Chapter A

Dimension, Units and Significant Figures

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A.1 - Introduction

What is Physics?

Sciences can be quantitative, descriptive or a combination of both. Geology and Biology are mostly descriptive. Physics and Chemistry are mostly quantitative. It hard to accurately express the breadth of physics as a field. It involves scales that vary from subatomic to the entire universe. Physics studies matter at all scales, its interactions and behavior. At the very small scale we have the elementary particles and the fundamental forces of nature. Physics studies the scales of nuclei, atoms and molecules; in that there can be significant overlap with chemistry. With thermodynamics, physics studies the thermal properties of matter; that is another area of overlap with chemistry which we will discuss at the end of the course.

Physics studies the fundamental forces of nature. There are four fundamental forces of nature: gravity, electromagnetism, the strong nuclear force and the weak nuclear force; the two nuclear force only are significant on the microscopic scales. The two macroscopic fundamental forces are studied in introductory physics courses; this semester we will discuss gravity and in Physics II electromagnetism is the central topic.

Length Scales in Physics

Let us consider length scales from the very small to the very large. The size of the universe is unknown; it even could be infinite. The largest distance we can talk about is the size of the part of the universe we can see, the observable universe. The very smallest distance we can talk about in physics is the Planck Length. Although we do not have a theory of gravity that is consistent with quantum physics, by knowing the values of the relevant constants we can understand at what length scales will be relevant for such a theory. That length scale is called the Planck Length. It is most likely that even the notion of distance has no meaning when smaller than the Planck length.

Description	Lengths or Distances
Diameter of Observable Universe	93 billion ly = 8.8×10^{26} m
Diameter of Milky Way galaxy	106 ly = 10^{17} m
Distance light travels in a year	1 ly = 9.5×10^{15} m
Mean Sun–Nuptune distance	4.5×10^{12} m
Mean Earth–Sun distance	1.5×10^{11} m
Radius of the Sun	7×10^{8} m
Mean Earth–Moon distance	3.8×10^{8} m
Radius of the Earth	6.4×10^{6} m
Diameter of a small dust particle	10^{-4} m
Diameter of a hydrogen atom	10^{-10} m
Typical diameter of a nucleus	8×10^{-15} m
Diameter of a proton	2×10^{-15} m
Planck Length	1.6×10^{-35} m

A.2 - Dimensional Analysis

When we give a value to something, like distance, time, mass or velocity, we must specify the appropriate units. Suppose we wanted to add two lengths in different units, for example 2 m + 3 ft. Clearly, since the units are different, we cannot just add the numbers but the expression does make sense; we could first convert ft to m (or m to ft) and then combine the two numbers.

Suppose instead we were asked to add 2 m + 3 s. This is a meaningless expression; we cannot add a length to a time. In the first case we added two quantities with different units but the same dimensions, two lengths, and the expression made sense. The second case consists of adding quantities of different units and different dimensions, a length and a time; this is meaningless and the expression is considered incorrect.

The three fundamental dimensions are length [L], mass [M] and time [T]. To give the dimension of a variable we use square brackets. For example, if x is some distance then it is dimensionally a length, and we write: $\dim(x) = [L]$. A velocity is length per time, so if v is a velocity we would write $\dim(v) = [\mathrm{L}]/[\mathrm{T}]$.

If two quantities are equal they must be dimensionally equal. Moreover, as illustrated previously, if we add two quantities they must be dimensionally the same. If $\alpha + \beta = \gamma$ is a correct expression then it must follow that the three variables have the same dimension.

$$\alpha + \beta = \gamma \implies \dim(\alpha) = \dim(\beta) = \dim(\gamma)$$

It is also the case that the dimension of the product of two variables is the product of their dimensions:

$$\dim(\alpha\beta) = \dim(\alpha)\dim(\beta)$$

A dimensionless quantity is some number, like π, 2 or $\sqrt{3}$, that would be expressed without units. If κ is dimensionless we write $\dim(\kappa) = 1$, since multiplying by a dimensionless quantity doesn't change the dimension of a quantity.

The condition that our expressions must be dimensionally correct puts a constraint on the final form of our expressions. In fact, in many cases dimensional analysis will uniquely determine our expressions up to multiplicative dimensionless constants. Suppose one is trying to recall an expression for the surface area of a sphere in terms of its radius. An area A is dimensionally a length squared and a radius r is a length. It follows that the expression for the area in terms of the radius must have the general form:

$$\dim(A) = [L]^2 \text{ and } \dim(r) = [L] \implies A = \kappa\, r^2 \text{ where } \dim(\kappa) = 1.$$

In the case the dimensionless constant is $\kappa = 4\,\pi$.

