Chapter A

Electric Charges, Forces and Fields

Blinn College - Physics 1402 - Terry Honan

A.1 - Electric Charge

Properties of Charge

There are two types of electric charge. Like charges repel and unlike attract.

We can label the types with signs. The choice of positive and negative was originally arbitrary, but our current convention was set a long time ago.

Electric charge is conserved.

The other fundamentally conserved quantities are: energy, linear momentum and angular momentum; these were discussed in the first semester mechanics course. A conserved quantity cannot be created or destroyed. To charge a body one can add charges to it or remove charges from it.

Electric charge is quantized.

Any charge is an integer multiple of the fundamental charge e. The unit of charge is C = Coulombs, which is considered a fundamental unit in the SI system.

$$Q = n\, e, \quad \text{where } e = 1.602 \times 10^{-19}\ \text{C}$$

The definition of the Coulomb will be given later, when we discuss the magnetic force between two conductors. Take the value of the fundamental charge e as our tentative definition.

Normal matter consists of protons, neutrons and electrons with charges shown here.

Particle	Charge
proton	$+e$
neutron	0
electron	$-e$

It is clear that any combination of these will satisfy the charge quantization rule $Q = n\, e$, where $n = 0, \pm 1, \pm 2, \pm 3, \ldots$

> **Units:** The SI unit for charge is: C

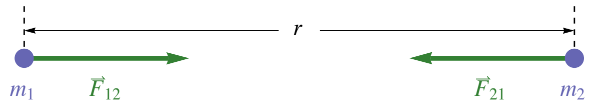
A.2 - Coulomb’s Law

The first understood of the fundamental forces was gravity, which was described by Newton with his law of universal gravitation. Coulomb successfully described the electrostatic force between charges by analogy to gravity.

Review of Gravity

Newton's law of gravity is an inverse square law between point masses. If m_1 and m_2 are point masses separated by distance r the magnitude of the force between them is

$$F = G\,\frac{m_1\, m_2}{r^2}.$$



The force on one mass due to another is toward the other. These forces satisfy Newton's third law and break up into equal and opposite pairs.

Coulomb's Law

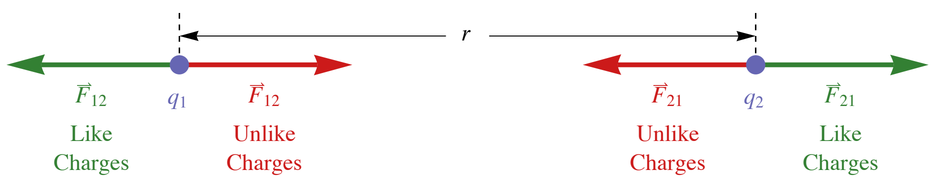
Coulomb found the force law for electrostatics by analogy to gravity. Mass is the gravitational analog of charge. There are two key differences between the electric and gravitational cases. Electric charge can be positive or negative but mass is always positive. The force between two masses is attractive but the force between like charges is repulsive. The electric force is an inverse square law between point charges. The magnitude of the force is

$$F = k_e\,\frac{|q_1|\,|q_2|}{r^2},$$

where the absolute values guarantee a positive result. The constant k_e is a universal constant, like Newton's gravitational constant G. It is related to another constant ε_0, which is usually taken as more fundamental.

$$\varepsilon_0 = 8.85\times10^{-12}\,\frac{\mathrm{C}^2}{\mathrm{N}\cdot\mathrm{m}^2}$$

$$k_e \equiv \frac{1}{4\,\pi\,\varepsilon_0} = 8.99\times10^{9}\,\frac{\mathrm{N}\cdot\mathrm{m}^2}{\mathrm{C}^2}$$



The force on one charge due to another is toward or away from the other charge. These forces satisfy Newton's third law and break up into equal and opposite pairs.

Example A.1 - Gravitational versus Electrical Attraction for a Hydrogen Atom

In addition to the attractive electric force between a proton and an electron there is also gravitational attraction as well. Is it important to consider gravity when studying the physics of the hydrogen atom?

What is the ratio of the gravitational to electric attraction between a proton and an electron?

Solution

The values of the relevant constants are :

$$\begin{aligned} k_e &= 8.99\times10^{9}\ \mathrm{N}\cdot\mathrm{m}^2/\mathrm{C}^2 \\ G &= 6.673\times10^{-11}\ \mathrm{N}\cdot\mathrm{m}^2/\mathrm{kg}^2 \\ e &= 1.602\times10^{-19}\ \mathrm{C} \\ m_{\text{electron}} &= 9.11\times10^{-31}\ \mathrm{kg} \\ m_{\text{proton}} &= 1.673\times10^{-27}\ \mathrm{kg} \end{aligned}$$

The electric and gravitational forces have magnitudes :

$$F_{\text{elec}} = k_e\,\frac{|Q_1|\cdot|Q_2|}{r^2} = k_e\,e^2/r^2$$

$$F_{\text{grav}} = G\,\frac{m_1\,m_2}{r^2} = G\,m_{\text{electron}}\,m_{\text{proton}}/r^2$$

We now take the ratio. Note that because both forces vary as inverse square laws, the ratio does not depend on the distance between particles, r.

$$\frac{F_{\text{grav}}}{F_{\text{elec}}} = \frac{G\, m_{\text{electron}}\, m_{\text{proton}} / r^2}{k_e\, e^2 / r^2} = \frac{G\, m_{\text{electron}}\, m_{\text{proton}}}{k_e\, e^2} = 4.41 \times 10^{-40}$$

The above number is dimensionless and is thus independent of units. This small numeric value shows that unless 40 digit accuracy is needed, we can ignore gravity when studying the hydrogen atom. The smallness of this value poses a fundamental question: How does some underlying theory that unifies gravity with the other forces, the strong nuclear force and the electroweak force, give rise to such a small dimensionless number. There is no such unified theory now but it is considered an ultimate goal of physics.

To find the force on a charge due to a distribution of charge one adds the forces due to each charge in the distribution. Force is a vector and this addition is then vector addition.

Example A.2 - Two Point Charges

An 80 μC is at (4 m, 0) and a $-50\,\mu$C is at the origin. What is the force on the 80 μC charge? What is the force on the -50 μC charge?

