Chapter D

Magnetism

Blinn College - Physics 1402 - Terry Honan

D.1 - Magnetic Dipoles and Magnetic Fields

Analogy to Electric Fields

There are two types of "magnetic charge" or poles, North poles **N** and South poles **S**. Playing with bar magnets demonstrates that like poles repel and unlike attract. This is analogous to the situation we had with electric charge. This analogy is a deep one and is called Electromagnetic duality. North and South poles are related to the magnetic field $\vec{B}$ as positive and negative electric charges are to the electric field.

$$\mathbf{N} \text{ and } \mathbf{S} \text{ are to } \vec{B}$$
$$\text{as}$$
$$+ \text{ and } - \text{ are to } \vec{E}$$

North poles experience a force in the direction of the magnetic field and south poles are pushed opposite the field.

Gauss's Law for Magnetism and the Absence of Isolated Poles

Isolated magnetic poles could exist but so far none have ever been observed. Gauss's law in the electric case $\Phi_e = \sum E_\perp A = q_{\text{enclosed}}/\varepsilon_0$, where Φ_e is the electric flux through a closed surface. We now have added the "e" subscript to the flux to distinguish the electric flux from the magnetic flux, which we will now consider. We can interpret Gauss's law as stating that electric field lines begin at isolated positive charges and end at isolated negative charges. The absence of isolated magnetic poles implies that magnetic field lines never begin or end; they either form closed loops or go off to infinity. The nonexistence of isolated magnetic poles implies that the right hand side of Gauss's law for magnetism is zero.

$$\Phi_m = \sum B_\perp A = 0 \ \text{(closed surface)}$$

Magnetic Dipoles

Recall that an electric dipole was some charge configuration with zero net (electric) charge, but a net separation of charge. Although we cannot have isolated magnetic pole, we can have magnetic dipoles. A permanent magnet is a magnetic dipole, there is as much north as there is south but they are separated.

If we apply Gauss's law for magnetism to a magnet and put a Gaussian surface around the North pole then there is magnetic flux leaving the surface at the end of the magnet. For the flux to be zero through this Gaussian surface the field lines inside the magnet must close back on themselves and form closed loops. Because of this if a bar magnet is cut in half then it doesn't split into a pair of isolated poles; it becomes two smaller dipoles.

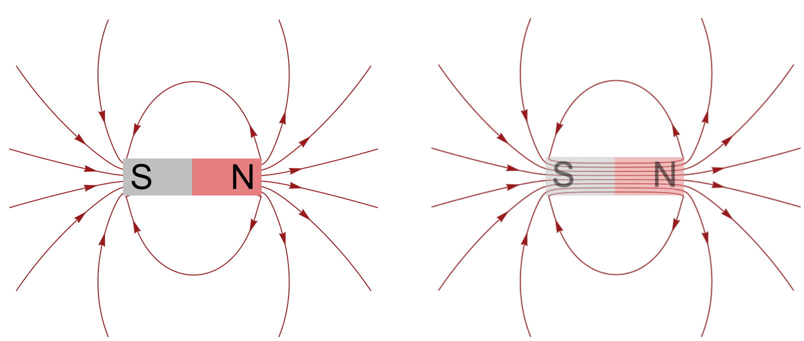
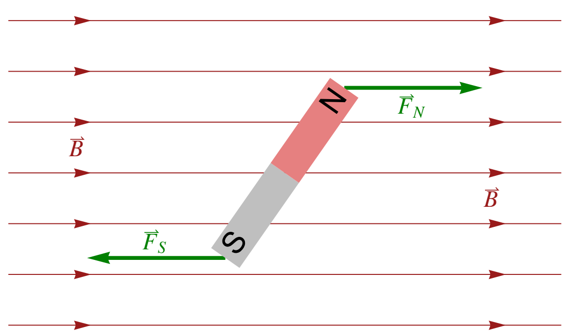


Image on the left shows a bar magnet and its magnetic field. On the right we see the field lines inside.

