Chapter O

Fluid Statics

Blinn College - Physics 1401 - Terry Honan

O.1 - Fluids and Density

There are three primary states of matter: solid, liquid and gas. Other more exotic states can occur under extreme temperatures and pressures; the most common of these is a plasma. We have intuition for these three states: A solid has a shape and a volume, although it can be deformed into a different shape. Liquids have a volume but no specific shape. A gas has no shape and will expand to fill any volume. Our intuition for solid, liquids and gases can be deceiving. For instance, at high pressures the distinction between liquids and gases often disappears. Also, things that are obviously solids to us, like glasses and plastics, can actually be very very slow-flowing liquids.

In this chapter we are studying fluids. A fluid is defined as a liquid or a gas. Lumping together these two distinct things may seem awkward at first but we will see that they share many common properties. The most important of these properties is pressure.

All matter can be ascribed a density, where density ρ is defined as a mass per unit volume.

$$\rho = \frac{m}{V} = \frac{\text{mass}}{\text{volume}}$$

Mass values in the metric system were defined based on properties of water; a cubic centimeter, which is the same as a milliliter, of water has one gram g of mass. A liter L of water, which is the same as a cubic decimeter, then has a kilogram of mass and cubic meter has a thousand kilograms of mass.

$$\rho_{\text{water}} = 1\ \frac{\text{g}}{\text{cm}^3} = 1\ \frac{\text{kg}}{\text{L}} = 1000\ \frac{\text{kg}}{\text{m}^3}$$

Water is a very useful reference density, since if something floats it is less dense than water and if it sinks it is more dense than water. The specific gravity is defined as the ratio of the density of a substance to the density of water; this is the same as the numerical value of density in units of g/cm^3.

Densities of Solids		
Substance	$\rho\ (\text{kg}/\text{m}^3)$	$\rho\ (\text{g}/\text{cm}^3)$
Gold	19 320	19.32
Lead	11 300	11.3
Silver	10 490	10.49
Iron or Steel	7800	7.8
Aluminum	2700	2.7
Granite	2700	2.7
Ice	917	0.917
Wood	300 – 900	0.3 – 0.9
Cork	270	0.27

Densities of Fluids		
Substance	$\rho\ (\text{kg}/\text{m}^3)$	$\rho\ (\text{g}/\text{cm}^3)$
Mercury	13 600	13.6
Glycerin	1260	1.26
Sea water	1025	1.025
Water	1000	1.000
Olive oil	920	0.92
Carbon dioxide	1.908	0.00198
Air	1.29	0.00129
Helium	0.18	0.00018
Hydrogen	0.090	0.000090

Densities of solids (left) and fluids (right) (reference)

Example O.1 - What is the Average Density of the Earth

The mass M and radius R of the earth are

$$M = 5.97\times 10^{24}\ \text{kg} \ \text{ and } \ R = 6.38\times 10^{6}\ \text{m}$$

Solution

Using the formula for the volume of a sphere we can get the result.

$$V = \frac{4}{3}\pi R^3 = 1.0878\times 10^{21}\ \text{m}^3 \implies \rho = \frac{M}{V} = 5490\ \frac{\text{kg}}{\text{m}^3}$$

O.2 - Pressure in a Fluid

Pascal's Principle

Pressure P is defined as force per unit area,

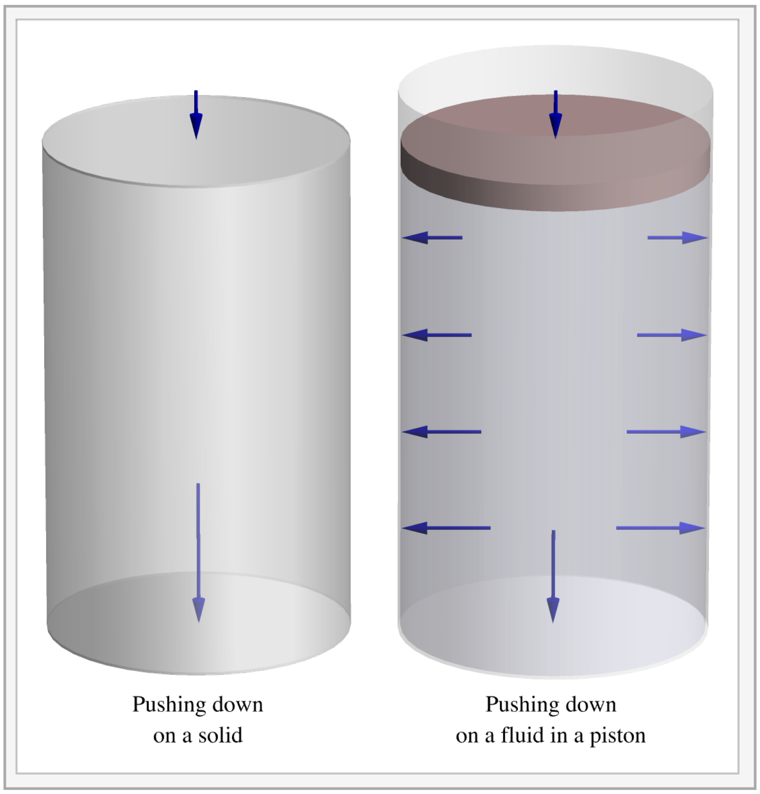
$$P = \frac{F}{A} = \frac{\text{Force}}{\text{Area}}$$

