = \sqrt{\frac{3 k_B T}{m}}$

$U = \frac{3}{2} N k_B T = \frac{3}{2} n R T$ (internal energy of monatomic ideal gas)

- **Work** $W = \pm$(area under P vs. V graph) (done by system)

A cycle is a closed path: $W = \pm$(area enclosed) (work for a cycle)

Constant P or small ΔV: $W = P \Delta V$
Ideal gas at constant T: $W = n R T \ln(V_f / V_i)$

- **First Law** $\Delta U = Q - W$
- **Entropy** $\Delta S = \frac{Q}{T}$ (const. T or small Q)

Const. T: $\Delta S = \frac{Q}{T}$, Changing T: $\Delta S = m c \ln \frac{T_f}{T_i}$

- **Second Law** For a thermally isolated system: $\Delta S_{\text{tot}} \geq 0$
- **Heat Engines** $Q_H = Q_C + W$, Efficiency: $e = \frac{W}{Q_H} = 1 - \frac{Q_C}{Q_H}$

Max. Eff.: $e_{\max} = 1 - \frac{T_C}{T_H}$ (Carnot Engine is H.E. of max. eff.)