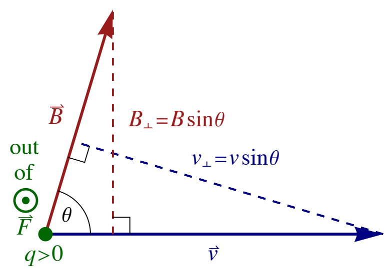
A bar magnet is a magnetic dipole and magnetic dipoles experience a torque in an external magnetic field.



D.2 - Force on Moving Charges and Currents

Electricity and magnetism are not separate forces, where electric fields just exert a force on electric charge and magnetic fields exert a force on magnets. Instead electricity and magnetism are aspects of the same force called electromagnetism. Magnetic fields cause forces on moving (electric) charges and currents.

Magnetic Force on Moving Charges



If a charge q is moving with a velocity $\vec{v}$ in a magnetic field $\vec{B}$, then the force $\vec{F}$ has a magnitude of

$$F = |q|\, v\, B\, \sin\theta = |q|\, v_\perp\, B = |q|\, v\, B_\perp$$

In this expression, θ is the angle between the $\vec{v}$ and $\vec{B}$ vectors; the component of $\vec{v}$ perpendicular to $\vec{B}$ is $v_\perp = v\sin\theta$, and $B_\perp = B\sin\theta$ is the component of the field perpendicular to the velocity.

The velocity and field vectors define a plane and the force is in the direction perpendicular to the plane. There are two possible perpendicular directions. To determine which gives the direction of the magnetic force, use the right had rule: put your thumb (of your right hand) in the direction of the velocity and your fingers in the direction of the magnetic field. Your palm points in the direction of the force on a positive charge. If the charge is negative then the force is opposite that.

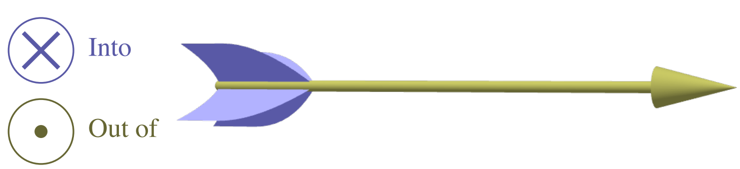


Figure: The convention we use to represent the third dimension relative to some two dimensional figure is to use a dot to represent "out of" and an × to represent "into". A useful way to remember this is with an arrow; if it points at you, it is a dot and away an ×.

Units: The SI unit for magnetic field is: tesla = T = $\frac{\text{N}}{\text{A}\cdot\text{m}}$

Example D.1 - The Earth's Magnetic Field

The earth's magnetic field at Bryan Texas has magnitude 47.3 μT. It is directed 2.85° east of true north and has a "dip angle", the angle below horizontal, of 59.4°. For the purposes of this example we will take magnetic north as north.

(a) What are the northward and downward components of the earth's field.

Solution

(b) What is the force (both magnitude and direction) on an electron moving downward at 2.5×10^6 m/s in this field? Give the direction as north, south, east, west, up or down.

Solution

Force on Currents

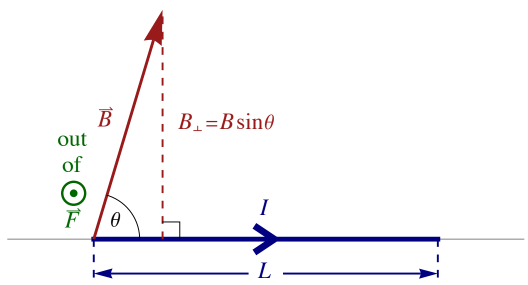
We want an expression for the force on a segment of wire of length L carrying a current I in a magnetic field $\vec{B}$. Model the current by positive charges flowing through the wire at a speed v in the direction of the current. For the segment of length L the time it takes for the charge to move the distance L is $\Delta t = L/v$. Since current is charge per time we can write the total amount of moving charge in this segment as $q = I\,\Delta t = I\,L/v$. We can then write an expression for the force on the segment.