In the SI system, we measure pressure in Pa = pascal.

Units: The SI unit for pressure is: $\text{Pa} = \text{N}/\text{m}^2$

Suppose you have a vertical cylindrical column of some solid. Pushing downward at the top of the column will increase the force per area on a horizontal surface inside the column, but it will not affect the force per area on any vertical surfaces. A downward force on the column will not produce any horizontal outward forces.

In the case of a fluid, a liquid or a gas, pushing downward on the top of a vertical cylindrical piston, the pressure (force/area) is the same for all orientations of a surface; pushing downward on a piston will also increase the outward force of the fluid in all directions.



This piston example illustrates Pascal's principle:

> Increasing the pressure by a fixed amount at one position of a closed container with a static fluid, increases the pressure everywhere in the fluid by the same amount.

Pressure pushes in all directions equally. The force on a flat surface of area A inside a surface is has the same magnitude of $F = PA$, independent of the orientation of the surface.

Atmospheric Pressure and Gauge Pressure

Atmospheric pressure varies. The current pressure a topic in discussion of the current weather; typically high pressure corresponds to good weather and low pressure to bad weather. When a standard value of atmospheric pressure is given, it is an average value; we will call this 1 atm.

$$1\ \text{atm} = 1.013 \times 10^5\ \text{Pa} = 14.7\ \text{lb}/\text{in}^2$$

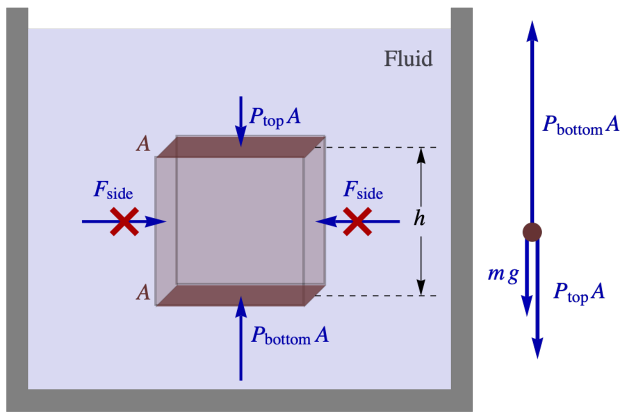
The listed value in pounds per square–inch or psi = $\mathrm{lb}/\mathrm{in}^2$ is familiar to most. Forces due to atmospheric pressure are huge. Why do we not notice them? A pressure of 1 atm acting on the large area of a wall gives a huge force but there is the same huge force acting on the other side of the wall to cancel it. If all the air in an adjacent room were evacuated making it a vacuum, then essentially any wall would collapse under those huge forces. It is pressure differences that cause forces.

Suppose a car's tire pressure is badly depleted and a pressure gauge reads 13 psi = 13 $\mathrm{lb}/\mathrm{in}^2$. Does that mean the pressure in the tire is less than outside? Clearly this is not the case; air will flow out of the tire. What the gauge reads is called the gauge pressure, which is the difference between the absolute pressure and atmospheric pressure. So in the tire example the pressure in the tire is larger than 1 atm by 13 psi. An absolute pressure of zero is a perfect vacuum. A gauge pressure of zero means the pressure is 1 atm.

$$\text{gauge pressure} = P - 1\ \text{atm}$$

Variation of Pressure with Depth

It is a well-known fact that as one goes deeper under water, the pressure increases. This is a general feature of fluids and it is caused by the added weight of the fluid above it.



Isolate a rectangular block of the fluid, shown in a pale red. The net force of the fluid sitting in the same fluid must be zero. Pressure increases with depth because of the additional weight of fluid above.

