Chapter E

Newton's Laws

Blinn College - Physics 1401 - Terry Honan

E.1 - Historical Context

Aristotle 384-322 BC

Rest is the natural state. The problem of physics was to explain why a body had a velocity.

It is a matter of common experience that after giving a body a push, it will eventually slow to a stop. When a body is pushed it is given an impetus; it continues to move until it loses that impetus. This notion is now known as incorrect.

Aristotle was a philosopher and physics was, for him, just natural philosophy. This perspective led to a de-emphasis of experiment. Physics was a subject one thinks about and writes about, but there was no need to test ideas with experiment. The history of science is filled with mistakes; what made Aristotle's mistakes so serious was that by de-emphasizing experiment no one found his errors for almost 2000 years.

As an example of the absurdity of Aristotelian physics, Aristotle claimed that heavier bodies fall faster that lighter ones. This seems intuitive at first; a brick falls faster than a feather. But he took it a step further by saying that the rate is *in proportion* to a body's mass. This was a precise mathematical statement and verifying it was as simple as dropping two objects of different mass. It is remarkable that for almost 2000 years no one thought of performing that experiment to test Aristotle's hypothesis.

Galileo Galilei 1564-1642

Galileo was the one who finally performed the free-fall experiment mentioned above. He observed that bodies of different mass fall with the same acceleration. In fact, most of the earlier material on kinematics of free fall and projectiles is due to Galileo realized that a feather fell slower than a hammer because of some extra force that we now know as friction.

Motion with constant velocity is the natural state. The problem of physics was to explain what caused changes in velocity, or what caused acceleration.

The reason why a body, when given a push on a horizontal surface, slows to a stop is due to friction. One can create motion with less friction and can imagine motion without any friction. In that case the body will continue to move indefinitely. Naturally bodies move with constant velocity unless some net force causes it to have an acceleration.

Galileo was a transitional figure. He demonstrated the inadequacy of Aristotelian physics. He also asked the right questions. The problem of understanding motion was no longer to explain why a body moves but to explain what caused changes in the motion.

Isaac Newton 1642-1727

Newton grew up in a Galilean world; the fallacies and misconceptions of Aristotelian physics were no longer a part of one's education. Galileo had properly posed the problem: What causes acceleration. Newton was the remarkable genius capable of providing the answer. He summarized dynamics in terms of three laws.

It would be hard to overstate the historical significance of Newton. In addition to fundamentally altering later intellectual developments, he had a profound influence on the practical aspects of common life; the technological explosion leading to the Industrial Revolution is directly related to his clear exposition of the science of mechanics.

E.2 - Newton’s First Law

Newton with his first law summarized what Galileo had done. The idea is that motion with a constant velocity is the natural state.

An object at rest tends to stay at rest. An object in uniform linear motion tends to maintain that motion. The word "tends" should be taken to mean: unless acted upon a net force.

The first sentence sounds more Aristotelian than Galilean. Uniform linear motion implies motion in a straight line at a constant speed, that is: constant velocity. Rest should only be viewed as a special case of constant velocity; it is a constant *zero* velocity.

A force is anything that pushes or pulls on an object. We will see that forces are vectors.

False Forces

Recall that a frame of reference is some coordinate system used to study motion. In an accelerated frame, for example inside an accelerating car, one feels one's self pushed opposite the direction of acceleration. When moving in a car in a straight line and braking, the acceleration is backward and one feels a false force pushing forward. When turning to the left, the acceleration is to the left and the false force is to the right. Generally, in any accelerated frame there is a false force opposite the acceleration.

What is the nature of these false forces? The point of the first law is that all bodies will tend to move with a constant velocity. When braking in a car, a person in the car will tend to keep moving at a constant velocity but the car slows around him. Relative to the car he is thrown forward. Similarly, when turning to the left in a car the person tends to continue in a straight line and the car moves to the left.

The Principle of Relativity

The principle of relativity is built in to Newtonian mechanics. It is a result of Galileo's observations on motion. This notion of relativity is known as *Galilean Relativity* and should be contrasted with *Special Relativity* which was introduced by Albert Einstein in 1905.

All inertial frames of reference are equivalent.