$$\Delta t = L/v \text{ and } q = I\,\Delta t = I\,L/v \implies F = q\,v\,B\sin\theta = \frac{IL}{v}\,v\,B\sin\theta = ILB\sin\theta$$

We can, as before, write $B_\perp = B\sin\theta$ as the perpendicular component of the field in the direction of the current. the magnitude of the force on a segment of wire of length L carrying a current I as

$$F = ILB\sin\theta = ILB_\perp$$

where θ is now the angle between the field and the current.



Since positive charges are moving in the direction of the current the right hand rule for the direction of the force is similar. Your thumb points in the direction of the current, your fingers in the direction of the field and then the force on the wire is in the direction of the palm.

Motion of Charged Particles

Any force that acts perpendicularly to the velocity of a particle doesn't affect the speed of the particle; it only alters its direction. This is always the case with the magnetic force, since it is perpendicular to the velocity. Suppose a particle with speed v is shot into a region of uniform magnetic field with the velocity perpendicular to the field then the magnitude of the force is just $F = |q|\,v\,B$. Since the speed and the magnitude of the force are constant and the force and velocity are perpendicular, the motion will be uniform circular motion. Using the acceleration for uniform circular motion $a_c = v^2/r$ and Newton's second law we get:

$$F = m\,a \implies |q|\,v\,B = m\,\frac{v^2}{r} \implies r = \frac{m\,v}{|q|\,B}\,.$$

If a charged particle moves in a uniform magnetic field with a velocity that is not perpendicular to the field, then the perpendicular component changes as before and the parallel component is unchanged. The resulting motion is a combination of linear and circular motion, giving a helix. Thus, the general shape of the path of a charged particle in a uniform magnetic field is helical.

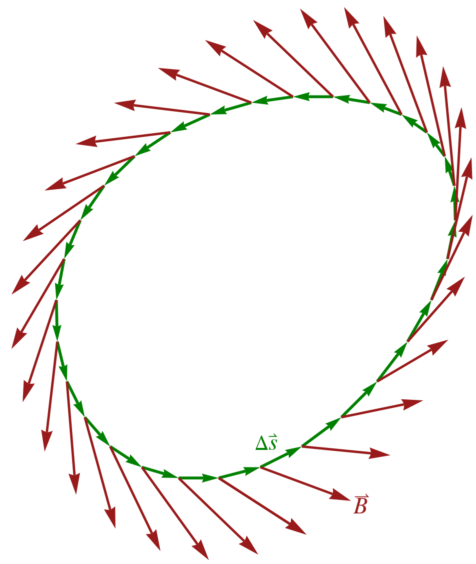
D.3 - Sources of Magnetic Fields - Ampere’s Law

Ampere’s Law

In Chapter 19 we saw that electric field exert forces on charges and that charges are the source of electric fields; we discussed in detail how to find electric fields due to charges. For magnetism, we have seen that there are forces on moving charges and currents in magnetic fields. We now need to show that moving charges and currents are sources of magnetic fields.

Ampere’s Law is related sums we have seen in two previous chapters. In Chapter 20 we saw that the electric potential and electric field were related by the sum over small displacements making a path: $\Delta V = -\sum E_{\parallel}\, \Delta s$. In Chapter 19 we had Gauss's law relating the flux, a sum over small surfaces that combine to form a closed surface, to the total charge enclosed by that surface; $\sum E_{\perp}\, A = q_{\text{enclosed}} / \varepsilon_0$. Ampere's law involves summing $E_{\parallel}\, \Delta s$ around a closed path, a loop, and relating that to the total current enclosed by that loop.

$$\sum B_{\parallel}\, \Delta s = \mu_0\, I_{\text{enclosed}} \qquad (\Delta \vec{s} \text{ is small})$$