Solution

Take $q_1 = -50\,\mu\text{C}$ and $q_2 = 80\,\mu\text{C}$. The distance from the origin to (4 m, 0) is $r = 4\,\text{m}$.

$$F = k_e \frac{|q_1|\,|q_2|}{r^2},$$

First calculate the magnitude of the force.

$$\begin{aligned} F &= k_e \frac{|q_1|\,|q_2|}{r^2} \\ &= 9.0 \times 10^9\, \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\, \frac{(80\times10^{-6}\,\text{C})\,(50\times10^{-6}\,\text{C})}{(4\,\text{m})^2} \\ &= 2.25\,\text{N} \end{aligned}$$

Since we have unlike charges, one positive and one negative, then the force is attractive, or toward the other charge; for q_1 that is the positive-x direction

$$\vec{F}_{12} = F\,\hat{x} = 2.25\,\text{N}\,\hat{x}$$

and for q_2 the force is in the negative-x direction.

$$\vec{F}_{21} = -F\,\hat{x} = -2.25\,\text{N}\,\hat{x}$$

A.3 - The Electric Field

The gravitational analog of electric field $\vec{E}$ is the gravitational field $\vec{g}$. To define the gravitational field, we find the force $\vec{F}$ on a test mass m_0 and divide the test mass into it.

$$\vec{g} = \frac{\vec{F}}{m_0}.$$

We define the electric field similarly. Find the force $\vec{F}$ on a test charge q_0 and divide the test charge into it.

$$\vec{E} = \frac{\vec{F}}{q_0}.$$

The direction of the electric field is the direction of the force on a positive test charge; thus, it points toward negative charges and away from positive charges.

Units: The SI unit for Electric Field is: N/C

Field of a Point Charge

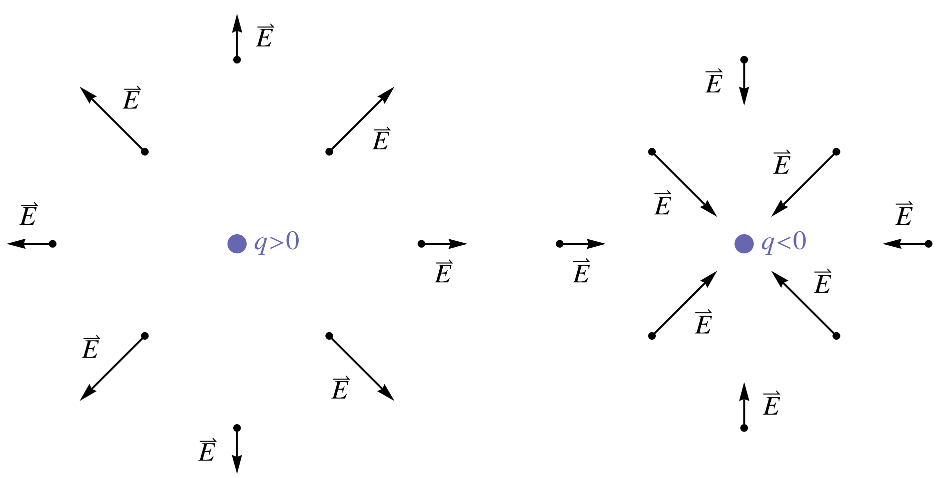
The electric field for a point charge Q can be found using the prescription above. Take the vector $\vec{r}$ to be the vector from Q to some point P. To get $\vec{E}$ due to Q at P, first find the force on a test charge q_0 placed at P using Coulomb's law.

$$F = k_e \frac{|q|\,|q_0|}{r^2}$$

Since $\vec{E} = \vec{F}/q_0$, we can write the magnitude of the electric field in terms of the magnitude of the force: $E = F/|q_0|$

$$E = k_e \frac{|q|}{r^2}$$

Since the direction of $\vec{E}$ is the direction of the force on a positive test charge. If q is positive then it repels a positive q_0 and if q is negative then it attracts a positive q_0. It follows that the direction of $\vec{E}$ is away from positive q and toward a negative q.



The electric field is directed away from positive charges and toward negative.

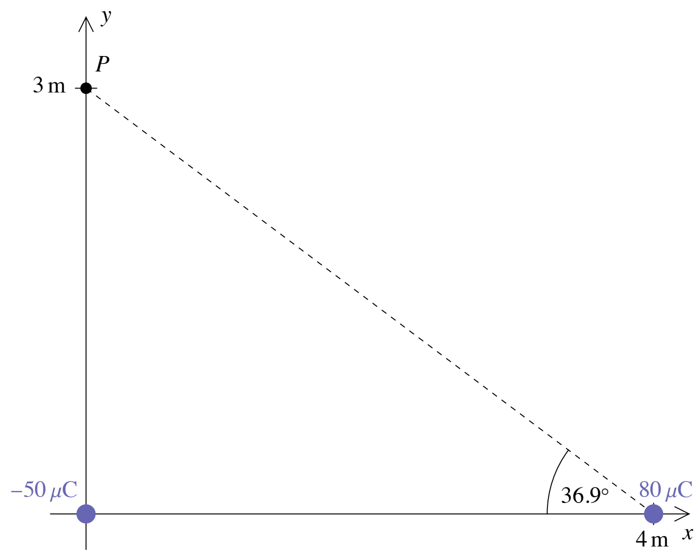
Principle of Superposition

If you have two or more point charges then the total electric field due to all the charges is the sum of the fields due to each charge. The total electric field $\vec{E}$ due to the charges $q_1, q_2, \ldots$ is

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \ldots$$

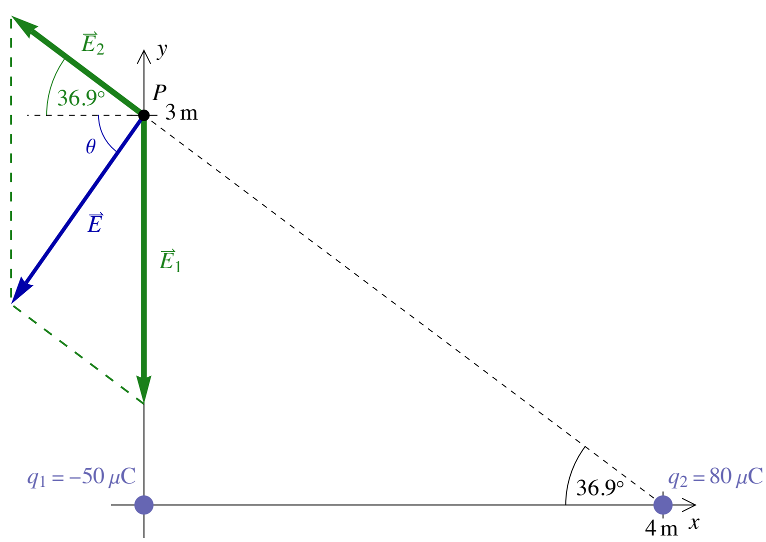
where $\vec{E}_1$ is the field due to q_1, $\vec{E}_2$is the field due to q_2, etc.

Example A.3 - Two Point Charges - Continued



Let us take the previous example a step further. With the same 80 μC at (4 m, 0) and −50 μC at the origin, find the electric field (vector) at the point P, at (0, 3 m). (Note that the angle of 36.9 ° is given to facilitate the calculation.) Also, find the magnitude and direction angle (measured counterclockwise from the x-axis) of the electric field.