Recall that a frame of reference is some coordinate system used to study motion. If someone (preferably a passenger) throws a ball straight upward in a car moving with a constant velocity, it will move in a way that is indistinguishable from free fall. To an observer on the side of the road the ball would move as a projectile but to both observers the acceleration would be the same, a downward acceleration of g. If the car is turning, or accelerating in any way, then there will be false forces and the motion will deviate from free fall.

An inertial frame is a rest frame *or* a non-rotating frame moving at a constant velocity with respect to a rest frame.

Basically, an inertial frame is a non-accelerated frame. The equivalence of inertial frames means that there is no preferred absolute rest frame. Moving with a constant velocity is indistinguishable from being at rest. Suppose some experiment is performed in a van moving at a constant velocity. (We assume the road is as smooth as possible.) The result of that experiment will be give the same result as if the van were at rest.

Space and time are absolute in Galilean relativity.

Absolute space means that space is three dimensional Euclidean space. The length of a meter stick is the same to all observers and is independent of relative motion. If x, y and z are coordinated relative to one frame and x', y' and z' are coordinates relative to a different frame the lengths (actually length-squared) being equal can be written as

$$\Delta x^2 + \Delta y^2 + \Delta z^2 = \Delta x'^2 + \Delta y'^2 + \Delta z'^2 \quad \text{(Absolute Space)}$$

Absolute time implies that the time between two events (two different positions and two different times) is the same for all observers. If Δt is the time between two events in one frame and $\Delta t'$ is the time relative to another frame, then

$$\Delta t = \Delta t' \qquad \text{(Absolute Time)}$$

Einstein's Special Relativity eliminated the notions of absolute space and time.

Although relativity had been a well-established principle of classical mechanics, it became clear in the middle of the nineteenth century that the laws of electromagnetism, which were summarized by Maxwell's four equations, violated this principle. It appeared that Maxwell's equations were valid in only one absolute rest frame. In 1905 Einstein realized that Maxwell's equations were valid in all inertial frames and he put relativity back into fundamental physics. To do this however he had to modify the notions of absolute space and time that were implicit in Galilean relativity.

In special relativity, lengths and times are different frames but, it turns out, that the length-squared minus the time-squared is the same in all frames, where we multiply time by the speed of light c to make the units work out.

$$\Delta x^2 + \Delta y^2 + \Delta z^2 - c^2 \Delta t^2 = \Delta x'^2 + \Delta y'^2 + \Delta z'^2 - c^2 \Delta t'^2$$

E.3 - Newton’s Second Law

One can state the second law with words, as in the case of the first law, but it just becomes a complicated sentence dictating a simple equation. It will then be given as an expression.

$$\vec{F}_{\text{net}} = m\,\vec{a} \tag{E.1}$$

The net force $\vec{F}_{\text{net}}$ is the vector sum of all forces acting on a body. m is the mass of the body and $\vec{a}$ is its acceleration. Newton's first law may be viewed as a consequence of the second. If $\vec{F}_{\text{net}} = \vec{0}$ then $\vec{a} = \vec{0}$ and this implies $\vec{v}$ is a constant.

Definition of Mass and Force

Force is a measure of the amount that something pushes or pulls. Mass is a measure of the amount of material. The second law may be considered a simultaneous definition of force and mass. To define a mass all we need is a notion of a reproducible force. For example consider a spring that is compressed by a fixed amount to a standard distance; if that spring is then compressed to that same standard distance will produce the same force and this is reproducible. If we call this force F_0 then we can act this same force on different masses m_0 and m, and measure their instantaneous accelerations. If m_0 gets an acceleration a_0 and m gets acceleration a. We can then write the ratio of the masses in terms of the ratio of the accelerations.

$$F_0 = m_0\,a_0 = m\,a \implies \frac{m}{m_0} = \frac{a_0}{a}$$

To verify that this can be used as a definition of mass imagine grabbing some rock and choosing that as the standard of mass; call it 1 Rock. If the mass used is this standard rock, then $m_0 = 1$ Rock; if the second mass gets an acceleration that is 1/2 that of the rock then we conclude that its mass is $m = 2$ Rocks.