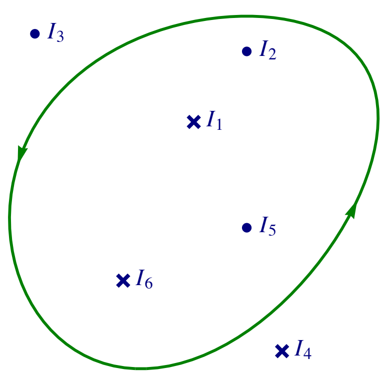
The sign convention for the currents in I_{enclosed} requires some discussion. There is a second right hand rule that relates a sense of circulation or rotation to a perpendicular direction; wrap the fingers of your right had in the direction of circulation and your thumb is in the perpendicular direction. In Physics I, this was used to relate a sense of rotation to a direction of the angular velocity vector; rotating clockwise (when viewed from above) corresponds to a downward angular velocity and counterclockwise gives upward.

We have introduced a new constant μ_0. This is a fundamental constant, like ε_0.

$$\mu_0 = 4\,\pi \times 10^{-7}\ \frac{\text{N}}{\text{A}^2}$$

Note that we can write this with different units: $\text{N} / \text{A}^2 = \text{T} \cdot \text{m} / \text{A}$.

Example D.2 - Finding I_{enclosed}



The diagram above shows a closed contour for Ampere's law. There are six currents which pass either into the page (✖) or out of it (•). What is I_{enclosed} for this contour?

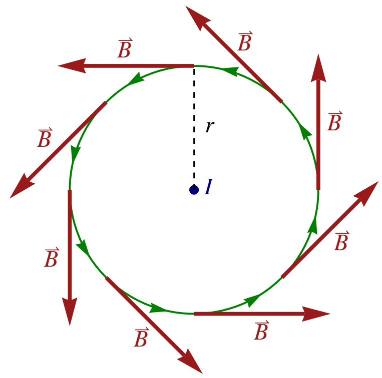
Solution

Since the contour is counterclockwise, the right hand rule connects that with currents out of the page. The "out of" currents are positive and the "into" currents are negative. Currents I_3 and I_4 are not enclosed and do not contribute. The result is:

$$I_{\text{enclosed}} = -I_1 + I_2 + I_5 - I_6$$

The Long Straight Wire

As we have seen, magnetic field lines either form closed loops or go off to infinity. For a long straight wire the field lines cannot go out to infinity, since that would imply a net magnetic pole along the wire. So in this case the field lines must circulate around the wire.



The magnetic field circulates around a long straight wire. Here the current *I* is out of the page, the counterclockwise contour for Ampere's law is shown in green and the magnetic fields are shown as vectors in red.

When summing around the contour $\sum B_{\parallel}\,\Delta s$ the magnetic field is parallel to the $\Delta\vec{s}$ vectors and is constant in magnitude. It follows that B can be taken out of the sum and $\sum\Delta s$ is the total arc length, the circumference $2\,\pi\,r$.

$$\sum B_{\parallel}\,\Delta s = \sum B\,\Delta s = B\sum\Delta s = B\,2\,\pi\,r$$

The enclosed current is just the wire's current $I_{\text{enclosed}} = I$. Ampere's law gives us the magnetic field due to the solenoid.

$$\sum B_{\parallel}\,\Delta s = \mu_0\,I_{\text{enclosed}} \quad\Longrightarrow\quad B\,2\,\pi\,r = \mu_0\,I$$

We have then the field a distance r from the long wire is

$$B = \frac{\mu_0\,I}{2\,\pi\,r}$$

To get the direction of the field use the circulation right had rule described above. The field lines circulate around the wire so put your thumb in the direction of the current and the fingers wrap around in the direction of the field.

