Chapter B

Electric Potential, Potential Energy and Capacitance

Blinn College - Physics 1402 - Terry Honan

B.1 - Definition of Electric Potential

Review of Potential Energy

Recall from the first semester course that a force is defined to be *conservative* when its work, $W = \sum F_{\shortparallel}\,\Delta s$, is independent of path, meaning that the work is the same for all paths between the same two endpoints.

In mechanics, energy is introduced through the work-energy theorem: $W_{\text{net}} = \Delta K$, where $K = \frac{1}{2}\,m\,v^2$ is defined as the kinetic energy. When a force is conservative we can write its work in terms of the endpoints of the path only. We define potential energy for a conservative force by

$$\Delta U = -W = -\sum F_{\shortparallel}\,\Delta s$$

The reason for the negative sign is to get a term to add to the kinetic energy in the work-energy theorem, moving the term from the work side to the energy side of the equation gives the minus sign.

Examples of conservative forces from the first semester course are gravity, both for a uniform field and for a varying field, and the elastic force of a spring. We will now see that the electrostatic force is conservative; this will allow the definition of electric potential energy.

Basic Definitions

First some terminology: We will define electric potential V as the potential energy per test charge, $V = U/q_0$. The similarity of the letters U and V combined with the similar names, potential versus potential energy, creates a confusion between these related but distinct notions. Voltage is potential difference ΔV; that is, differences in potential. When the sign of ΔV is ignored, we will sometimes use the standard, although somewhat ambiguous, notation where voltage is written as just V, instead of the more precise $|\Delta V|$.

> **Units:** The SI unit for Electric Potential and Voltage is: V = Volt

The diagram shows the underlying relationships between the force, field, potential energy and potential. The potential V, at the lower right, is what we are introducing here.

$$\begin{array}{ccc}
\vec{\boldsymbol{F}} & \longleftarrow \quad \vec{E} = \dfrac{\vec{F}}{q_0} \text{ and } \vec{F} = q\,\vec{E} \quad \longrightarrow & \vec{\boldsymbol{E}} \\
\uparrow & & \uparrow \\
\Delta U = -\sum F_{\shortparallel}\,\Delta s & & \Delta V = -\sum E_{\shortparallel}\,\Delta s \\
\downarrow & & \downarrow \\
\boldsymbol{U} & \longleftarrow \quad V = \dfrac{U}{q_0} \text{ and } \Delta U = q\,\Delta V \quad \longrightarrow & \boldsymbol{V}
\end{array}$$

Recall that we define the electric field by considering the force on a test charge and by dividing the charge into the force, $\vec{E} = \vec{F}/q_0$. The potential is defined from the potential energy similarly. The potential is defined as the potential energy per test charge.

$$V = \frac{U}{q_0}$$

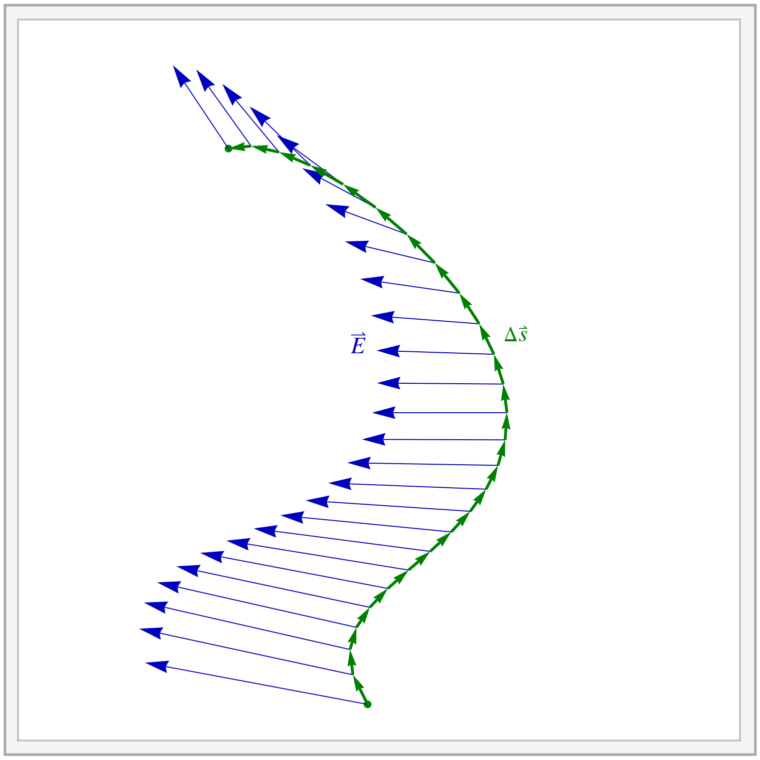
The zero of potential energy is arbitrary and so is the zero of potential. When a charge q is moved across a potential difference ΔV we get a change in potential energy given by