To show how the second law is a definition of force we act different forces on the same mass, m_0.

$$F_0 = m_0\,a_0 \text{ and } F = m_0\,a \implies \frac{F}{F_0} = \frac{a}{a_0}$$

Units: The SI unit for Force is: $\text{N} = \text{newton} = \text{kg m}/\text{s}^2$

E.4 - Newton’s Third Law

Newton's third law is crucial for a proper understanding of force. If $\vec{F}_{12}$ is the force of body 1 on body 2 and $\vec{F}_{21}$ is the force of 2 on 1, then

$$\vec{F}_{21} = -\vec{F}_{12} \tag{5.2}$$

This is often stated in the language of action-reaction pairs: to every action there is an equal and opposite reaction. Equal and opposite means that one vector is the negative of the other.

At first glance the third law is counter-intuitive. When a small car gets in a collision with a large truck the force of the truck on the car is the same, in magnitude, as that of the car on the truck. The same force on a smaller mass has a larger effect, meaning that since $F = m\,a$ the acceleration is larger. For an extreme example, the gravitational force of the Earth on a person is equal in magnitude to the force of the person on the Earth. Since the mass of the Earth is *much* larger the acceleration of the Earth is negligible.

Tension, Normal Forces and Surface Friction

Suppose ten people pull on a rope in a tug-of-war. Each person can pull with a force of F_0. Suppose five pull on one side and five pull on the other. The total force pulling in either direction is $5\,F_0$, which is then the tension in the rope. Suppose instead all ten pull on the same side against a tree. The force pulling, and thus the tension, is now $10\,F_0$. In this second case the rope pulls on the tree with the $T = 10\,F_0$ and by the third law the tree pulls back with the same force; the tree then is equivalent to another ten people pulling on the other side. At any position P in a rope, the left part of the rope pulls to the left with a force T and the right part pulls to the right with a force of T. A spring scale that stretches, like a fish scale, reads this tension force in a rope.

Forces between surfaces break up into two components: perpendicular to a surface is the normal force and parallel to a surface are friction forces. The word normal means perpendicular. When a person stands on a floor he pushes *down* on the floor with a normal force N equal to his weight. The floor then pushes *up* on the person with the same magnitude force. Suppose instead that a person leans against a wall. The person pushes into the wall with normal force of N and the wall pushes back on the person with the same magnitude normal force. Friction forces also break up into equal and opposite pairs. When an object slides to a rest on a floor, the floor pushes backward on the object with a force known as *kinetic friction*; the object then exerts a forward friction force on the floor. When a walking person pushes backward on the floor with a *static friction* force, the floor pushes forward with the same force.

E.5 - Applying the Second Law

Free-body Diagrams

The left hand side of the second law, $\vec{F}_{\text{net}}$, is the vector sum of all forces acting on a body. To help with this we draw a free-body diagram; this is a vector diagram showing all forces acting on the body. To draw a free-body diagram we must include contact forces and action-at-a-distance forces.

Include all contact forces.

Contact forces are due to all things in contact with the body. Imagine a surface around the body; the contact forces are due to everything piercing that surface. If something is standing on a floor or leaning against a wall then we draw the corresponding normal forces and perhaps friction forces. Air is in contact with a body and its effects, if they need to be considered, will be contact forces.

Include field forces

The forces that can act on a body without touching it are the *field forces*. These are the fundamental forces of nature: gravity, electromagnetism, the weak nuclear force and the strong nuclear force. The two nuclear forces act only over short distances and, for purposes of classical mechanics, can be neglected. It should be pointed out that the electromagnetic and weak nuclear forces are no longer considered separate forces; in 1968 the Electroweak unification was published and it was experimentally verified in the early eighties.