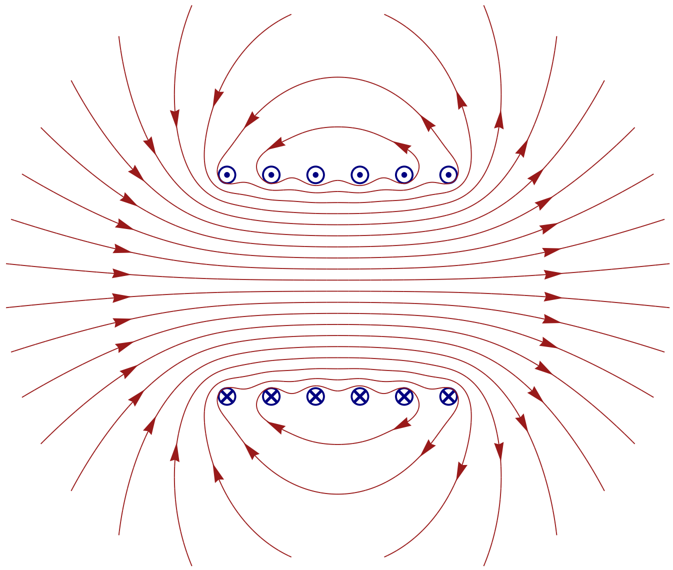
Example D.3 - The Field of a Long Straight Wire

A horizontal wire runs east-west with a 20 A current to the west.

(a) What are the magnitude and direction of the magnetic field 3 cm directly below the wire. Give the direction answer as north, south, east, west, up or down.

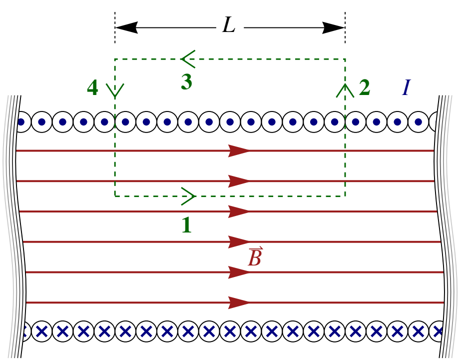
Solution

The Long Solenoid



Cross section of a six-turn solenoid

The figure above shows a cross section of a solenoid with 6 turns and a finite length. A long solenoid is infinitely long and the wires are tightly packed.



Cross section of a long solenoid

For an (infinitely) long solenoid take the current to be I and the density of turns to be n.

$$n = \frac{\text{\# of turns}}{\text{length}}$$

The field inside the solenoid is uniform and the field outside is zero. (The field outside approaches zero as the length become infinite.) Choose the contour to be four segments as shown

$$\sum B_{\parallel}\,\Delta s = \sum_{1} B_{\parallel}\,\Delta s + \sum_{2} B_{\parallel}\,\Delta s + \sum_{3} B_{\parallel}\,\Delta s + \sum_{4} B_{\parallel}\,\Delta s$$
$$= B\,L + 0 + 0 + 0$$

There are $n\,\ell$ turns through the contour giving

$$I_{\text{enclosed}} = n\,L\,I$$

It follows from Ampere's law that the field anywhere inside a long solenoid is

$$B = \mu_0\,n\,I.$$

To get the direction of the field use the circulation right hand rule. Now the current circulates and the field is straight, so wrap your fingers around the current loops in the direction of the current and your thumb is in the direction of the field inside.

Example D.4 - A Long Solenoid

Consider a solenoid with a circular cross-section with a radius of 3 cm and a length of 75 cm, 200 turns with a vertical central axis carrying a 12 A current that is clockwise when viewed from above. Take up to be the z-direction. You may consider the length long compared to the radius.

(a) What is the magnetic field inside the solenoid. Give both magnitude and direction.