Consider a fluid and isolate a rectangular block of the same fluid, shown in red above. The rectangular block has an area A at the top and the bottom and a height h. Take the pressures at the top and bottom of the block to be P_{top} and P_{bottom}, respectively, and the forces at the top and bottom are $P_{\text{top}}\,A$ and $P_{\text{bottom}}\,A$. The net force on a rectangular block of fluid sitting in the same fluid must be zero. Pressure exerts forces on the sides of the rectangular block, but those forces will cancel. Since this is a case of static equilibrium, the net force must vanish. Equating the total force upward $P_{\text{bottom}}\,A$ with the total downward force $P_{\text{top}}\,A + W$ gives:

$$P_{\text{bottom}}\,A = P_{\text{top}}\,A + W$$

where W is the weight of fluid in the block. The mass in the block is $m = \rho V$ and the volume is $V = Ah$.

$$W = mg = \rho V g = \rho A h g$$

Combining these two expressions and canceling the area A gives the expression for variation of pressure with depth.

$$P_{\text{bottom}} = P_{\text{top}} + \rho g h$$

For water open to the air, the pressure at the top is 1 atm. At what depth h_0 does the pressure increase by another 1 atm.

$$1\ \text{atm} = \rho_{\text{water}}\, g\, h_0$$

Using the density of water we can find h_0.

$$1\ \text{atm} = 1.013\times 10^5\ \text{Pa},\ \ \rho_{\text{water}} = 1000\ \frac{\text{kg}}{\text{m}^3}\ \ \text{and}\ \ g = 9.8\ \frac{\text{m}}{\text{s}^2}\ \Longrightarrow\ h_{0,\text{water}} = \frac{1\ \text{atm}}{\rho_{\text{water}}\, g} = 10.3\ \text{m}$$

With water then, every additional h_0 = 10.3 m of depth gives another atmosphere of pressure. So at a depth of h_0 the pressure is 2 atm, at 2 h_0 it is 3 atm, etc.

Example O.2 - Water Pressure at the Titanic Wreck

The wreck of the titanic is about 3800 m below the surface. Taking the density of sea water to be 1027 kg/m^3 what is the pressure at the wreck? Give the answer in both Pa and atm.

Solution

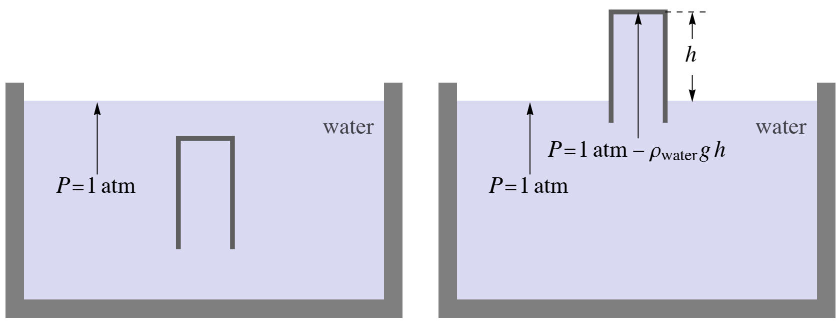
This is a straight-forward application of the formula for variation of pressure with depth. Take the top position to be at the surface P_{top} = 1 atm, and the $P_{bottom} = P$

$$\rho = 1027\ \frac{\text{kg}}{\text{m}^3}\ ,\ h = 3800\ \text{m}\ \text{ and }\ P_{top} = 1\ \text{atm} = 1.013\times 10^5\ \text{Pa}$$

$$P = P_{bottom} = P_{top} + \rho g h = 3.83\times 10^7\ \text{Pa} = 379\ \text{atm}$$

Torricelli Barometer

Imagine a sink full of water and a glass in the sink. With the glass full of water invert the cup and lift it with the lip still below the surface. The pressure at the surface is 1 atm, so the pressure at the top of the inverted glass is less by $\rho g h$, where h is the height of the glass.

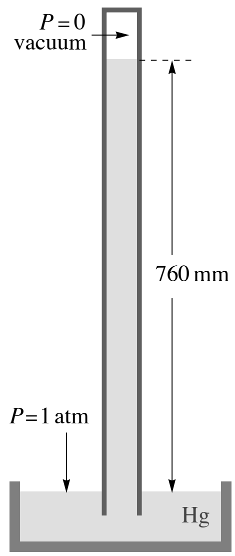


What would happen if the glass were taller than h_0 = 10.3 m? The absolute pressure cannot be negative so the maximum height of the column of water is h_0; above that would be a vacuum. This could be made into a barometer, a device to measure atmospheric pressure. If a column of water is taller than h_0 = 10.3 m, then watching the small changes in the height is measuring the atmospheric pressure. Because it is so tall, it would not be a practical device. Using a more dense fluid would improve this.