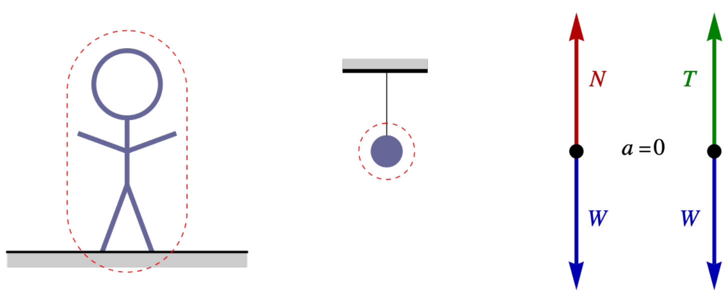
In Newton's time these forces were considered action-at-a-distance, that is distant objects directly interacting. This Newtonian notion implies an instantaneous interaction over arbitrarily large distances. We now understand interaction in terms of the notion of a field. The idea is this: Particles create fields, fields propagate at a finite speed by dynamical rules and then fields exert forces on other particles.

Accelerations do not belong in free-body diagrams.

The free-body diagram gives the left-hand side of the second law. The right-hand side involves the acceleration. It is important that to note that $m\,\vec{a}$ is not a force and does not belong in the free-body diagram. It is a good idea to draw the acceleration next to the diagram, though. Being systematic with directions, as we were in earlier chapters, is more trouble than it is worth. The easiest method is to choose the direction of the acceleration as positive.

One Dimensional Examples

A Standing Person / A Hanging Weight



It follows from the second law that the normal force equals the gravitation force on the person, his weight.

$$F_{\text{net}} = m\,a \implies N - W = 0 \implies N = W.$$

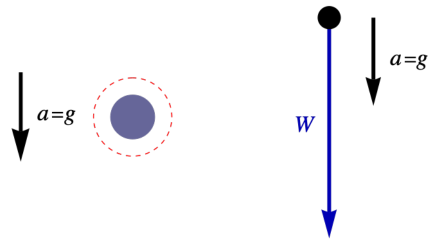
A bathroom-type spring scale, which is compressed, reads the normal force. This, in this stationary case, is just the weight.

For the hanging mass it is the same but there is a tension force pulling up instead of the normal force pushing up.

$$F_{\text{net}} = m\,a \implies T - W = 0 \implies T = W.$$

Tension is read by the other type of spring scale, one that you stretch. Think of a fish scale or a hanging produce scale in a grocery.

Free Fall and Weight

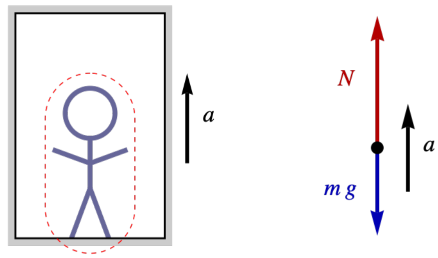


When and object is in free-fall, the only force acting on a body is its weight. As we established in Chapter B it has a downward acceleration of g. Choosing downward as positive the second law implies that the weight equals the mass times the acceleration due to gravity.

$$F_{\text{net}} = m\,a \implies W = m\,g$$

This expression applies generally to give the weight of a body of mass m in a gravitational field. The strength of that field is g, the acceleration due to gravity. Do not think of g as an acceleration but as the proportionality between weight (a force) and mass, which is dimensionally an acceleration. If gravity is the only force acting on a body, as is the case of free-fall, then the downward acceleration is g.

Elevators and Apparent Weight



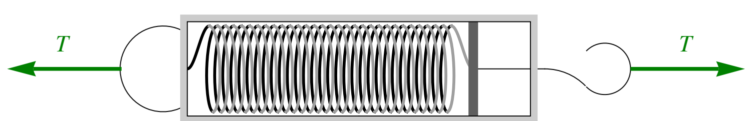
$$F_{\text{net}} = m\,a \implies N - m\,g = m\,a \implies N = m\,g + m\,a$$

The normal force here, which can still be read by a spring scale, is called the apparent weight. Similarly, a mass being lifted by a rope an upward acceleration will experience the same apparent weight, but the express would be for the tension instead of the normal force.

Note that for an object in free-fall $a = -g$, so its apparent weight is zero. This is called weightlessness. An orbit is a perpetual state of free-fall. There is no apparent weight in orbit. Weightlessness does not mean there is no gravity; in a low-earth orbit and astronaut's weight is just a little less than his earth weight. The point is that the astronaut feels no gravity; the apparent weight is zero.