$$\Delta U = q\,\Delta V.$$

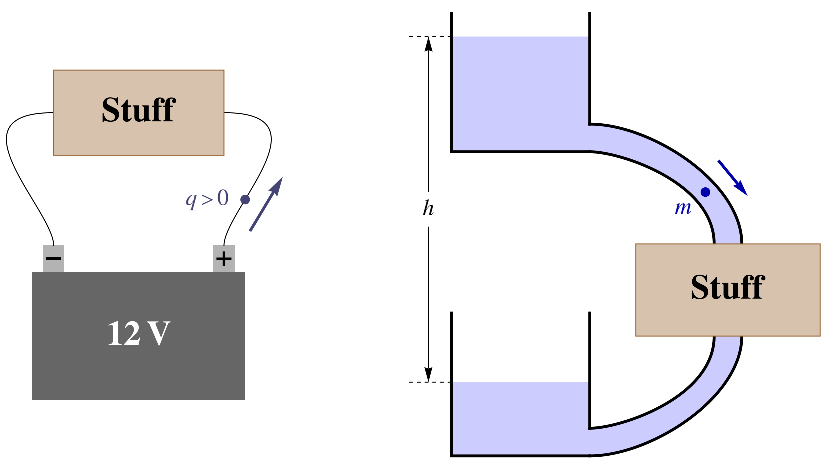
If the above expressions are taken as the definition of V then dividing both sides of the ΔU formula gives an expression for the voltage (potential difference) as an integral of the electric field over a contour

$$\Delta V = -\sum E_{\parallel}\,\Delta s$$

This is the fundamental expression showing how potential can be found from electric fields.



A Physical Example and Gravitational Analog



Charge flowing from the positive terminal to the negative terminal of a car battery (left) and its gravitational analog with a mass *m* of water flowing from an elevated tank to a lower one.

Consider the 12 V battery in a car connected across some load: the starter, headlights, car stereo, etc. Charge flows from the positive terminals to negative terminals, where when we discuss the flow of charge we mean the flow of positive charge. When a positive charge q flows from the positive terminal to the negative terminal the potential difference (voltage) is negative

$$\Delta V = V_f - V_i = V_- - V_+ = -12\text{ V}.$$

It follows that since the charge q is positive, the change in the potential energy ΔU is negative.

$$q > 0 \implies \Delta U = q\,\Delta V = q\times(-12\text{ V}) < 0$$

Here U is the electrical energy stored in the battery. This is decreasing and that is providing energy to the load; it turns the starter, powers the headlights or powers the car stereo.

There is a very good gravitational analogy for this. Consider water in an elevated tank, where water is allowed to flow over a paddle wheel before falling into a lower tank. If h is the (positive) height difference between the water levels of the two tanks, then when a mass m flows from the top to the bottom tank there is a drop in potential energy.

$$\Delta U = m\,g\,\Delta y = -m\,g\,h < 0$$

This setup represents a gravitation analog of a battery. Through the paddle wheel it provides a (theoretical) maximum of mgh of energy that can harnessed for any application.

In both cases we are losing potential energy; the battery and its gravitational analog are being depleted. To replenish the energy of the gravitational setup we may mechanically pump water from the bottom to the top, where the external work of the pump adds to the potential energy. Moving a mass m of water from the bottom to the top gives

$$W_{\text{external}} = \Delta U = m\,g\,\Delta y = +m\,g\,h > 0.$$

In a car the alternator, a type of a generator, converts external work provided by the car's engine into electrical energy by pushing (positive) charge to higher potential.

$$W_{\text{external}} = \Delta U = q\,\Delta V = q\times(+12\text{ V}) > 0$$

Example B.1 - Accelerating Charge

(a) What is the speed of an electron after accelerating from rest across a 12 V potential difference?