Solution



Take $q_1 = -50\,\mu$C and $q_2 = 80\,\mu$C. The distance from q_1 at the origin to P at (3 m, 0) is $r_1 = 3$ m. We can use the Pythagorean theorem to find r_2 the distance from q_2 at (4 m, 0) to P.

$$r_2 = \sqrt{(4\text{ m})^2 + (3\text{ m})^2} = 5\text{ m}$$

Now find the magnitudes of $\vec{E}_1$ and $\vec{E}_2$.

$$E_1 = k_e \frac{|q_1|}{r_1^2} = 9.0\times 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\ \frac{50\times 10^{-6}\text{ C}}{(3\text{ m})^2} = 50\,000\text{ N/C}$$

$$E_2 = k_e \frac{|q_2|}{r_2^2} = 9.0\times 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\ \frac{80\times 10^{-6}\text{ C}}{(5\text{ m})^2} = 28\,800\text{ N/C}$$

From the magnitudes and angles we can find the components of the fields and then the vectors. $\vec{E}_1$ has only a negative y-component

$$E_{1x} = 0 \ \text{ and } \ E_{1y} = -E_1 = -50\,000\ \frac{\text{N}}{\text{C}} \implies \vec{E}_1 = -50\,000\ \frac{\text{N}}{\text{C}}\ \hat{y}$$

For $\vec{E}_2$, its x-component is negative and adjacent to the angle and its y-component is positive and opposite the angle

$$E_{2x} = -E_2 \cos 36.9\,° = -73\,030\ \frac{\text{N}}{\text{C}}\ \text{ and }\ E_{2y} = E_2 \sin 36.9\,° = 17\,290\ \frac{\text{N}}{\text{C}}$$

$$\Longrightarrow\ \vec{E}_2 = -73\,030\ \frac{\text{N}}{\text{C}}\ \hat{x} + 17\,290\ \frac{\text{N}}{\text{C}}\ \hat{y}$$

Adding these together gives the total field.

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = E_x\,\hat{x} + E_y\,\hat{y} = -73\,000\ \frac{\text{N}}{\text{C}}\ \hat{x} - 32\,700\ \frac{\text{N}}{\text{C}}\ \hat{y}$$

For the magnitude of the field

$$E = \sqrt{E_x^2 + E_y^2} = 80\,000\ \frac{\text{N}}{\text{C}}$$

To find the direction angle, we first find the angle shown in the diagram.

$$\theta = \tan^{-1}\frac{E_y}{E_x} = 24.1\,°$$

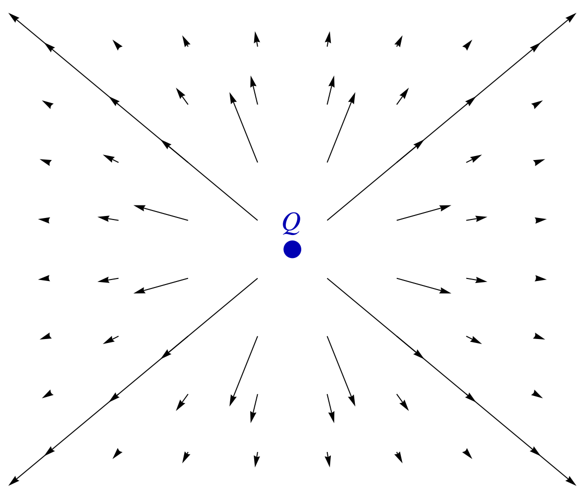
For the angle measured counter-clockwise from the positive-x direction we add 180°.

$$\theta_E = \theta + 180\,° = 204.1\,°$$

A.4 - Field Diagrams and Electric Flux

Field Diagrams

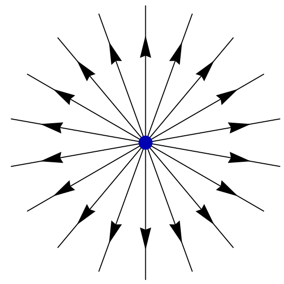
The electric field is a vector field. This means that at each position in space there is a vector. A typical way of representing a general vector field is to draw a grid with a vector at each point in the grid.



Point Charge Field - Vectors on a Grid

Such diagrams, however, can get quite complicated in the case of electric fields; one gets a tangled mess of overlapping arrows. The convention that we use is to draw continuous curves showing only the direction of the field at some position. The field lines begin at positive charges and end at negative charges. We will see that the density of lines is a measure of the strength (magnitude) of the field.

Consider the field of a positive point charge; the field lines point radially away from the charge.



Point Charge Field - Continuous Curves

Different concentric spheres with the charge at the center will have the same number of lines passing through them. The area of a sphere is $A = 4\pi r^2$, so if we take the number of lines per area we get:

$$\frac{\text{\# of lines}}{\text{Area}} \propto \frac{1}{r^2}$$

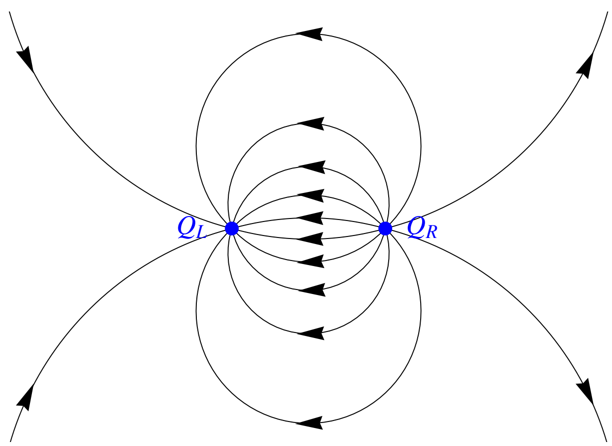
where "$\propto$ " is the proportional symbol. The electric field magnitude is also proportional to $\frac{1}{r^2}$. It follows that

$$\text{mag of field} = E \propto \frac{\text{\# of lines}}{\text{Area}}.$$

This gives the graphical interpretation of the magnitude of the field. When the lines are close together the field is strong and when they are far apart it is weak.

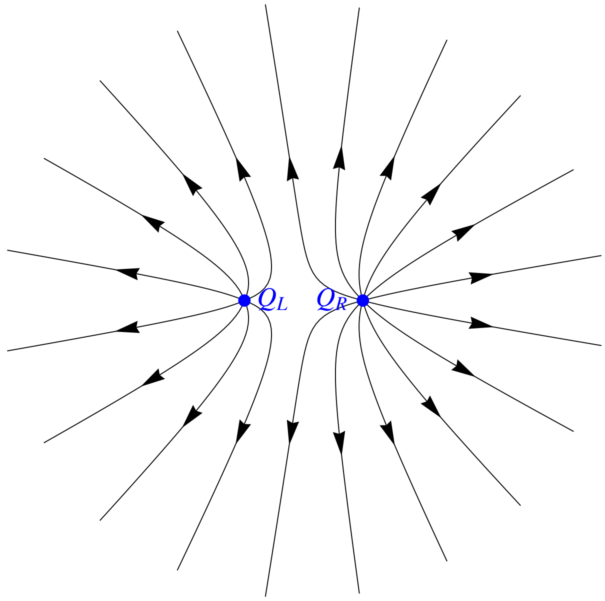
Electric Dipole

$Q_L = -Q$ is the charge on the left and $Q_R = +Q$ is the charge on the right.