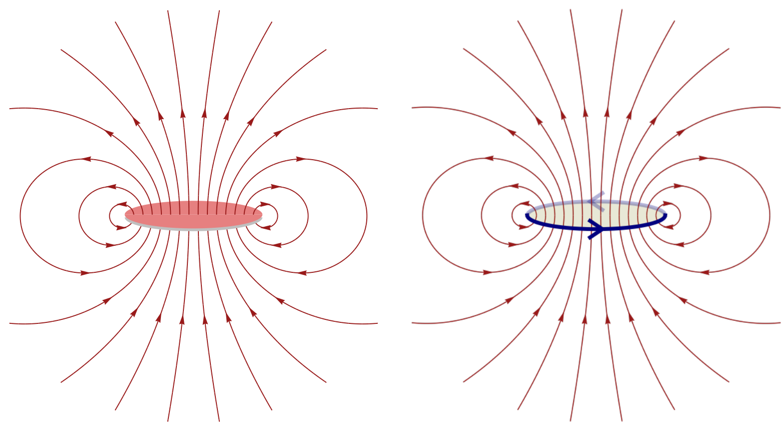
Solution

D.4 - Current Loops As Magnetic Dipoles and Torque

Electromagnets and Permanent Magnets as Dipoles

Circular Current Loop and Flat Round Magnet

Magnetic dipoles experience a torque in a magnetic field. A torque is a rotational force and a dipole will rotate to align with a field. We have seen that permanent magnets are dipoles. We will now see that current loops and coils are magnetic dipoles as well. To first see this compare the fields of permanent magnets to coils (electromagnets) of the same shape.



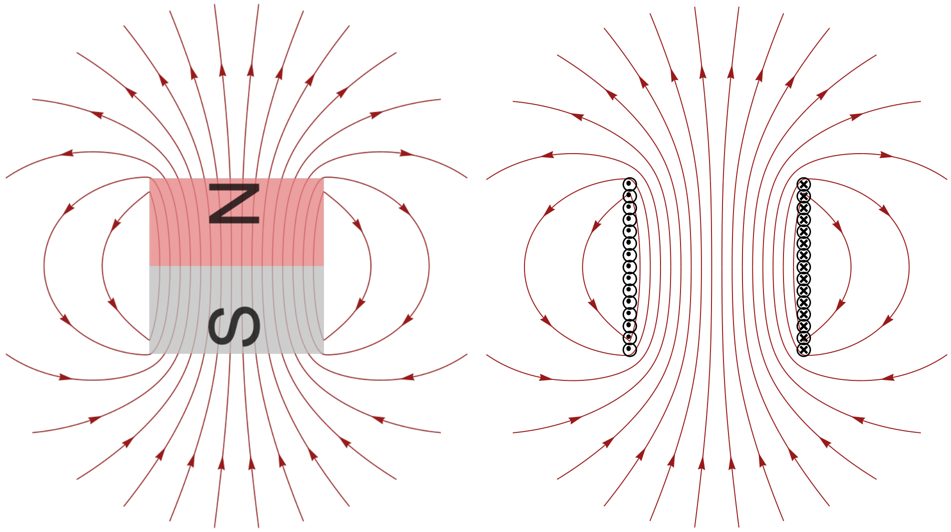
On the left is a flat circular magnet (like a refrigerator magnet) with north on the top. On the right is circular current loop of the same shape and size.

We will also give a formula for the magnetic field at the center of a loop of radius R with current I.

$$B = \frac{\mu_0 I}{2R}$$

To get the direction of the field at the center use the right hand rule for circulation. Wrap your fingers with the current and the thumb points in the direction of the field.