Solution

The potential difference is the voltage. The relevant constants are the elementary charge and the electron mass.

$$\begin{aligned} V &= |\Delta V| = 12\text{ V} \\ e &= 1.602\times 10^{-19}\text{ C} \\ m &= m_{\text{electron}} = 9.11\times 10^{-31}\text{ kg} \end{aligned}$$

We use conservation of energy to find the speed. In part (b) we will pay attention to signs but to simplify this problem, let us ignore signs by taking absolute values.

$$K_i + U_i = K_f + U_f \implies 0 = \Delta K + \Delta U \implies |\Delta K| = |\Delta U|$$

Since $K = \frac{1}{2}\,m\,v^2$ and $K_i = 0$ we have $|\Delta K| = \frac{1}{2}\,m\,v_f^2$. When a charge $q = -e$ is moved across a ΔV we have $\Delta U = q\,\Delta V = -e\,\Delta\text{V}$. Taking absolute values gives $|\Delta U| = e\,|\Delta V| = e\,V$. This gives our result.

$$|\Delta K| = |\Delta U| \implies \frac{1}{2}\,m\,v_f^2 = e\,V \implies v_f = \sqrt{\frac{2\,e\,V}{m}} = 2.05\times 10^6\,\frac{\text{m}}{\text{s}}$$

(b) What is the sign of the potential difference ΔV?

Solution

$\Delta K > 0$ so it follows that $\Delta U < 0$. Since $\Delta U = q\,\Delta V = -e\,\Delta\text{V}$ then $\Delta V > 0$.

B.2 - Electric Potential for Charges

We have found expressions for electric fields due to charge distributions, first considering a single point charge and then several. We will now do the same for potentials. Since potential is a scalar we will see that the potential calculations are simpler because they lack the vector complications in the field calculations.

Point Charge

Begin with the general expression for the potential difference ΔV due to a field by summing over small displacements $\Delta\vec{s}$, while moving from $\vec{r}_i$ to $\vec{r}_f$. Note that $E_{\parallel}\,\Delta s = E\,\Delta s\cos\theta = E_r\,\Delta r$, where Δr is the component of $\Delta\vec{s}$ in the outward radial direction.

$$\Delta V = -\sum E_{\parallel}\,\Delta s = -\sum E\,\Delta s_{\parallel} = -\sum E_r\,\Delta r$$

We know that the field of a point charge q has the magnitude $E = k_e\,|q|/r^2$ and we also saw that its outward radial component is $E_r = k_e\,q/r^2$. Combining this with the expression above, gives an expression for the change in potential when moving in the field of a point charge.

$$\Delta V = -\sum k_e\,\frac{q}{r^2}\,\Delta r \;\text{ from }\; r_i \text{ to } r_f$$

This sum over small displacements can be done using an integral from calculus; the result is written in terms of r_i and r_f, the initial and final radial distances from the charge q.

$$\Delta V = -k_e\,q\left(\frac{1}{r_f} - \frac{1}{r_i}\right)$$

We are looking for some function $V(r)$, describing the potential as a function of position. Since it must satisfy $\Delta V = V(r_f) - V(r_i)$, it is unique up to an arbitrary constant. The simplest choice is

$$V(r) = k_e\,\frac{q}{r}$$

where the arbitrary constant is chosen to make the potential zero at infinity

$$V(\infty) = 0 \quad \text{or more precisely} \quad \lim_{r\to\infty} V(r) = 0.$$

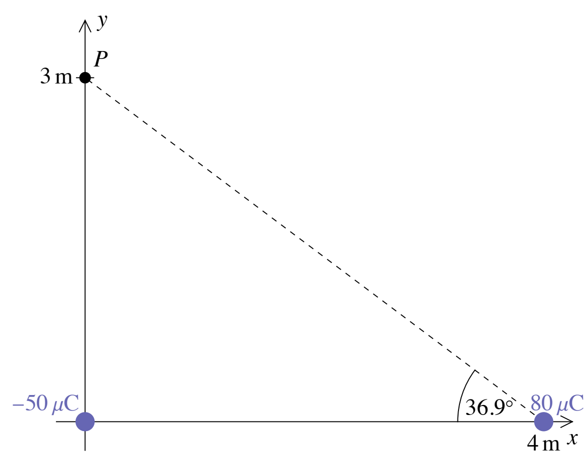
Distribution of Charges

In the previous chapter, we saw that the electric field for a collection of charges is the sum of the fields for each charge. The analogous result is true for the elctric potential. If you have two or more point charges then the total electric potential due to all the charges is the sum of the potentials for to each charge. The total electric potential V due to the charges $q_1, q_2, \ldots$ is