A much more dense fluid is mercury, with a density of $\rho_{mercury} = 13\,600\ \text{kg}/\text{m}^3$. This is the basis of the barometer invented by Torricelli.

$$\rho_{mercury} = 13\,600\ \frac{\text{kg}}{\text{m}^3}\ \Longrightarrow\ h_{0,mercury} = \frac{1\ \text{atm}}{\rho_{mercury}\, g} = 0.760\ \text{m} = 760\ \text{mm}$$

Pressure is sometimes measured in mmHg or millimeters of mercury with a standard value of 760 mmHg for atmospheric pressure.



The Torricelli Barometer

Example O.3 - Blaise Pascal's Very French Barometer

Blaise Pascal was a mathematician, physicist, philosopher and theologian. He did important early experiments using Torricelli's barometer; as an example, he took a Torricelli barometer up a mountain and showed that pressure decreases with altitude. A much more whimsical experiment involved building a barometer using Bordeaux wine instead of mercury. If the density of the wine was $984\,\mathrm{kg}/\mathrm{m}^3$, then what was the height of his barometer column?

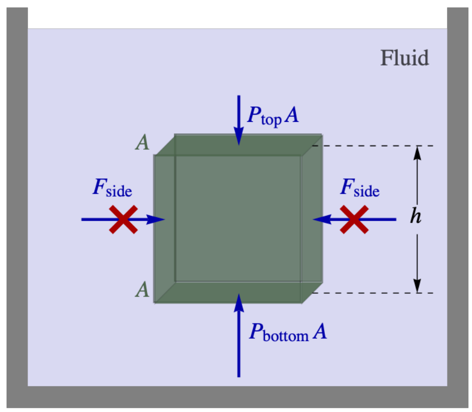
Solution

$$1\text{ atm} = 1.013\times 10^5\text{ Pa} = \rho_{\text{wine}}\, g\, h_{0,\text{wine}} \implies \rho_{\text{wine}} = 984\,\frac{\text{kg}}{\text{m}^3} \implies h_{0,\text{wine}} = \frac{1\text{ atm}}{\rho_{\text{wine}}\, g} = 10.5\text{ m}$$

O.3 - Buoyancy and Archimedes' Principle

Archimedes (287 - 212 BC) was a remarkable polymath and genius. Much of his work was lost to history, but some somewhat recently discovered documents illustrate how advanced he was for the time; he had anticipated and used much of what later became calculus almost two millenia before its invention. The common story of the discovery of Archimedes' principle is perhaps apocryphal but is too good to not repeat. While taking a bath, he discovered the principle of buoyancy. Thrilled by this discovery, he ran into the street naked screaming “Eureka”.

The buoyant force B is the net upward force on a floating or submerged object caused by the variation of pressure with depth. To see there is a net upward force consider the submerged block in the diagram below.



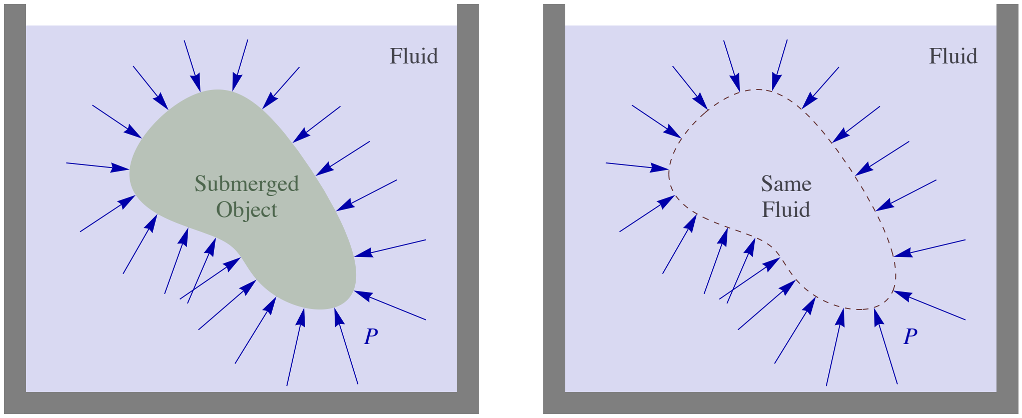
The buoyant force B on rectangular block is caused by the variation of pressure with depth, since the force pushing up on the bottom is larger than the force pushing down at the top.