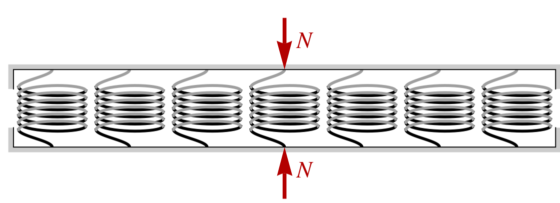
Spring Scales and Balance Scales

Spring scales measure weight. On the moon, the acceleration due to gravity is about one-sixth that on the earth. A spring scale on the moon would give one-sixth the earth weight. There are two types of spring scales.



A tension spring scale measures the tension in a rope or string.

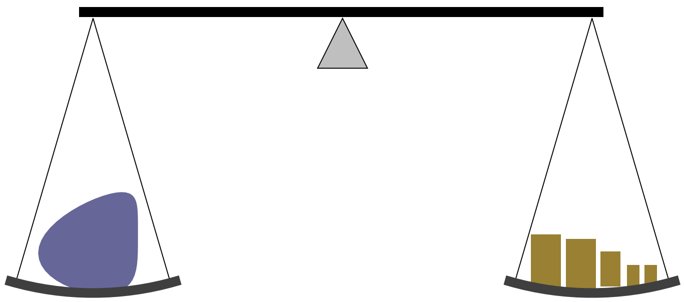
A tension spring scale stretches under tension and the amount the spring stretches depends on the tension; it can be used to measure tension. Hanging something from one measures its weight. Think of a fish scale.



A compression spring scale measures a normal force.

A bathroom-type scale is a spring scale that compresses. The amount it compresses varies with the normal force and thus, it measures the normal force between surfaces. Standing on one gives a person's weight.

A balance scale is fundamentally different; it measures mass. The simplest balance scale has two well-balanced trays pivoting symmetrically on a fulcrum. An unknown mass is placed on one side and known masses are added to the other side until there is balance.

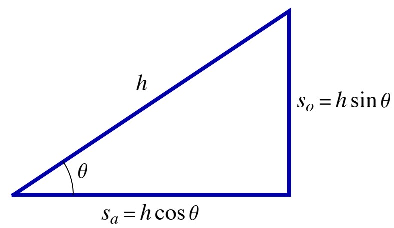


A balance scale measures mass.

It should be clear that the setup above would give the same result on the moon as on the earth; it measures mass not weight. Although the simple balance shown above is conceptually clearest, there are many other common types. For example, a triple-beam balance is common in physics labs; an unknown mass is placed on a tray and masses are moved along bars until there is a balance. Also a doctor's office-type scale uses the same principle and also measures mass.

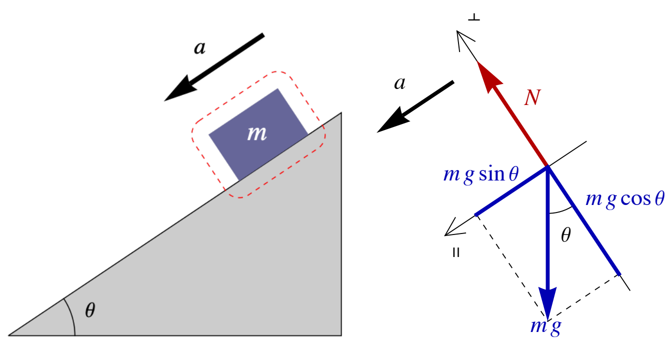
Two Dimensional Examples

When given a two dimensional problem we first draw a free-body diagram. Next, we resolve all forces and the acceleration into a coordinate system representing a pair of perpendicular directions. Applying the second law to both perpendicular directions gives a pair of equations. Resolving all forces (or accelerations) into the perpendicular directions requires only the simple trigonometry from a right triangle; the force (or acceleration) will the the hypotenuse h of the right triangle and the opposite and adjacent sides will be $h \sin\theta$ and $h \cos\theta$, respectively.



Block on a Frictionless Inclined Plane

Consider the case of a block sliding down a frictionless inclined plane. We will, as is the convention, always refer to the angle of an incline as the angle of the surface measured from horizontal. First we draw the free-body diagram. The only thing touching the block is the surface of the incline. Recall that surface forces break up into two components: perpendicular to the surface is the normal force N. The friction forces are parallel to the surface; since there is no friction we have the normal force as the only contact force. Then we add in the weight $m\,g$ which acts, of course, downward. Note that the angle between the surface's normal and vertical is the same as between the surface and horizontal.