$$V = V_1 + V_2 + \ldots$$

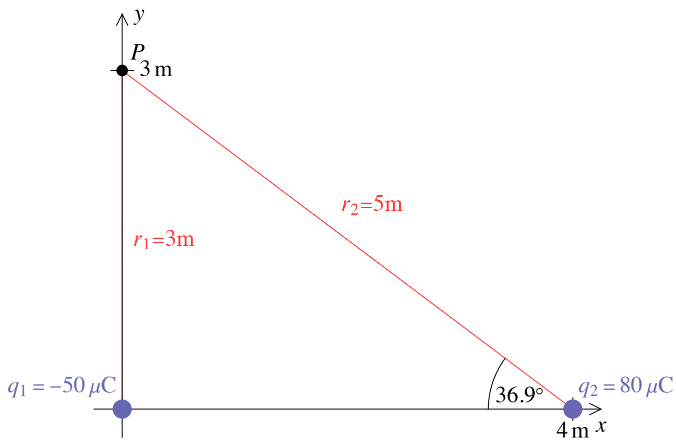
where V_1 is the field due to q_1, $V_2$is the field due to q_2, etc.

Example B.2 - Two Point Charges - Continued Again



Previously, we found the electric field at P. Now, given the same 80 μC at (4 m, 0) and −50 μC at the origin, find the electric potential at the point P, at (0, 3 m).

Solution



Take $q_1 = -50\,\mu$C and $q_2 = 80\,\mu$C. The distance from q_1 at the origin to P at (3 m, 0) is $r_1 = 3$ m. We can use the Pythagorean theorem to find r_2 the distance from q_2 at (4 m, 0) to P.

$$r_2 = \sqrt{(4\text{ m})^2 + (3\text{ m})^2} = 5\text{ m}$$

Now find the magnitudes of V_1 and V_2.

$$V_1 = k_e\frac{q_1}{r_1} = 9.0\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\ \frac{-50\times10^{-6}\text{ C}}{3\text{ m}} = -150\,000\text{ V}$$

$$V_2 = k_e\frac{q_2}{r_2} = 9.0\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\ \frac{80\times10^{-6}\text{ C}}{5\text{ m}} = 144\,000\text{ V}$$

The total V is the sum

$$V = V_1 + V_2 = -6000\text{ V}$$

Note that this calculation was much simpler than finding the electric field; since we are just adding scalars. Also the angle is not relevant here. Also notice that the signs of the charges matter and the sign of the potential due to a charge is the sign of the charge.

B.3 - Potentials and Electric Fields

Potential from Electric Field

We have seen that the fundamental expression that gives the potential difference from the electric field is

$$\Delta V = -\sum E_{\parallel}\,\Delta s\ .$$

This is the change in the potential between the endpoints of some contour. A simple special case of this is a uniform field. If $\vec{E}$ is uniform (meaning spatially constant) then it comes out of the sum.

$$\Delta V = -E_{\parallel}\,\Delta\ s$$

where $\Delta\,\vec{s}$ is the vector from the starting position to the final position.

Another important special case is the infinitesimal form; if one moves along a small displacement $d\vec{s}$, the infinitesimal change in the potential is given by

$$\Delta V = -E_{\parallel}\,\Delta s\ \ (\text{small }\Delta s)$$

Equipotentials

Equipotentials are surfaces of constant potential.

Potential is a scalar field, meaning that at each position in space there is a scalar function. A way of representing scalar fields in a two dimensional graph is to draw contour of constant values. On weather maps, the scalars of temperature and pressure are represented by isotherms and isobars, which are contours of constant temperature and pressure. In three dimensions we have surfaces of constant values instead of contours. A surface of constant potential is called an equipotential and these will appear as contours in two dimensional diagrams.

Field lines are perpendicular to equipotentials.

To understand how equipotentials are related to electric field lines consider the infinitesimal form of the potential expression $\Delta V = -E_{\parallel}\,\Delta s$. Take $\Delta\vec{s}$ to be some small displacement along the equipotential. Along the equipotential $\Delta V = 0$, this is because the potential is constant along an equipotential. We then get $0 = E_{\parallel}\,\Delta s$ which implies that the field is perpendicular to the $\Delta\vec{s}$. Since this is true for any direction along the equipotential it follows that the field is always perpendicular to the equipotential.

Electric field lines point toward lower potential.

Begin with the expression $\Delta V = -E_{\parallel}\,\Delta s$ and take $\Delta\vec{s}$ to be some infinitesimal displacement in the direction of an electric field line. Since $\Delta\vec{s}$ is in the same direction as the field then $E_{\parallel}\,\Delta s > 0$ which implies that $\Delta V = -E_{\parallel}\,\Delta s < 0$. This tells us that field lines always point toward lower electric potential.