Since the forces on the sides of the block must cancel, the net upward force, the buoyant force B, becomes $B = P_{\text{bottom}}\, A - P_{\text{top}}\, A$. Using the variation of pressure with depth we get $P_{\text{bottom}} - P_{\text{top}} = \rho_{\text{fluid}}\, g\, h$. Since the volume of the block is $V = A\, h$. Combining these results gives an expression for the buoyant force F_b.

$$B = \rho_{\text{fluid}}\, V\, g$$

This is Archimedes principle: the buoyant force is the weight of the displaced fluid.

The derivation above illustrates that the variation of pressure with depth, but this mathematical result could not motivate anyone to run down the street naked screaming “Eureka”. A more general derivation involves replacing the submerged object with the fluid. The force of water pressure pushing up on the submerged object is the buoyant force. Now replace the submerged object with an imaginary surface, shown dashed in the diagram below to the right, that contains the same fluid. The force on the fluid sitting in the same fluid bust be zero, so the buoyant force must equal the weight of fluid that occupies that volume. Thus, the buoyant force is the weight of the displaced fluid.



Example O.4 - Just the Tip of the Iceberg

What fraction of an iceberg's total volume is above the water? Take the density of seawater to be $1027\,\text{kg}/\text{m}^3$ and the density of ice to be the tabulated value given earlier, $\rho_{\text{ice}} = 917\,\text{kg}/\text{m}^3$.

Solution

We are given the density of ice, the density of sea water and we will need g.

$$\rho_{\text{ice}} = 917\,\text{kg}/\text{m}^3 \;\text{ and }\; \rho_{\text{sw}} = 1027\,\text{kg}/\text{m}^3.$$

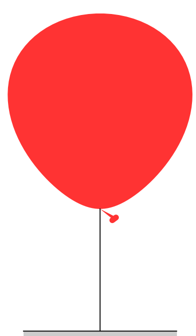
Archimedes principle says that the total weight of the ice W_{ice} with a total volume V_{tot} must equal the weight of the displaced seawater, W_{disp}, with a volume V_{disp}.

$$W_{\text{ice}} = \rho_{\text{ice}}\, V_{\text{tot}}\, g \;\text{ and }\; W_{\text{disp}} = \rho_{\text{sw}}\, V_{\text{disp}}\, g \implies W_{\text{ice}} = W_{\text{disp}} \implies \rho_{\text{ice}}\, V_{\text{tot}}\, g = \rho_{\text{sw}}\, V_{\text{disp}}\, g$$

From this we can find the fraction of the ice below the water and then the fraction above.

$$\begin{pmatrix}\text{fraction}\\ \text{below}\end{pmatrix} = \frac{V_{\text{disp}}}{V_{\text{tot}}} = \frac{\rho_{\text{ice}}}{\rho_{\text{sw}}} = 0.893 \implies \begin{pmatrix}\text{fraction}\\ \text{above}\end{pmatrix} = \frac{V_{\text{tot}} - V_{\text{disp}}}{V_{\text{tot}}} = 1 - \frac{V_{\text{disp}}}{V_{\text{tot}}} = 1 - \frac{\rho_{\text{ice}}}{\rho_{\text{sw}}} = 0.107$$

Example O.5 - Lighter than Air



A helium balloon has a volume of $12\,\text{L} = 0.012\,\text{m}^3$ of helium gas with a density of $0.123\,\text{kg}/\text{m}^3$. The rubber of the balloon has a mass of $1.4\,\text{g}$. The balloon is held from below by a string. What is the tension in the string? Take the density of air to be $1.20\,\text{kg}/\text{m}^3$.

Solution

We are given volume V, the mass of the rubber m_{rubber}, the density of helium ρ_{helium}, the density of air ρ_{air} and we will also need g.

$$V = 0.012\,\text{m}^3,\; m_{\text{rubber}} = 1.4\,\text{g} = 1.4\times 10^{-3}\,\text{kg},\; \rho_{\text{helium}} = 0.123\,\text{kg}/\text{m}^3,$$
$$\rho_{\text{air}} = 1.20\,\text{kg}/\text{m}^3 \;\text{ and }\; g = 9.8\,\text{m}/\text{s}^2$$

The total mass of the balloon is the sum of the mass of the balloon's rubber and the helium in it. Multiplying by g gives the balloon's total weight.

$$m_{\text{tot}} = m_{\text{rubber}} + \rho_{\text{helium}}\, V = 0.003956\ \text{kg} \implies W = m_{\text{tot}}\, g = 0.03877\ \text{N}$$

The buoyant force B is the weight of the displaced air.

$$B = \rho_{\text{air}}\, g\, V = 0.14112\ \text{N}$$

The free-body diagram for the balloon consists of the upward buoyant force and both the tension and weight acting downward. The tension then becomes

$$T = B - W = 0.102\ \text{N}$$