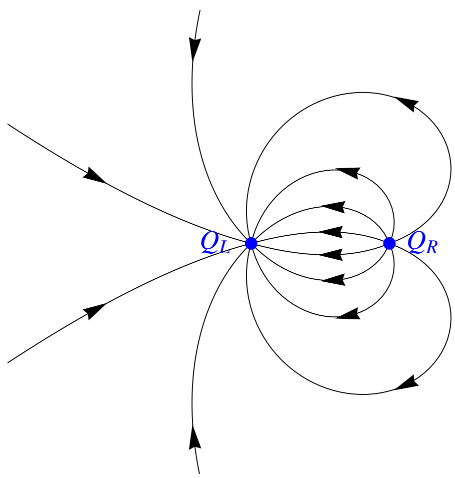
Two Positive Charges

$Q_L > 0$ is the charge on the left and $Q_R > 0$ is the charge on the right. Also $Q_L < Q_R$.



One Positive, One Negative, Net Charge Negative

$Q_L < 0$ is the charge on the left and $Q_R > 0$ is the charge on the right. Also $Q_L + Q_R < 0$.



Electric Flux

Electric flux is a measure of the number of field lines passing through a surface. We will develop a definition of flux gradually, starting with special cases and generalizing.

Flux of a Uniform Field through a Flat Surface

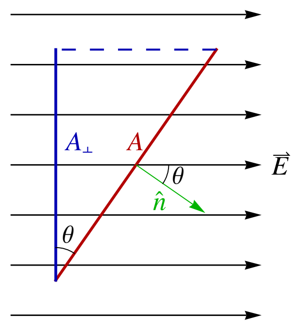
We saw in the discussion of electric field diagrams that the number of field lines per area is a measure of the strength of the field. Strictly, by area we mean $A_\perp$, the part of the area perpendicular to the field.

$$E \propto \frac{\text{\# of lines}}{A_\perp}.$$

We want the flux Φ to be a measure of the number of lines through a surface $\Phi \propto (\text{\# of lines})$ so we can define the flux in the case of a uniform field and a flat surface to be

$$\Phi = E\,A_\perp = E\,A\cos\theta$$

The angle θ is measured between the electric field and the normal (perpendicular) to the surface. The $\hat{n}$ shown in the diagram is the unit normal vector, that is the unit vector perpendicular to the surface.



Putting the $\cos\theta$ with the E gives $E_\perp = E\cos\theta$ and we get

$$\Phi = E_\perp\, A$$

There is a sign ambiguity in the flux. Every surface has two sides or faces; if in the picture above, we chose the opposite unit normal vector $-\hat{n}$ then the angle would change to $180° - \theta$ and the flux Φ would change sign.

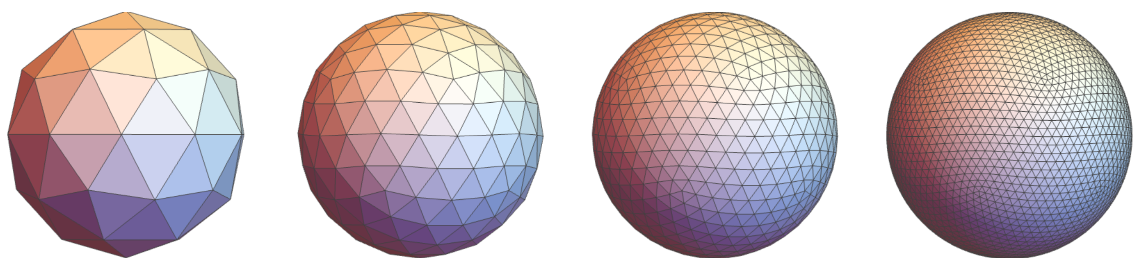
The electric flux through a surface depends only on the component of the field through the surface. If the electric field is parallel to the surface, the flux is zero. Remember that flux is a measure of the number of lines passing through a surface and parallel field lines do not pass through the surface.

Flux in General

The expression for flux must now be generalized to the case of a general field that varies spatially and a surface that is not flat. To do this, break the surface into (infinitely) many small flat pieces. The flux through one small piece is $\Phi = E_\perp\, A$, where $E_\perp$ is the perpendicular component of the field at that small A. The total flux through a surface is found by summing over all these small areas.

$$\Phi = \sum E_\perp\, A$$

Here we are using the typical notation where the symbol "$\sum$" represents a sum. When we say small, we are implying the limit as the areas go to zero. We will never actually evaluate these huge sums, except in simple symmetric cases where the sum becomes trivial.



This shows successively finer grids approximating a sphere. The sum in the definition of flux is over each triangle in a grid.

The sign ambiguity in the flux is still present with this general formula since all surfaces have two sides, but for the important case of closed surfaces we can choose the convention to use the outside surface; this will be important in out discussion of Gauss's Law, where we will consider the flux for closed surfaces only.

Point Charge at the Center of a Sphere

Consider the case of a point charge q at the center of a sphere of radius R. What is the flux through the sphere?

$$\Phi = \sum E_\perp\, A$$

The electric field due to a point charge has magnitude $E = k_e\, |q| / r^2$. When we remove the absolute value from the charge then we will use E_r to denote the radial (or outward) component of the field $E_r = k_e\, q / r^2$; if the q is negative then E_r is negative, implying that it is directed inward.

The field is now perpendicular to the surface of the sphere and along the surface or the sphere we have $r = R$, the radial (outward) component has the constant value

$$r = R = \text{constant} \quad \Longrightarrow \quad E_\perp = E_r = k_e\, \frac{q}{R^2} = \text{constant}$$

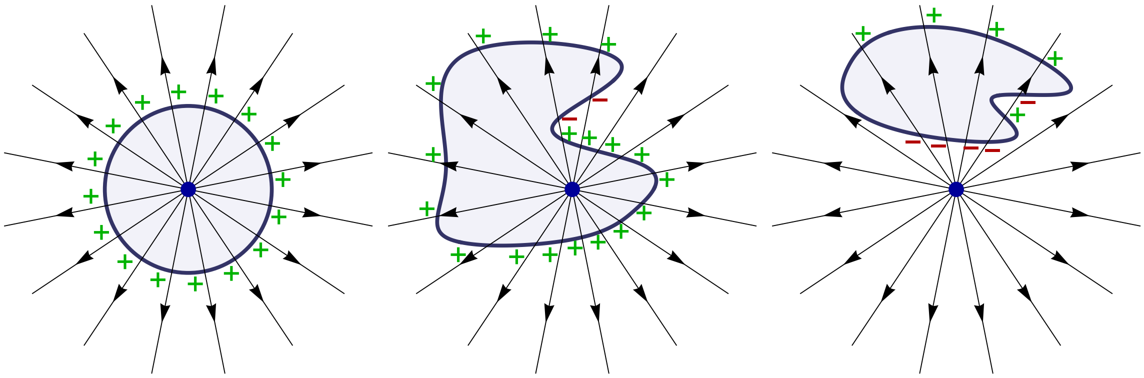
Because it is constant we can take it out of the sum (factoring it out of the sum). Using the total surface area of a sphere of radius R we get the result:

$$\Phi = \sum E_\perp\, A = \sum k_e\, \frac{q}{R^2}\, A = k_e\, \frac{q}{R^2} \sum A = k_e\, \frac{q}{R^2}\, A_{\text{tot}} = k_e\, \frac{q}{R^2}\, 4\pi R^2 = 4\pi k_e\, q = \frac{q}{\varepsilon_0}$$