Electric Field from Potential

To go from the potential to the electric field, begin with the expression for a small displacement $\Delta\vec{s}$, $\Delta V = -E_{\parallel}\,\Delta s$. To get the component of the electric field parallel to $\Delta\vec{s}$, just solve for $E_{\parallel}$.

$$\Delta V = -E_{\parallel}\,\Delta s \quad\Longrightarrow\quad E_{\parallel} = -\frac{\Delta V}{\Delta s}$$

To find E_x move by Δx in the x-direction.

$$E_x = -\frac{\Delta V}{\Delta x}$$

Similarly, one can find the y and z components of the electric field.

Conductors in Electrostatics - II

Potential is constant throughout a conductor in electrostatics.

The condition that the electric field is zero implies that the potential is constant. Consider some path entirely inside a conductor. Summing to get the potential difference gives

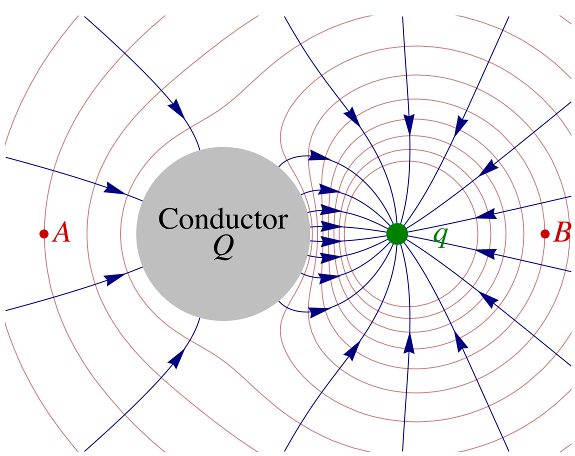
$$\Delta V = -\sum E_{\parallel}\,\Delta s = 0$$

Note that this argument does not apply to disconnected conductors. If there are several disconnected conductors each one will have its own potential.

It follows that the surface of a conductor is always an equipotential. The condition that the field is perpendicular to a conductor may now be seen as a consequence of fields being perpendicular to equipotentials.

Example B.3 - Field Lines, Equipotentials and Conductors

The figure shows a point charge q and a conductor with a net charge Q. The field lines are dark blue and the equipotentials are dark red. Two points, A and B are also labeled.



(a) What are the signs of q, Q and $Q+q$?

Solution

For q the field lines point toward it, so it is negative. For the conductor's charge Q, there are more field lines leaving than entering so $Q>0$. For the total charge the field lines come in from infinity so the total charge is negative $Q+q<0$.

(b) Where is the electric field larger, at point A or point B?

Solution

At point B the field lines are closer together to the field is stronger there, $E_A < E_B$. (This refers to the last chapter's material.)

(c) Where is the electric potential larger, at point A or point B?

Solution

This is more subtle. Both points A and B are on equipotentials. Follow the equipotential of point A around to the other side of the conductor and point B. Since electric field lines pass from the B equipotential to the A equipotential and since field lines point toward lower potential we have $V_A > V_B$.

B.4 - Potential Energy of Charges

We have seen that the potential energy difference when a charge q is moved across a potential difference ΔV is

$$\Delta U = q\,\Delta V.$$

To find the potential energy of a configuration we will start with zero potential energy when all the charges are far apart. For a two charge configuration begin with q_1 in place and move charge q_2 from infinity to its position a distance r from q_1. The potential due to q_1 is varying from 0 to $k_e\,\frac{q_1}{r}$.

$$U = \Delta U = q_2\left(k_e\,\frac{q_1}{r} - 0\right)$$

This gives the expression for two charges separated by a distance r

$$U = k_e\,\frac{q_1\,q_2}{r}.$$

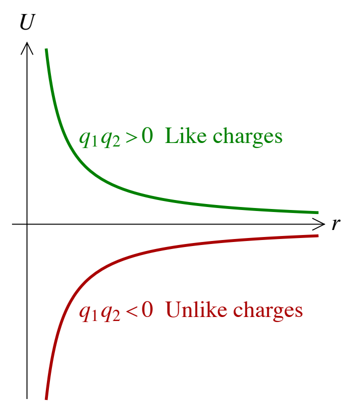


Figure - Potential Energy for Two Point Charges

Example B.4 - Energy and Electrons Released from Rest

Two electrons are initially at rest a distance of 5.0×10^{-10} m. If they are released, then what is their speed when separated by a large distance, i.e. at infinity? (Both electrons will have the same speed.)