A.3 - Systems of Units

Two quantities with the same dimension can have different units. For instance, we can measure length in ft or in m. We will soon discuss generally how to convert between different units. To avoid excessive conversions we use systems of units. Within a system if two quantities have the same dimension they will have the same units. By using a system we can avoid carrying through units while performing difficult calculations.

The metric system was introduced at the time of the French revolution. The intent was to base units on powers of ten and not on obscure factors like 1 ft = 12 in. Within the metric system there are two systems we will consider. For the most part this semester we will use the SI (Systeme International) system, which is also known as the MKS system. Here lengths are measured in m = meter, masses are in kg = kilogram and time is measured in s = second. In the CGS system we use cm = centimeter, g = gram and s. The British Engineering isystem (BE) uses for length, mass and time dimensions: ft, slugs and s. Note that lb = pound are units of force (or weight) and not a unit of mass.

The following table shows different quantities, their dimensions and units, in each of our three systems. The three fundamental dimensions are Length, Mass and Time, and within a system the corresponding units are the fundamental units. In addition to the fundamental units there are derived units; examples of derived units in SI are the newton, the joule, the watt and the pascal.

Quantity	Dimension	SI Unit	CGS Unit	BE Unit
Length	L	m	cm	ft
Mass	M	kg	g	slug
Time	T	s	s	s
Velocity	L/T	m/s	cm/s	ft/s
Acceleration	L/T^2	m/s^2	cm/s^2	ft/s^2
Force ($F = m\,a$)	$\mathrm{M\,L}/\mathrm{T}^2$	$\mathrm{N} = \mathrm{kg}\cdot\mathrm{m}/\mathrm{s}^2$ (newton)	dyne = $\mathrm{g}\cdot\mathrm{cm}/\mathrm{s}^2$	$\mathrm{lb} = \mathrm{slug}\cdot\mathrm{ft}/\mathrm{s}^2$ (pound)
Work or Energy ($W = F\,d$)	$\mathrm{M\,L}^2/\mathrm{T}^2$	J = N·m (joule)	erg = dyne·cm	ft·lb
Power ($\mathcal{P} = W/\text{time}$)	$\mathrm{M\,L}^2/\mathrm{T}^3$	W = J/s (watt)	erg/s	ft·lb/s
Pressure ($P = F/\text{Area}$)	$\mathrm{M}/(\mathrm{L}\cdot\mathrm{T}^2)$	$\mathrm{Pa} = \mathrm{N}/\mathrm{m}^2$ (pascal)	$\mathrm{dyne}/\mathrm{cm}^2$	$\mathrm{lb}/\mathrm{ft}^2$

When using a system of units we only need to consider units at the beginning and end of every problem. At the start of a problem we convert all relevant quantities into our system. The calculation can then be performed without considering units; if an equations is correct it must be dimensionally correct and the units will automatically work out. At the end of a problem we then need to restore the appropriate units for the quantity that was calculated.

A.4 - The SI System

The second, meter and kilogram

The SI base units we will use most are seconds, meters and kilograms. Over the years these definitions evolved to allow for more precise measurements.

The original definition of a second was in terms of a day, $60 \times 60 \times 24$ seconds is one day and a day is the average time for each rotation of the earth relative to the sun. This was not appropriate for accurate measurements and the second was redefined in terms of the current most accurate way of measuring time, atomic clocks. The most common atomic clock is based on a radiation frequency of a "hyper-fine structure" transition in the common isotope of Cesium, $^{133}\mathrm{Cs}$. Since 1967, a second has been defined as exactly 9192631 cycles of this radiation frequency.

Originally, the meter was defined in terms of the dimensions of the earth; the distance from the north pole to the equator (along a meridian through Paris) was 10 000 kilometers. This was, of course, difficult to reproduce, so the standard meter was converted to the distance between two marks on a metal bar at some fixed temperature. As science progressed, and scientific precision increased, a better standard was needed. Early in the twentieth century it was redefined in terms of a fixed number of wavelengths of a specific emission line of a krypton atom. Since 1983, the modern definition is now in terms of the speed of light. We define the speed of light to be some exact value, given below, and that then defines the meter in terms of the atomic clock definition of a second.