Note that this is independent of the radius of the sphere. This must be the case; flux is a measure of the number of field lines passing through a surface and any field line that passes through one sphere will pass through another of a different radius.

Point Charge and any Closed Surface

Now suppose that the sphere so that the q was not at the center. Any line that passes through the original sphere would still pass through so the flux must be the same: $\Phi = q/\varepsilon_0$. If we deform the sphere into any curved surface that contains the the charge then the flux is still the same.



If the charge is outside of the sphere then the flux would be zero. When a line enters a surface it is a negative contribution and when it leaves it is a positive contribution so it gives a net flux of zero. Summarizing the flux due a point charge q and any closed surface.

$$\Phi = \begin{cases} q/\varepsilon_0 & \text{when } q \text{ is inside closed surface} \\ 0 & \text{when } q \text{ is outside closed surface} \end{cases}$$

A.5 - Gauss's Law and Calculating Fields

Gauss's Law

Now extend the previous discussion and consider a closed surface and all of the charges in the universe. If a charge is inside the surface it contributes q/ε_0 to the flux and if a charge is outside it contributes zero. If we define q_{enclosed} as the total charge enclosed by the surface the we can write an expression for the flux through any closed surface.

$$\Phi = \frac{1}{\varepsilon_0} q_{\text{enclosed}} \qquad \text{(Gauss's Law)}$$

This expression is called *Gauss's Law*.

The surface we use for Gauss's law is a closed surface. We refer to the closed surface used as a Gaussian surface. When field lines leave the surface that is a positive contribution to the flux and when they leave it is a negative contribution. Field lines only begin at positive charges or infinity and end at negative charges or infinity. What Gauss's law says is that counting the field lines entering or leaving a surface tells you how much charge is enclosed by the surface.

Calculating Fields with Gauss's Law

Gauss's law is useful as a method to calculate electric fields in cases of symmetry. To find the field at some position P we will choose a Gaussian surface that reflects the symmetry and passes through the point P so that the flux can be written as $\Phi = E \times (\text{some area})$. These vague general comments should become clearer as we progress.

Charge Densities

Often instead of specifying a given charge we talk about charge densities. There are three different charge densities to consider.

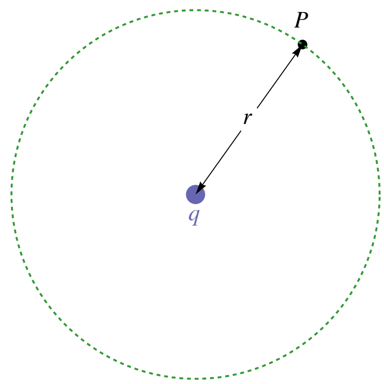
- $\lambda = \frac{\text{charge}}{\text{length}}$ = linear charge density $\Longleftrightarrow$ charge = $\lambda \times$(some length)
- $\sigma = \frac{\text{charge}}{\text{area}}$ = surface charge density $\Longleftrightarrow$ charge = $\sigma \times$(some area)
- $\rho = \frac{\text{charge}}{\text{volume}}$ = volume charge density $\Longleftrightarrow$ charge = $\rho \times$(some volume)

Spherical Symmetry

A distribution of charge is spherically symmetric if it is the same under any rotation about the origin. The simplest example of a spherical distribution is a point charge at the origin. Other examples are a uniform solid sphere or a hollow spherical shell; under any rotation a sphere looks the same. We will first consider the simplest example of a point charge and then find a general expression for all spherical symmetry.

Although we know the electric field for a point charge already we will now recalculate that result using Gauss's law and this will illustrate the use of Gauss's law to find fields. We start with a point charge q at the origin and we want the electric field a distance r from the charge. Take our point P to be at r. Spherical symmetry implies that the electric field only has a radial component E_r and that that radial component must be constant over a sphere of radius r. We choose the Gaussian surface to be a sphere of radius r.

$$\Phi = \sum E_\perp A = \sum E_r A = E_r \sum A = E_r A_{\text{tot}} = E_r 4\pi r^2$$



The Gaussian surface is shown as green and dashed. The only charge inside is q so $q_{\text{enclosed}} = q$.

The charge that is enclosed inside our Gaussian surface (the sphere of radius r) is the only charge q.

$$q_{\text{enclosed}} = q$$

Applying Gauss's law gives the expression for E_r we had previously.

$$\Phi = \frac{1}{\varepsilon_0} q_{\text{enclosed}} \quad\Longrightarrow E_r 4\pi r^2 = \frac{1}{\varepsilon_0} q \quad\Longrightarrow E_r = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = k_e \frac{q}{r^2}$$

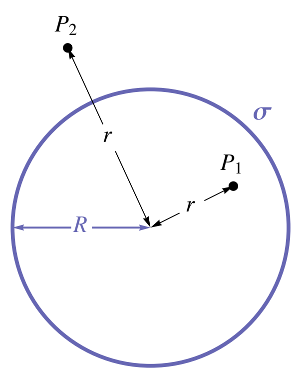
We can now get a general result for the problem of spherical symmetry. The expression for the left hand side of Gauss's law, the expression for flux, depends only on the spherical symmetry so it applies generally. The q_{enclosed} will vary with the problem at hand.

$$\Phi = \frac{1}{\varepsilon_0} q_{\text{enclosed}} \quad\Longrightarrow E_r 4\pi r^2 = \frac{1}{\varepsilon_0} q_{\text{enclosed}} \quad\Longrightarrow E_r = \frac{1}{4\pi\varepsilon_0}\frac{q_{\text{enclosed}}}{r^2} = k_e \frac{q_{\text{enclosed}}}{r^2}$$

In this expression, q_{enclosed} is the total charge enclosed by (or inside) a sphere of radius r.

Example A.4 - Thin-shelled Hollow Sphere

Consider a thin-shelled or radius R with a uniform surface charge density σ. (This is a hollow sphere of negligible thickness.) What is the electric field as a function of r, the distance from the center? (Give answers for the two cases: $r < R$ and $r > R$.)