Solution

Call r_i the initial distance. The relevant constants are the elementary charge and the electron mass.

$$r_i = 5.0 \times 10^{-10}\ \text{m}$$
$$e = 1.602 \times 10^{-19}\ \text{C}$$
$$m = m_{\text{electron}} = 9.11 \times 10^{-31}\ \text{kg}$$

We use conservation of energy to find the speeds. Since both electrons will have the same speed we have

$$K = 2 \times \frac{1}{2}\, m\, v^2$$

The potential energy is the potential between two point charges.

$$U = k_e \frac{q_1\, q_2}{r} = k_e \frac{(-e)\,(-e)}{r} = k_e \frac{e^2}{r}$$

Since the charges are initially at rest $K_i = 0$ and since they are a large distance apart in the end, $U_f = 0$

$$K_i + U_i = K_f + U_f \implies 0 + k_e \frac{e^2}{r_i} = 2 \times \frac{1}{2}\, m\, v_f^2 + 0 \implies v_f = \sqrt{\frac{k_e\, e^2}{m\, r_i}} = 712\,000\ \frac{\text{m}}{\text{s}}$$

B.5 - Capacitance

A capacitor is just any configuration involving two conductors. Begin with the two conductors neutral and then connect some DC voltage source (a battery, for instance) with a voltage $V = |\Delta V|$ across them. Charge will flow until an electrostatic state is reached. The conductor connected to the positive terminal will gain some charge $+Q$ and the other will gain charge $-Q$. There is a proportionality between charge and voltage.

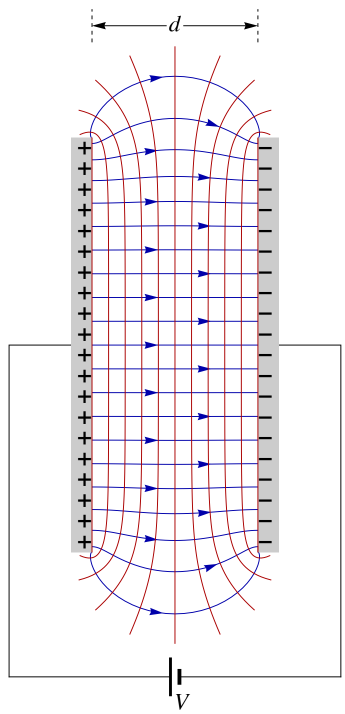
$$Q = C\,V \qquad (C \text{ is the capacitance.})$$

The capacitance between any pair of conductors depends on the geometry of the conductors, meaning that it depends on their size, shape, relative orientation and relative distance. We can measure the capacitance for any configuration but we can only calculate it in simple cases of symmetry.

Units: The SI unit for Capacitance is: farad = F = C / V

The Parallel Plate Capacitor

There is a pair of parallel plates with cross-sectional area A separated by d. We assume that d is small compared to the smallest linear dimension in A. For instance, if the cross-section is circular the separation is small compared to the radius and if rectangular it is small compared to the smaller of the length and width. The electric field is perpendicular to the plate and the equipotentials are parallel to the plates.



Using Gauss's law we related the field at the surface of a conductor to the surface charge density

$$E = \frac{\sigma}{\varepsilon_0} \quad \text{and} \quad \sigma = \frac{Q}{A} \quad \Longrightarrow \quad E = \frac{Q}{\varepsilon_0\, A}.$$

The voltage is the potential difference. Consider a path $\Delta\vec{r}$ between the plates in the direction opposite to the field.

$$V = \Delta V = -E_{\shortparallel}\,\Delta s = E\,d = \frac{Q}{\varepsilon_0\, A}\,d$$

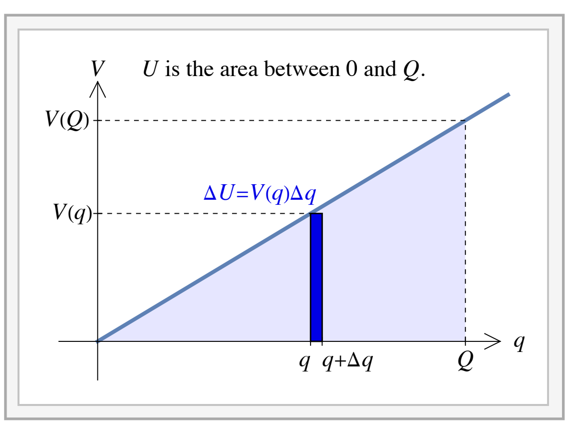
Using the definition of capacitance $Q = C_0\,V$ we can find C_0.

$$C_0 = \frac{\varepsilon_0\, A}{d}$$

B.6 - Energy

Energy in Capacitor

Move charge between the plates so that the charge q varies from 0 to Q, $0 \le q \le Q$. When the charge is q a small charge Δq is moved across the plates. The voltage as a function of charge is $V(q) = q/C$.