Since surface forces split naturally into parallel and perpendicular components, it is typically best to choose the coordinates to be parallel and perpendicular to the surface. The normal force is already resolved along the perpendicular direction. Whenever there is an inclined plane at angle θ resolving the weight into parallel and perpendicular components always gives:

$$|W_\perp| = m\,g\,\cos\theta \text{ and } |W_{\parallel}| = m\,g\,\sin\theta.$$

The acceleration only has a parallel component and that is just a. Applying the second law to the parallel direction we get

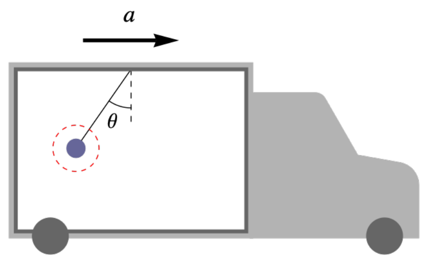
$$F_{\text{net},\parallel} = m\,a_{\parallel} \implies m\,g\,\sin\theta = m\,a \implies a = g\,\sin\theta.$$

Solving the second law in the direction perpendicular to a surface *always* gives the normal force.

$$F_{\text{net},\perp} = m\,a_{\perp} \implies N - m\,g\cos\theta = 0 \implies N = m\,g\cos\theta.$$

If we are just solving for the acceleration this value isn't needed; we will see that in examples involving friction that the normal force will be of interest.

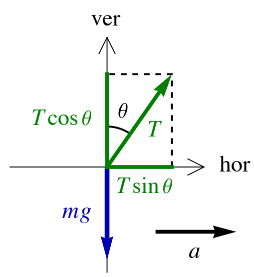
Example E.1 - The Accelerating Pendulum as an Accelerometer



Anything hanging in a vehicle can be used as an accelerometer, meaning it that can measure acceleration. Relate the hanging angle, as measured from vertical as shown, to the acceleration.

Solution

Here we will choose our perpendicular axes to be horizontal (in the direction of the acceleration) and vertical. Note that the forces do not balance here; they should not balance because there is an acceleration.



After resolving the tension into horizontal and vertical components, we apply the second law to each direction and get:

$$F_{\text{net,hor}} = m\,a_{\text{hor}} \implies T\sin\theta = m\,a$$

$$F_{\text{net,ver}} = m\,a_{\text{ver}} \implies T\cos\theta - m\,g = 0 \implies T\cos\theta = m\,g.$$

From these two expressions we can relate the acceleration to the angle. Divide the second expression into the first and using $\tan\theta = \sin\theta/\cos\theta$ we get:

$$a = g\tan\theta$$

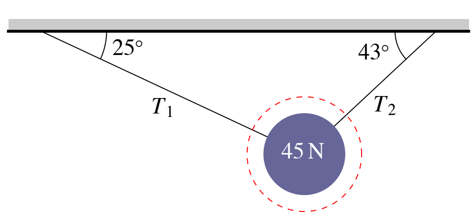
Zero Acceleration

In any application of the second law where the acceleration is zero, it follows that the forces balance.

$$\vec{F}_{\text{net}} = \vec{0}$$

Solving such a problem is not different than problems with accelerations; the second law is resolved into a pair of perpendicular directions to give a pair of equations. The simplification is that instead of summing the forces to zero in some direction, one can balance them. Replace, for instance $F_{\text{net,ver}} = 0$ with $F_{\text{up}} = F_{\text{down}}$ or replace $F_{\text{net,hor}} = 0$ with $F_{\text{left}} = F_{\text{right}}$.