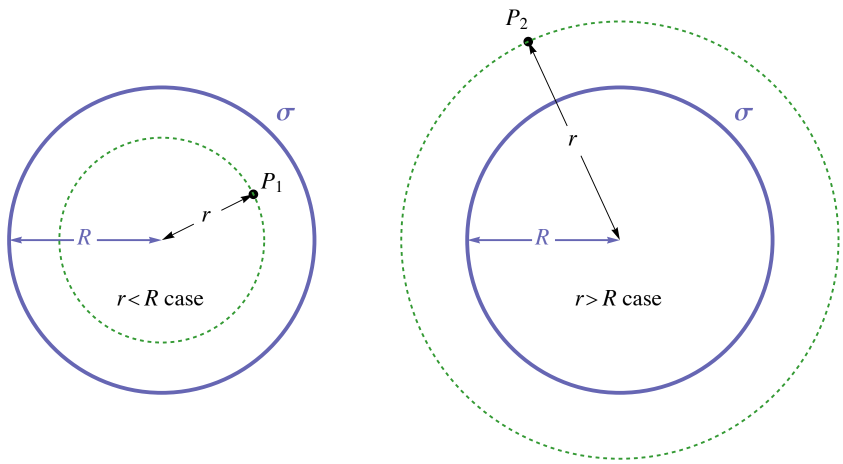
P_1 represents the $r < R$ case and P_2 represents the $r > R$ case

Solution

In the expression we derived above for spherical symmetry

$$E_r = k_e \frac{q_{\text{enclosed}}}{r^2}$$

the only thing we need is to find q_{enclosed} as a function of r. Consider first the case of $r < R$. All of the charge is outside of the Gaussian surface.



The Gaussian surface is shown in green and dashed for each of the two cases, $r < R$ and $r > R$.

For the $r < R$ case there is no charge inside the Gaussian surface so it follows that $q_{\text{enclosed}} = 0$. We can then solve for the field.

$$E_r = 0$$

The $r > R$ case has charge inside the Gaussian surface; all of the charge on the spherical shell is inside the Gaussian surface. We are given the surface charge density and must find the charge from that. As discussed above to get a charge from σ we use: $Q = \sigma \times$(some area) and the relevant area is the surface area of a sphere of radius R, which is $4\,\pi\,R^2$.

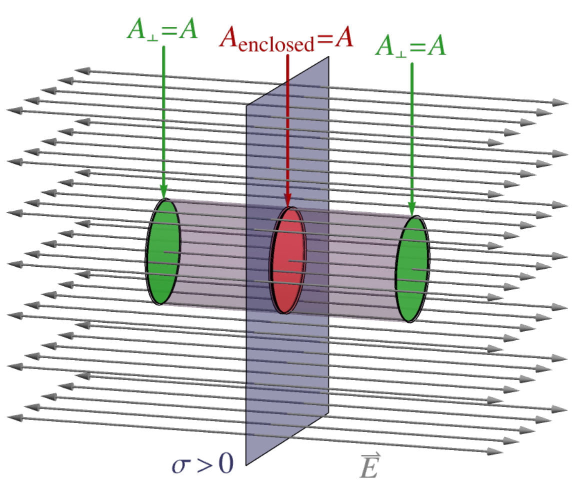
$$q_{\text{enclosed}} = \sigma\, 4\,\pi\,R^2$$

$$E_r = k_e \frac{q_{\text{enclosed}}}{r^2} = k_e \frac{\sigma\, 4\,\pi\,R^2}{r^2}$$

An alternative way of writing this, in terms of ε_0

$$E_r = k_e \frac{\sigma\, 4\,\pi\,R^2}{r^2} = \frac{1}{4\,\pi\,\varepsilon_0} \frac{\sigma\, 4\,\pi\,R^2}{r^2} = \frac{\sigma\,R^2}{\varepsilon_0\, r^2}$$

Planar Symmetry - The Infinite Plane of Charge



Now consider an infinite plane with a uniform surface charge density σ. We choose our Gaussian surface to be a tube with ends as shown above; take the cross-section of the tube to have area A. The electric field is perpendicular to the surface and is directed away from it, taking σ to be positive. The field parallel to the sides of the tube, so the flux there is zero. The only contribution to the flux is at the two ends, labeled as $A_\perp$

and shown in green. At either end the field is perpendicular and the flux is $E A$ so the total flux leaving the Gaussian surface is $\Phi = 2 E A$. To finde q_{enclosed} we need to find how much charge is inside our Gaussian surface. That area of charge is shown in red and labeled A_{enclosed}. The charge is the that multiplied by σ. $q_{\text{enclosed}} = \sigma A_{\text{enclosed}} = \sigma A$. Gauss's law then gives us the field.

$$\Phi = \frac{1}{\varepsilon_0} q_{\text{enclosed}} \implies 2 E A = \frac{1}{\varepsilon_0} \sigma A \implies E = \frac{\sigma}{2 \varepsilon_0}$$

A.6 - Conductors, Insulators, Charging and Electrostatic Attraction

Conductors and Insulators

- Conductors

  Inside a conductor there are freely moving charges. In the most common conductors, metals, the charge carriers are electrons; most of the electrons are tied to their atoms but a small number of electrons from each atom are shared by all atoms in a *sea of electrons*, which are free to move. In semiconductors, the conduction mechanism is different. The charge carriers are either electrons jumping between atomic sites or positively charged holes (A hole is lack of an electron.) jumping between atoms; they are called n-type and p-type semiconductors for negative (electrons) and positive (holes) charge carriers.

- Insulators

  Inside an insulator there are no freely moving charges. All the electrons are tied to their atom or to bonds between atoms. Common examples of insulators are glasses, ceramics, rubbers and plastics.

Conductors in Electrostatics

In the first several chapters we are studying electrostatics where we consider properties of electric charges, forces and fields when nothing is allowed to move. Moving charges are currents and currents are not allowed in electrostatics.

- Inside a conductor $\vec{E} = \vec{0}$.

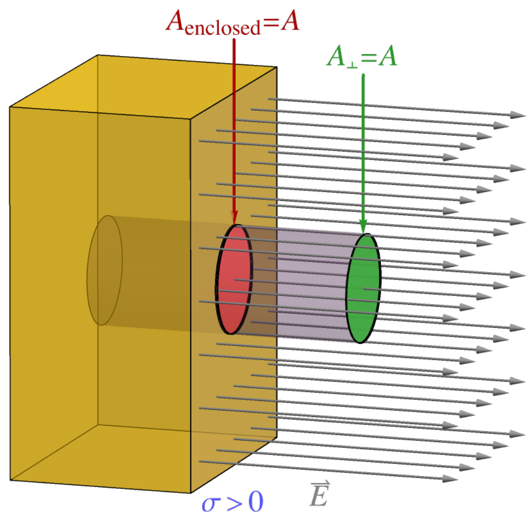
  If there is an electric field inside a conductor, any free charges will move. This means that an electric field inside a conductor implies a current. Since currents are not allowed in electrostatics, it follows that the electric field inside a conductor must be zero.