When Δq is moved across $V(q) = q/C$ the change in energy is $\Delta U = V(q)\,\Delta q$; this has the graphical interpretation as the area of the small rectangle in the diagram, shown in dark blue. The total energy when all the charge is moved is the total area which is the area of a triangle, shown in lighter blue.

$$U = \text{Area} = \frac{1}{2}\,\text{base} \times \text{height} = \frac{1}{2}\,Q\,V(Q) = \frac{1}{2}\,Q\,\frac{Q}{C} = \frac{Q^2}{2\,C}$$

Since $Q = C\,V$ we can rewrite the expression as

$$U = \frac{1}{2}\,C\,V^2 = \frac{Q^2}{2\,C} = \frac{1}{2}\,Q\,V.$$

Energy in an Electric Field

The energy in a capacitor is stored in the electric field between the plates. In general, wherever there is an electric field there is energy stored in that field. The energy density u is the energy per volume. We will derive an expression for the energy density in a field by using what we know about capacitors. In a parallel plate capacitor $C_0 = \frac{\varepsilon_0\,A}{d}$ and $V = E\,d$ giving the energy is

$$U = \frac{1}{2}\,C_0\,V^2 = \frac{1}{2}\,\frac{\varepsilon_0\,A}{d}\,(E\,d)^2.$$

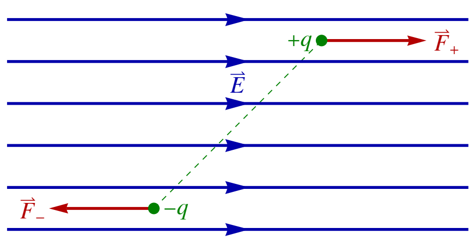
The volume between the plates is $A\,d$ giving $u = \frac{U}{A\,d}$ and thus

$$u_E = \frac{1}{2}\,\varepsilon_0\,E^2.$$

B.7 - Polarization and Dielectrics

Dipoles

A dipole is some charge configuration with zero net charge but a net charge separation. Consider two charges $-q$ and $+q$ sitting in a uniform electric field. The positive charge experiences a force in the direction of the field and the negative charge experiences a force opposite the field. This will create a net torque, a rotational force, that will tend to align the dipole with the field.



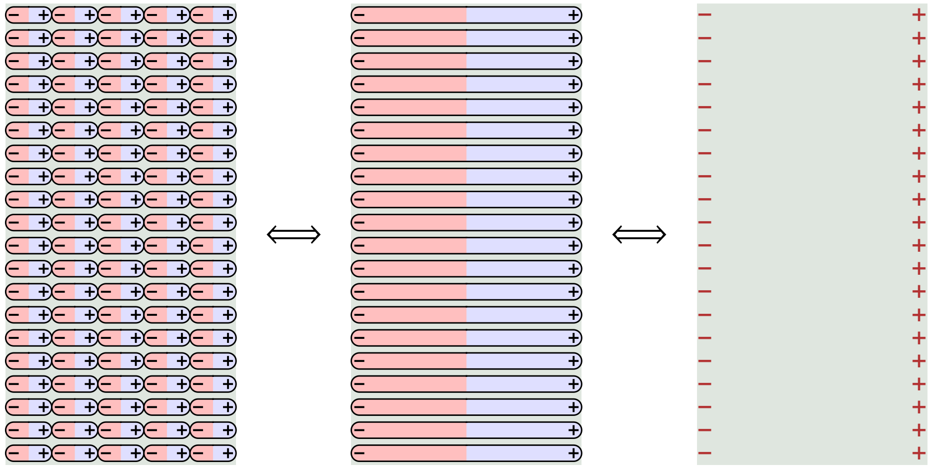
There is a torque on a dipole that will tend to align it with an electric field.

We can define a dipole moment for the configuration above to be $p = q\,d$, where d is the distance between the charges. We will only be qualitative in describing dipoles.

Many molecules are naturally polar. For example, think of a water molecule; the negatively charged electrons from the hydrogen atoms tend to spend more time around the oxygen atom making the oxygen side of the molecule negatively charged. Many other molecules or atoms are not inherently dipoles but get polarized and gain a dipole moment when in an electric field.