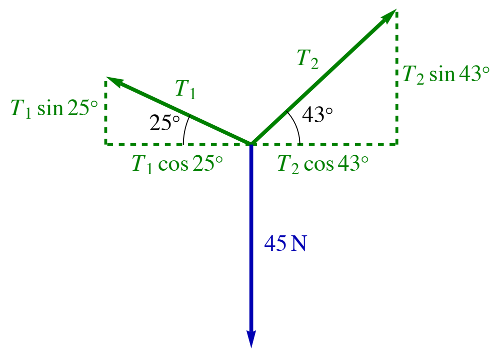
Example E.2 - A Weight Hanging from Two Ropes



A 45-N weight is hung from a ceiling by two ropes as shown. What are both tensions, T_1 and T_2?

Solution

The first step, as always, is to draw a good free-body diagram. Then resolve the forces into a pair of perpendicular directions, in this case horizontal and vertical.



There is no acceleration so the horizontal and vertical force balance. Horizontally, we have $T_1 \cos\theta_1$ to the left and $T_2 \cos\theta_2$ to the right.

$$F_{\text{net,hor}} = m\,a_{\text{hor}} = 0 \implies T_1 \cos 25\,° = T_2 \cos 43\,°$$

In the vertical direction there are $T_1 \sin\theta_1$ and $T_2 \sin\theta_2$ acting up and W down.

$$F_{\text{net,ver}} = m\,a_{\text{ver}} = 0 \implies T_1 \sin 25\,° + T_2 \sin 43\,° = 45\text{ N}$$

Using the horizontal expression we can solve for T_2 and then insert that into the vertical expression to solve for T_1.

$$T_2 = T_1 \frac{\cos 25\,°}{\cos 43\,°} \implies T_1 \sin 25\,° + T_1 \frac{\cos 25\,°}{\cos 43\,°} \sin 43\,° = 45\text{ N}$$

$$\implies T_1 = \frac{45\text{ N}}{\sin 25\,° + \frac{\cos 25\,°}{\cos 43\,°} \sin 43\,°} = 35.5\text{ N}$$

It is then easy to find T_2.

$$T_2 = T_1 \frac{\cos 25\,°}{\cos 43\,°} = 44.0\text{ N}$$

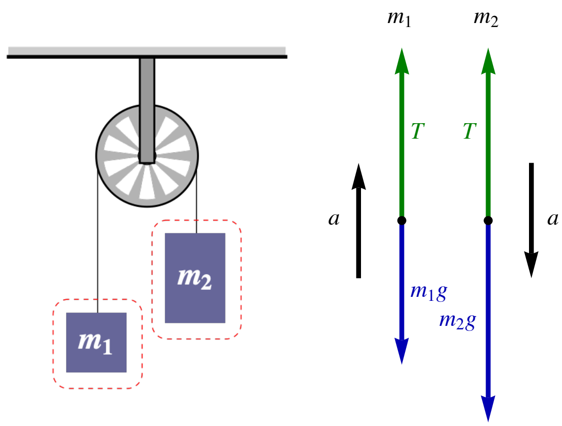
Problems with More than One Mass

Newton's second law applies to every mass in the universe. When considering a mechanics problem with multiple masses we follow the procedure outlined above for each mass; there is a separate free-body diagram for each mass.

Atwood's Machine

An ideal pulley is frictionless and light. The approximation of a pulley being frictionless is often appropriate. For a pulley to be light its mass must be small compared to the other masses in the problem. With an ideal pulley the tension is the same on both sides of the pulley.

Atwood's machine is a standard example. It consists of two masses m_1 and m_2 connected by a string over an ideal pulley. Here we take $m_1 < m_2$. It is clear that m_1 will accelerate upward and m_2 will accelerate downward. We make the assumption that the string is ideal; this means that it has negligible mass and does not stretch. In this case, the ideal string assumption implies that whatever distance $m_1$moves upward, m_2 moves downward. Since the acceleration is the second derivative of the position we can then conclude that two accelerations are equal in magnitude.