The original definition of the gram and kilogram was in terms of lengths and properties of water; a cubic centimeter of water was one gram. For more precision, a standard kilogram was created; some artifact that defined the kilogram. This artifact changed over time but the use of some artifact as a standard kilogram persisted until 2019.

The 2019 Redefinition of the SI System

In May 2019 the SI system of units was redefined. By choosing an exact value for the speed of light we were able to define a meter. Now all SI base units are defined entirely in terms of fundamental constants whose values are chosen as exact. Planck's constant, which is now referred to officially as the Planck constant, is the fundamental constant of quantum physics. The table below shows its units involve meters, seconds and kilograms, so by choosing its value to be exact we can now define the kilogram. Although, theoretically, this could have been done when Planck first introduced his constant in 1900, as a practical matter there would have been no way to use this definition to accurate calibrate a scale to kilograms. Experimental advances late in the twentieth century made it possible to create such a scale to accurately measure kilograms; this scale is a very complicated apparatus referred to as a Kibble balance, named after its inventor.

Fundamental Constant Name	Symbol Constant	Exact Numerical Value of Constant	SI Units of Constant and Base Units	Defined Base Unit
HFS freq. $^{133}\mathrm{Cs}$	Δf_{Cs}	9 192 631	$\mathrm{Hz} = \mathrm{s}^{-1}$	s = second
Speed of Light	c	299 792 458	ms^{-1}	m = meter
Planck Constant	h	$6.62607015 \times 10^{-34}$	$\mathrm{J \cdot s} = \mathrm{kg \cdot m^2 s^{-1}}$	kg = kilogram
Boltzmann Constant	k_{B}	1.380649×10^{-23}	$\mathrm{J/K} = \mathrm{kg \cdot m^2 s^{-2} K^{-1}}$	K = kelvin
Avogadro Constant	N_{A}	$6.02214076 \times 10^{23}$	mol^{-1}	mol = mole
Elementary Charge	e	$1.60217634 \times 10^{-19}$	$\mathrm{C} = \mathrm{A \cdot s}$	A = ampere
Luminous Efficacy	K_{cd}	683	$\mathrm{cd/W} = \mathrm{cd \cdot kg^{-1} m^{-2} s^3}$	cd = candela

In our discussion of thermodynamics at the end of the semester we will introduce two more SI base units: the kelvin, which is an absolute temperature scale, and the mole which is a very basic unit in chemistry. Now the kelvin is defined by choosing the Boltzmann constant to have an exact value and the mole is defined by setting the value of the Avogadro constant (the new name for Avogadro's number) to an exact value.

The other two base SI units are the ampere and the candela. The ampere is the SI unit of electric current and will be central to the Physics II discussion of electromagnetism; it is now defined by choosing the elementary charge e to an exact value. (Note that the electron's charge is $-e$ and the protons charge is $+e$.) The candela is unimportant for our elementary physics courses; it is a measure of the total light output of a source and its constant, the luminous efficacy, relates the total brightness to the power output of radiant energy.

A.5 - Conversion of Units

Conversion of units is typically a straightforward matter. There are more complicated cases where a systematic approach to conversions is needed. The idea here is simple; to convert we multiply by one, in some form. If $a = b$ is some conversion factor then $a/b = 1$.

Example A.1 - Write *g* in terms of miles and hours

The acceleration due to gravity is $9.80\ \mathrm{m/s^2}$. Convert this to $\mathrm{mi/hr^2}$.

Solution

We need to convert from m to mi and from s to hr. For the length conversion we can combine conversions from m to km and then from km to mi.

$$1\ \text{km} = 1000\ \text{m} \ \text{ and } \ 1\ \text{mi} = 1.609\ \text{km} \ \Longrightarrow \ 1\ \text{mi} = 1609\ \text{m}$$

We can similarly combine two conversions to get a s to hr conversion.

$$1\ \text{hr} = 60\ \text{min and } 1\ \text{min} = 60\ \text{s} \ \Longrightarrow \ 1\ \text{hr} = 3600\ \text{s}$$

Since in $9.80\ \text{m}/\text{s}^2$, m is in the numerator we must multiply by a form of one where m is in the denominator. Similarly, since s is in the denominator and squared, we must multiply by a form of one with s in the numerator and since it is squared, we must square the conversion.

$$9.80\ \frac{\text{m}}{\text{s}^2} \times \frac{1\ \text{mi}}{1609\ \text{m}} \times \left(\frac{3600\ \text{s}}{1\ \text{hr}}\right)^2 = 78\,900\ \frac{\text{mi}}{\text{hr}^2}$$

A.6 - Significant Figures

When you get results from a computer or calculator you will typically have far more digits than is proper. A rule of thumb for keeping track of the accuracy of numbers is to carefully keep track of significant figures. When presented with a number you must be able to identify the significant digits. In scientific notation all digits are significant, assuming there are no leading zeroes. For example, 6.8040×10^{-8} has five significant digits.