Dielectrics and Polarization Charge

A dielectric is a medium placed between the conductors of a capacitor. A dielectric will enhance capacitance. It is either the polar nature of the molecules or their capacity to be polarized that makes a dielectric. A good dielectric is one with strongly polar molecules. The alignment of dipole moments inside a dielectric creates a polarization charge. When the dipoles align, as in the diagram below, the interior positive and negative charges cancel creating the equivalent of long dipoles across the dielectric. This is equivalent to a buildup of charges on the sides of the dielectric; this effective charge is called a polarization charge. We will see that this has the effect of diminishing an electric field inside a dielectric.



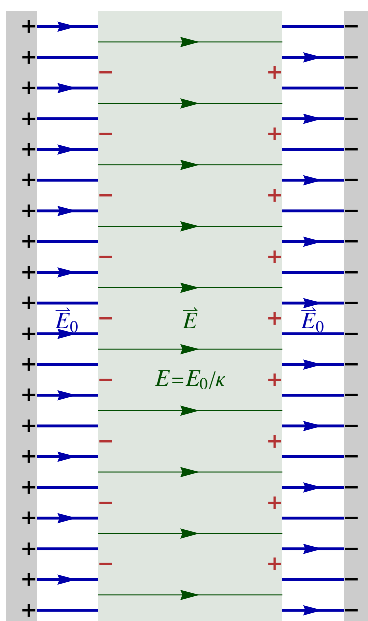
Dielectrics and Capacitance

Begin with a parallel plate capacitor with a uniform field magnitude E_0 between the plates. The field is related to the charge and charge density between the plates by

$$E_0 = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}.$$

If a conducting slab is placed between the plates the field inside the conductor will be $E = 0$. For this to happen charge in the conductor moves to the edges to cancel the charge in the capacitor. If a dielectric slab is placed between the plates the alignment of the dipoles is equivalent to a partial shielding of the capacitors charge and the field in the slab is diminished to a smaller value $E < E_0$. The dielectric constant κ is defined as the constant $\kappa \geq 1$ given by

$$E = \frac{E_0}{\kappa}.$$



Now take the added slab to fill the entire region between the plates while the charge on the plates is held constant. (This is done by disconnecting the capacitor from any voltage source.)

$$Q = Q_0$$

Voltage is related to the field by the standard relation $\Delta V = -\sum E_{\parallel} \Delta s$, which in this case becomes $V = Ed$; this implies that the voltage changes as the slab is added.

$$V = E\,d \text{ and } V_0 = E_0\,d \implies V = \frac{V_0}{\kappa}$$

$$C = \frac{Q}{V} \text{ and } C_0 = \frac{Q_0}{V_0} \implies C = \kappa\,C_0$$

This is the main result; adding a dielectric between the conductors enhances the capacitance by a factor called the dielectric constant κ. The dielectric constant is a material dependent constant.

Example B.5 - Parallel Plate Capacitor

A parallel plate capacitor consists of two circular conductors with a 3.0 cm radius, separated by 0.40 mm. It is filled with an unknown dielectric. When it is connected across a 12 V car battery, 8.5 nC of charge flows to either plate.

(a) What is its capacitance?

Solution

The plate separation, charge and voltage are given. We also need the constant ε_0.

$$d = 0.40\times 10^{-3}\ \text{m}\ ,\ \ Q = 8.5\times 10^{-9}\ \text{C}\ ,\ \ V = 12\ \text{V}\ \ \text{and}\ \ \varepsilon_0 = 8.85\times 10^{-12}\ \frac{\text{C}^2}{\text{N m}^2}$$

We can find the cross-sectional area from the radius.

$$r = 0.030\ \text{m} \implies A = \pi\,r^2 = 0.002874\ \text{m}^2$$

All we need to find the capacitance is V and Q.

$$Q = C\,V \implies C = \frac{Q}{V} = 7.0833\times 10^{-10}\ \text{F} = 7.08\times 10^{-10}\ \text{F}$$

(b) What is the dielectric constant for this capacitor?

Solution

Here we need the dimensions of the capacitor to find the empty capacitance.

$$C_0 = \varepsilon_0\,\frac{A}{d} = 6.2557\times 10^{-11}\ \text{F}$$

Using the definition of the dielectric constant we can find the dielectric constant κ.

$$C = \kappa\,C_0 \implies \kappa = \frac{C}{C_0} = 11.3$$

(c) What is the energy stored in this capacitor?

Solution

$$U = \frac{1}{2}\,C\,V^2 = \frac{Q^2}{2\,C} = \frac{1}{2}\,Q\,V$$

These are the expressions for the energy stored in a capacitor. Since we are given Q and V it is easiest to use the last expression.

$$U = \frac{1}{2}\,Q\,V = 5.1\times 10^{-8}\ \text{J}$$