- There is no excess charge inside a conductor. All excess charge is on the surface of a conductor.

  Consider a Gaussian surface entirely inside a conductor. Since the electric field is zero, Gauss's law implies that $q_{\text{enclosed}} = 0$. This means there is no excess charge in *any* region inside a conductor. There can be excess charge on a conductor, though. All excess charge is on the surface.

- The electric field is perpendicular to the surface of a conductor and it is proportional to the surface charge density, $E = \sigma / \varepsilon_0$.

For the same reason that the field is zero inside a conductor, it must be perpendicular to the surface of a conductor. If there is a component parallel to the surface of a conductor then that will induce surface currents and this violates the assumptions of electrostatics.



The region to the left represents a conductor. All the charge is at the surface of the conductor; the positive surface charge density σ is shown in blue. $A_\perp$ is the part of the surface were there is a flux. The part of the charge inside the conductor is labeled A_{enclosed}; this will be used to find q_{enclosed}.

Similarly to the example with an infinite planar sheet we have planar symmetry, but now all the charge is at the surface of the conductor and there is no field inside the conductor. Using the same Gaussian surface as before we now get that since the field is zero inside the conductor, so the inside end gives zero flux. This leaves a flux of $E\,A$ at the outside end.

If the surface charge density at that position is σ then the charge enclosed by the Gaussian surface is $q_{\text{enclosed}} = \sigma\, A_{\text{enclosed}} = \sigma\, A$.

$$\Phi = \frac{1}{\varepsilon_0}\, q_{\text{enclosed}} \implies E\,A = \frac{1}{\varepsilon_0}\,\sigma\,A \implies E = \frac{\sigma}{\varepsilon_0}$$

The field is perpendicular to the surface of the conductor and is proportional to the surface charge density at that position. Note that if the surface charge density is negative, the field points into the conductor.

Electrostatic Attraction

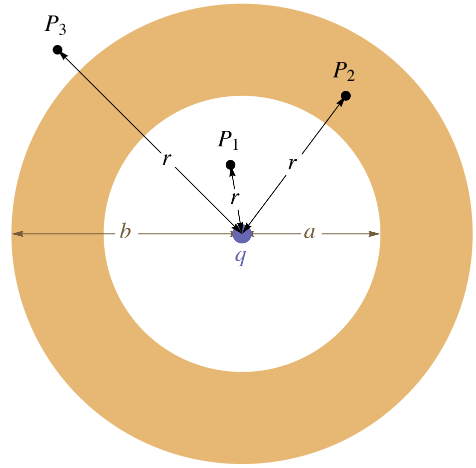
When a charge is brought near a neutral conductor the charge rearranges the charges in the conductor. Consider a positive charge near a conductor. Negative charges in the conductor move near the positive charge outside and the positive charges move to the other side. There is still as much positive as negative charge in the conductor but the positive charges are further. The attractive force between the outside positive charge and the nearer negative charges is then larger than the repulsive force of the positive which are further. There is a net attractive force. We say that the conductor is polarized. Even insulators can become polarized when a charge is brought near, giving a net attractive force.

Charging Objects

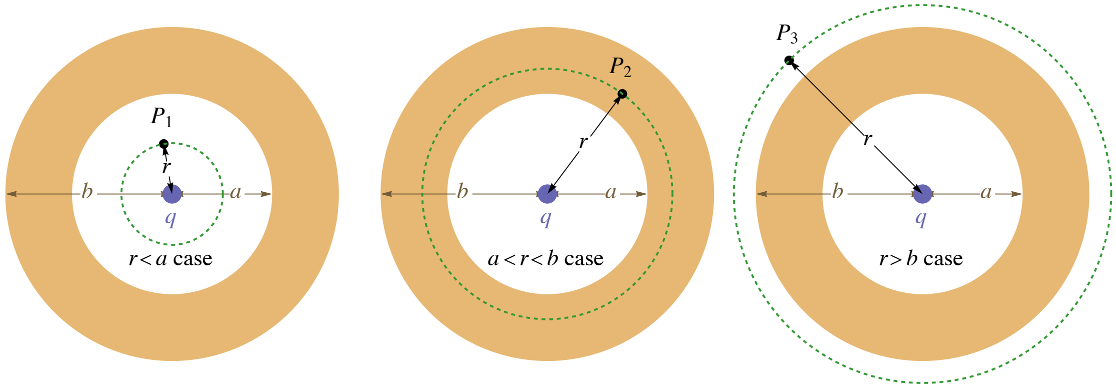
When two materials are rubbed together charge jumps from one to the other. This is how clothes in a dryer get charged. When you rub comb is run through your hair charge jumps from the comb to the hair.

A.7 - Conductors, Shielding and Gauss's Law

We consider another spherical Gauss's law example. Consider a point charge q at the center of a hollow conducting sphere. Take the sphere to have an inside radius a and an outside radius b. We will first consider the case where the conductor is neutral.



To solve for the field as a function of distance from the center we have to consider the three cases, $r < a$, $a < r < b$ and $r > b$.



The Gaussian surface is shown in green and dashed for each of the three cases, $r < a$, $a < r < b$ and $r > b$.

We have established the formula for the (outward) radial component of the electric field in our discussion of Gauss's law with spherical symmetry

$$E_r = k_e \frac{q_{\text{enclosed}}}{r^2}$$

where q_{enclosed} is the charge inside the Gaussian surface.

- $r < a$ case:

  q_{enclosed} is the charge inside the Gaussian surface through P_1. The only charge is $q_{\text{enclosed}} = q$. We can then write down the expression for the radial component of $\vec{E}$.

$$E_r = k_e \frac{q}{r^2}$$

- $a < r < b$ case:

  Between a and b we are now inside a conductor and the electric field is zero, $\vec{E} = \vec{0}$ and the radial component of $\vec{E}$ is also zero.

$$E_r = 0$$

  All of the charge is at the surface of a conductor but this conductor has two surfaces and there must be charge on both. Because of the field vanishing inside a conductor and Gauss's law, it must be true that for the Gaussian surface through P_2 we have $q_{\text{enclosed}} = 0$. This means that at $r = a$, the inside surface of the hollow conductor, the charge must cancel the charge at the center. It follows that at $r = a$ there is a uniformly distributed charge of $-q$.