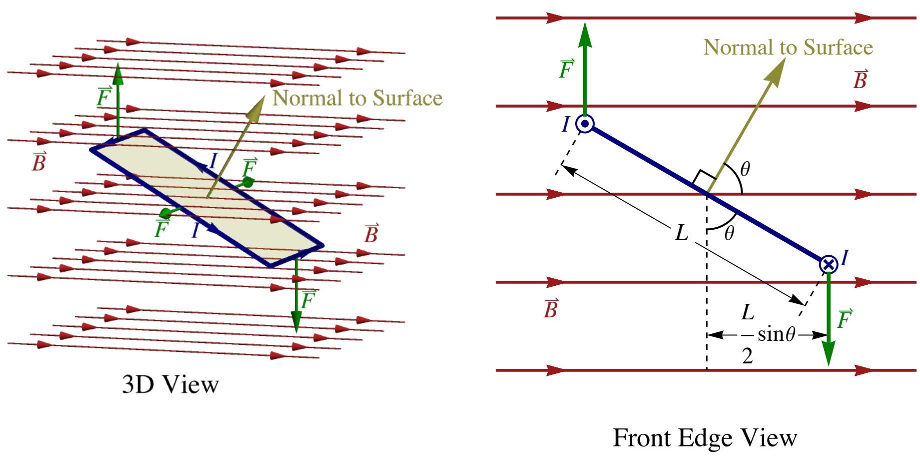
Tightly Wound Solenoid and Magnet



A solenoid with tightly packed wires (right) is equivalent to a permanent magnet of the same shape.

Torque on a Current Loop

Now consider an $L \times W$ flat rectangular loop with a current I sitting in a uniform field as shown below. The angle between the normal to the loop and the magnetic field is θ. There are two normals to the surface but we choose the normal to be consistent with the circulation right hand rule; wrap your fingers in the direction of the current around the loop and the thumb says which normal vector to choose.



Two views of an $L \times W$ current loop with current I in a magnetic field $\vec{B}$. The angle between the field and the normal to the loop is θ. The forces on the long sides of length L are outward and produce no torque. The forces on the short sides of length W creates a torque that tends to align the normal to the loop with the field.

The forces on the two long sides (length L) are outward and produce no torque. The short sides (length W) do produce a torque. the forces on the short sides both have the same magnitude.

$$F = I\,L\,B_\perp = I\,W\,B$$

From Physics I, recall that the torque τ is given by $\tau = r_\perp F$, where $r_\perp$is the component of $\vec{r}$ perpendicular to the force, where $\vec{r}$ the vector from the origin to where the force acts. $r_\perp$ is known as the lever-arm. In this case we get the torque on one segment τ_{segment} is

$$r_\perp = \frac{L}{2}\sin\theta \implies \tau_{\text{segment}} = r_\perp F = \frac{L\,F}{2}\sin\theta = \frac{L\,I\,W\,B}{2}\sin\theta$$

There are two segments and the torque on both are the same, so on the loop we have $\tau = 2\,\tau_{\text{segment}}$. We will write the result in terms of the area of the loop $A = L\,W$. It follows now that the total torque on the loop is

$$\tau = I\,A\,B\,\sin\theta = I\,A\,B_\perp.$$

In the second expression, $B_\perp$ is the component of the field perpendicular to the normal. The loop is a magnetic dipole and will rotate so that the normal rotates to align with the loop.

Torque on an *N*-turn Coil

We can build up a general flat loop by breaking the loop into rectangles. It follows that the total torque on the flat loop is the sum over the torques on all the small rectangles and that gives the same formula $\tau = I\,A\,B\,\sin\theta$., but now with A being the total area. Now consider an N-turn coil or solenoid. The total torque now gets multiplied by the number of turns N.

$$\tau = N\,I\,A\,B\,\sin\theta = N\,I\,A\,B_\perp$$

In this formula, A is the cross-sectional area of each loop. The coil will rotate so that the unit normal of each loop (given by the right hand rule) will rotate to align with the field.

Example D.5 - A Long Solenoid (continued)

Consider a solenoid with a circular cross-section with a radius of 3 cm and a length of 75 cm, 200 turns with a vertical central axis carrying a 12 A current that is clockwise when viewed from above. Take up to be the z-direction. You may consider the length long compared to the radius.

(b) If this solenoid sits in a magnetic field of $\vec{B} = \left(2.3\,\hat{x} - 3.5\,\hat{z}\right)$ mT, then what is the torque on the solenoid due to this field?

Solution