We now apply the second law to each mass. It is usually easiest to choose the direction of the acceleration to be the positive direction. Doing that here we will choose up to be positive for m_1 and down for m_2. Using this convention we get:

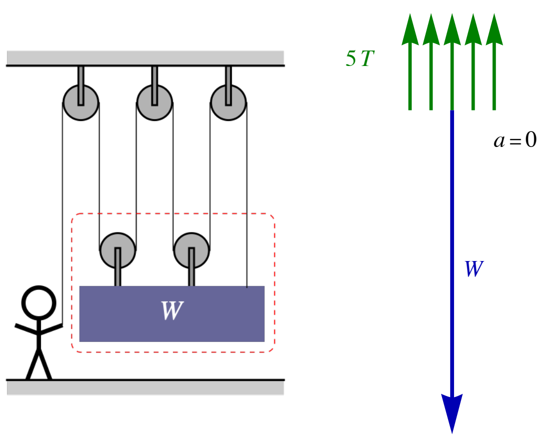
$$F_{\text{net},1} = m_1\, a_1 \implies T - m_1\, g = m_1\, a$$

$$F_{\text{net},2} = m_2\, a_2 \implies m_2\, g - T = m_2\, a$$

In these two expressions the tension and acceleration are the unknowns. The easiest method for solving for the acceleration is to add the two expressions, eliminating the tension. This gives:

$$a = \frac{m_2 - m_1}{m_1 + m_2}\, g.$$

A System of Pulleys



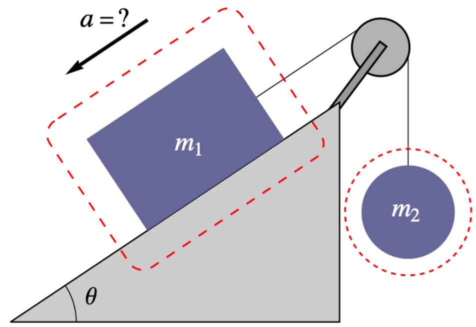
To find the net force lifting on the weight look at all the ropes pulling upward on the object. As a rule when pulleys are attached to a mass always consider the pulleys as part of the mass as shown. There are five ropes leaving the object. Assuming all pulleys are ideal, we get that there is a total force of five T pulling upward.

$$F_{\text{net}} = m\, a \implies 5\,T - W = 0 \implies T = \frac{W}{5}$$

This is an example of a mechanical advantage. With this pulley arrangement a heavy weight can be lifted with a force that is 1/5 the weight of the object. One is not getting something for nothing here. The assumption that the rope doesn't stretch implies that lifting the weight a certain distance involves pulling five times the distance on the rope. This is generally the case with a mechanical advantage and is the basis of simple machines. One can apply a smaller force but it must act over a larger distance.

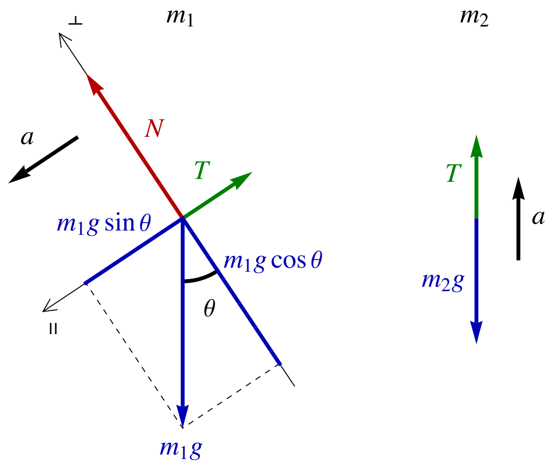
Example E.3 - A More Complex Arrangement

Consider the arrangement shown below. Assuming the incline is frictionless and the pulleys are ideal, what is the acceleration of m_1 down the incline?



Solution

The assumption that the string does not stretch allows us to relate the accelerations. Whatever distance m_1 slides down the incline, m_2 moves upward by that same amount. It follows that the accelerations of m_1 and m_2 are the same magnitude.



Since there is no friction and we don't need the normal force we can avoid the resolution of forces on m_1 perpendicular to the surface. The second law applied to the parallel direction gives:

$$F_{\text{net},\,1,\,\parallel} = m\,a_{1,\,\parallel} \implies m_1\,g\,\sin\theta - T = m_1\,a$$

Applying the second law to m_2 gives:

$$F_{\text{net},2} = m\,a_2 \implies T - m_2\,g = m_2\,a$$

The simplest way to find a is to eliminating T by adding the two expressions above.

$$a = \frac{m_1\,\sin\theta\ -\ m_2}{m_1\ +\ m_2}\,g$$