Beyond scientific notation, the only subtlety is the digit zero; all non-zero digits are always significant. Zeros to the left of the first non-zero digit are not significant. This becomes an issue when there is a decimal expression; a number like 0.00005020 has four significant digits. All those leading zeros are just placeholders; writing this in scientific notation makes it clear: 5.020×10^{-5}. In large numbers without decimal points, all zeros to the right are not significant. For example, 78000 has two significant digits but 78000., with the decimal point, has five; if the first zero is intended to be significant then you should use scientific notation and write 7.80×10^5.

Example A.2 - Significant Digits

How many significant figures are there in: **(a)** 31 000 , **(b)** 31 000. , **(c)** 0.0031000 , **(d)** 0.310 and **(e)** 3.10×10^4

Solution

For **(a)** the following zeros are not significant, so there are two significant digits, In case **(b)**, the decimal point at the end makes all the zeros significant, so it has five. The leading zeros in **(c)** are not significant but the trailing ones are. Similarly, **(d)** has three. **(e)** has three.

Multiplication, Division and Functions

When multiplying or dividing two or more values, the smallest number of significant figures in the values is the smallest number in the result. When you apply a function, like for instance the square, square root, a trig function, a log or an exponential, that function does not affect the number of significant figures in the answer.

Example A.3 - Significant Figures of a Function

In Chapter 2 we will see that the time of fall t for an object dropped from a height h is given by $t = \sqrt{2\,h/g}$, where g is the acceleration due to gravity. Suppose h and g are:

$$h = 15.4\ \text{m} \ \text{ and } \ g = 9.806\ \frac{\text{m}}{\text{s}^2}$$

Calculate t to the appropriate number of significant figures.

Solution

h has three significant figures and g has four, so the answer should be rounded to three. When using a calculator you will get many more digits than are appropriate. Suppose you get ten digits from the calculator: 1.772268073 . With that you round to three.

$$t = 1.77\ \text{s}$$

Addition and Significant Figures

When adding or subtracting values, what matters is the last significant decimal place of each number. the appropriate answer will have the largest or crudest value of the of the last significant decimal places. Suppose you have the values:

Number	Number of Significant Digits	Last Significant Decimal Place
2.0105	5	10^{-4}
28 000	2	10^{3}
1200.	4	10^{0}
0.0023050	5	10^{-7}

Example A.4 - Addition and Significant Figures

(a) Calculate 2.0105 + 2800 + 1200. + 0.0023050 to the appropriate number of significant figures.

Solution

The largest of the last significant places is 10^3, so that is the last significant decimal place of the sum. The calculator gives 2 920 201 281 so this makes the answer:

$$29\,000$$

(b) Now calculate 2.0105 + 0.0023050 to the appropriate number of significant figures.

Solution

The largest of the last significant places is 10^{-4}, so that is the last significant decimal place of the sum. The calculator gives 2 920 201 281 so this makes the answer:

$$2.0173$$

Problems with Using Significant Figures

Although the procedure for using significant figures is straight-forward and widely taught, it should be mentioned that it is merely a crude rule of thumb for propagating uncertainty and errors. For example, suppose a result should have three significant digits in the answer; if there is a long calculation to get that result and if each intermediate result is rounded to three digits, then the final result will have less than three significant figures. Compare the numbers 1.01 and 0.99; both are essentially equally accurate to within one percent, however the first has three significant figures nominally and the second has two. If you have a leading digit of one, you have essentially one less significant digit than if your leading digit is a nine. To avoid this round-off issue you should keep extra significant digits for intermediate steps of a calculation and only round in the end.

Another place where significant figures can give weird results is with angles. Compare the angles $-5\,°$ and $355\,°$. Both measure the same direction to the same accuracy, but the first has only one significant figure and the second has three.