- $r > b$ case:

  Outside of the conductor we now consider the Gaussian surface through P_3. q_{enclosed} is the charge inside that Gaussian surface and since the conductor is neutral, meaning it has zero net charge, we have only the charge at the center: $q_{\text{enclosed}} = q$ and we get:

$$E_r = k_e \frac{q}{r^2}$$

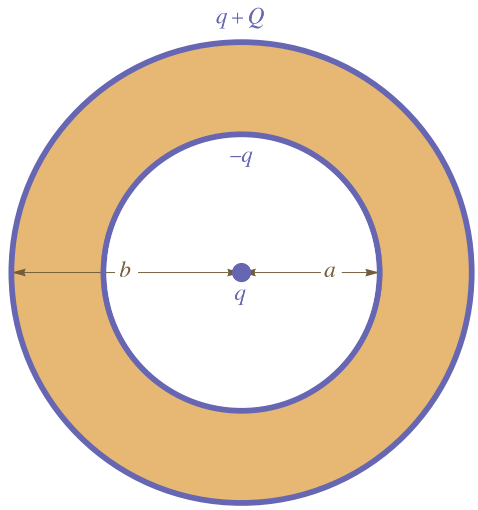
Summarizing, for the case of a point charge inside a neutral conductor, although the net charge on the conductor is zero there is $-q$ on the inside surface (at a) and $+q$ on the outside surface (at b).

Now consider the case where there is a net charge of Q on the conductor. The $r < a$ and $a < r < b$ cases are the same but the outside case is now different.

- $r > b$ case:

  Outside of the conductor we again consider the Gaussian surface through P_3 but now because the conductor has a charge then q_{enclosed} is the total charge inside that Gaussian surface and that is the total charge: $q_{\text{enclosed}} = q + Q$ and we get:

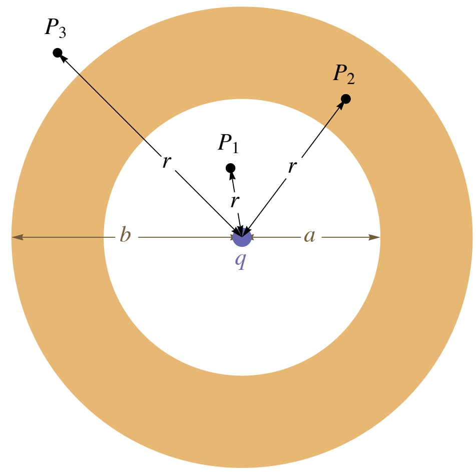
$$E_r = k_e \frac{q+Q}{r^2}$$



Example A.5 - A Point Charge Inside a Hollow Conducting Sphere

Let us return to the previous case and work it with numbers. Take the sphere to have an inside radius of 4 cm and an outside radius of 6 cm. We have now the values

$$a = 4\text{ cm} = 0.04\text{ m} \quad \text{and} \quad b = 6\text{ cm} = 0.06\text{ m}$$



(a) First we look a the case where the point charge at the center is $q = 5\,\mu\text{C}$ and the conductor is neutral. What is E_r, the radial component of $\vec{E}$, at a distances of $r = 3$ cm, $r = 5$ cm and $r = 7$ cm. Also describe the charge distribution by giving both the charge and surface charge density at both surfaces of the conductor, at 4 cm and 6 cm.

Solution

At $r = 3$ cm, we have the $r < a$ case.

$$E_r = k_e \frac{q}{r^2} = 9.0 \times 10^9 \frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}} \frac{5 \times 10^{-6}\,\mathrm{C}}{(0.03\,\mathrm{m})^2} = 5 \times 10^7 \frac{\mathrm{N}}{\mathrm{C}}$$

At $r = 5$ cm, we have the $a < r < b$ case.

$$E_r = 0$$

At $r = 7$ cm, we have the $r > c$ case.

$$E_r = k_e \frac{q}{r^2} = 9.0 \times 10^9 \frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}} \frac{5 \times 10^{-6}\,\mathrm{C}}{(0.07\,\mathrm{m})^2} = 9.18 \times 10^6 \frac{\mathrm{N}}{\mathrm{C}}$$

For the charges and charge densities: At $r = 4$ cm

$$\text{charge} = -q = -5\,\mu\mathrm{C} \quad \text{and} \quad \sigma = \frac{\text{charge}}{\text{area}} = \frac{\text{charge}}{4\,\pi\,r^2} = \frac{-5 \times 10^{-6}\,\mathrm{C}}{4\,\pi\,(0.04\,\mathrm{m})^2} = -2.49 \times 10^{-4} \frac{\mathrm{C}}{\mathrm{m^2}}$$

At $r = 6$ cm

$$\text{charge} = q = 5\,\mu\mathrm{C} \quad \text{and} \quad \sigma = \frac{\text{charge}}{\text{area}} = \frac{\text{charge}}{4\,\pi\,r^2} = \frac{5 \times 10^{-6}\,\mathrm{C}}{4\,\pi\,(0.06\,\mathrm{m})^2} = 1.10 \times 10^{-4} \frac{\mathrm{C}}{\mathrm{m^2}}$$

(b) Now consider the case where the point charge at the center is $q = -6\,\mu$C and the conductor has a net charge of $Q = 4\,\mu$C. What is E_r, the radial component of $\vec{E}$, at a distances of $r = 3$ cm, $r = 5$ cm and $r = 7$ cm. Also describe the charge distribution by giving both the charge and surface charge density at both surfaces of the conductor, at 4 cm and 6 cm.

Solution

At $r = 3$ cm, we have the $r < a$ case.

$$E_r = k_e \frac{q}{r^2} = 9.0 \times 10^9 \frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}} \frac{-6 \times 10^{-6}\,\mathrm{C}}{(0.03\,\mathrm{m})^2} = -6 \times 10^7 \frac{\mathrm{N}}{\mathrm{C}}$$

At $r = 5$ cm, we have the $a < r < b$ case.

$$E_r = 0$$

At $r = 7$ cm, we have the $r > c$ case.

$$E_r = k_e \frac{q + Q}{r^2} = 9.0 \times 10^9 \frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}} \frac{(-6 + 4) \times 10^{-6}\,\mathrm{C}}{(0.07\,\mathrm{m})^2} = -3.67 \times 10^6 \frac{\mathrm{N}}{\mathrm{C}}$$

For the charges and charge densities: At $r = 4$ cm

$$\text{charge} = -q = +6\,\mu\mathrm{C} \quad \text{and} \quad \sigma = \frac{\text{charge}}{\text{area}} = \frac{\text{charge}}{4\,\pi\,r^2} = \frac{6 \times 10^{-6}\,\mathrm{C}}{4\,\pi\,(0.04\,\mathrm{m})^2} = 2.98 \times 10^{-4} \frac{\mathrm{C}}{\mathrm{m^2}}$$

At $r = 6$ cm

$$\text{charge} = q + Q = -2\,\mu\mathrm{C} \quad \text{and} \quad \sigma = \frac{\text{charge}}{\text{area}} = \frac{\text{charge}}{4\,\pi\,r^2} = \frac{-2 \times 10^{-6}\,\mathrm{C}}{4\,\pi\,(0.06\,\mathrm{m})^2} = -4.42 \times 10^{-5} \frac{\mathrm{C}}{\mathrm{m^2}}$$