Chapter E

Faraday's Law and Inductance

Blinn College - Physics 1402 - Terry Honan

Faraday's law describes how a generator works. A changing magnetic flux through a conducting loop induces an EMF (Electromotive Force) around the loop. Faraday's law will become our fourth of Maxwell's equations.

E.1 - Faraday's Law

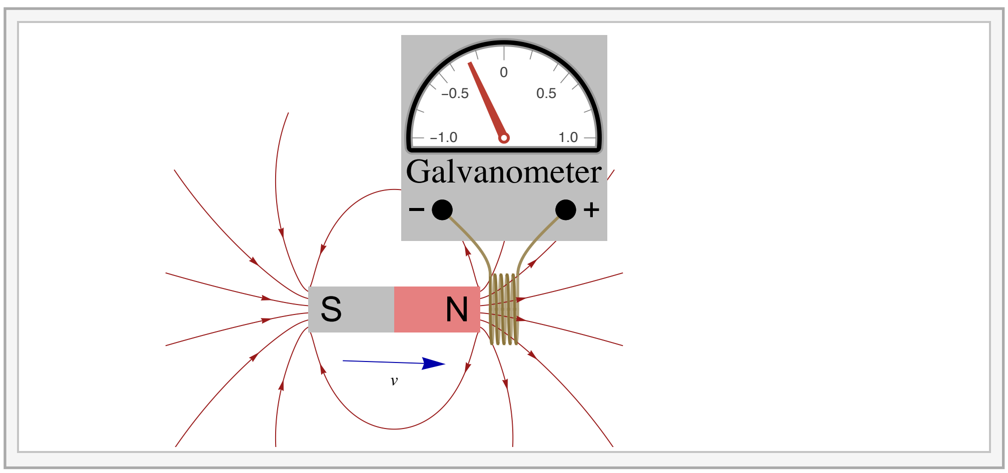
In the previous chapter we introduced magnetic flux in our brief discussion of Gauss's law for magnetism. The flux is $\Phi_m = \sum B_\perp A$. Now we will omit the m subscript.

$$\Phi = \sum B_\perp A$$

IF we have a flat surface and a uniform magnetic field then the flux becomes

$$\Phi = B_\perp A = BA\cos\theta$$

where we defined θ as the angle between the normal to a surface and the magnetic field.



We can demonstrate magnetic induction by moving a magnet toward a solenoid connected to a galvanometer, where a deflection of the galvanometer's needle indicates a current. Moving the magnet toward the solenoid induces an EMF in the solenoid, which creates a current in the solenoid-galvanometer circuit. Pulling the magnet away from the solenoid induces a current in the opposite direction. If the magnet is at rest in the solenoid no current is induced. Moreover, increasing the number of magnets increases these effects. It is clear that the induced EMF depends on the change in the flux. If $\mathcal{E}_{\text{ave}}$ is the average induced EMF and Φ is the magnetic flux through each loop of the solenoid then we get the proportionality

$$\mathcal{E}_{\text{ave}} \propto \Delta\Phi.$$

If the speed of the magnet is increased the effect is enhanced and slowing it diminishes it. This suggests an inverse proportionality with the time.

$$\mathcal{E}_{\text{ave}} \propto \frac{1}{\Delta t}.$$

Since in a coil all the loops are connected in series it follows that $\mathcal{E}_{\text{ave}}$ is proportional to the number of loops N.

Combining these proportionalities gives

$$\mathcal{E}_{\text{ave}} \propto N\,\frac{\Delta\Phi}{\Delta t}$$

and it turns out that the proportionality becomes an equality when we consider the magnitude of the average induced EMF $|\mathcal{E}_{\text{ave}}|$ and the absolute value of the change in flux with time

$$\mathcal{E}_{\text{ave}} = N\left|\frac{\Delta\Phi}{\Delta t}\right|.$$

If we consider the polarity (sign) of the induced EMF then this adds a sign, which is known as Lenz's law.

$$\mathcal{E}_{\text{ave}} = -N\frac{\Delta\Phi}{\Delta t}.$$

We will discuss Lenz's law in detail later.

Letting Δt go to zero these expressions become

$$|\mathcal{E}| = N\left|\frac{\Delta\Phi}{\Delta t}\right| \quad \text{and} \quad \mathcal{E} = -N\frac{\Delta\Phi}{\Delta t} \quad (\text{small } \Delta t)$$

This is analogous to the definitions of average and instantaneous velocity in kinematics where $v_{\text{ave}} = \frac{\Delta x}{\Delta t}$ and $v = \frac{\Delta x}{\Delta t}$ (small Δt).

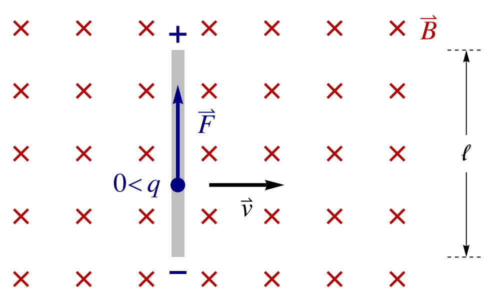
E.2 - Motional EMF

If a conductor moves in a magnetic field there is a magnetic force on the charge carriers. This magnetic force does work on the charge carriers. The EMF is the work per charge.

$$\mathcal{E} = \frac{W}{q}.$$

Translation of a Conducting Rod in a Uniform Field

Consider a conducting rod of length ℓ translating in a uniform magnetic field. Take the rod, its velocity and the field to be mutually perpendicular.



If q is some charge carrier then the force on it is

$$F = q\,v\,B_{\perp} \implies F = q\,v\,B$$

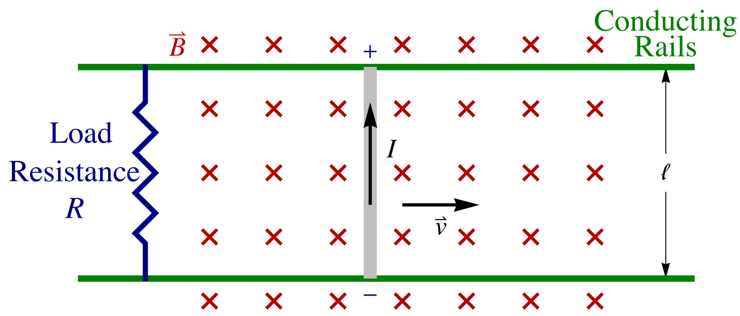
The work is

$$W = F\,\Delta r_{\parallel} = F\,\ell = q\,v\,B\,\ell.$$

Using $\mathcal{E} = W/q$ gives the induced EMF across the rod

$$\mathcal{E} = v\,B\,\ell$$

How can one measure this EMF? If a voltmeter is connected across the ends of the rod and it is moved with it then the same EMF is induced in the leads to the meter and it reads zero. To avoid this, imagine the rod moving with its ends sliding along a conducting rail. If the voltmeter is connected between the rails then it would read this voltage.



A DC Generator and Conservation of Energy

In fact, what we have here is a simple DC generator. Instead of connecting a voltmeter between the rails we could connect anything and it would be given a steady DC voltage. If a DC motor is connected then this motor could do work. We must address the question of conservation of energy. Where does this energy come from?

To see this place a load resistor R across the conducting rails. Ohm's law gives the current through the load

$$I = \frac{\mathcal{E}}{R}.$$

The rate of power dissipation in the load, which is the power output of the generator, is

$$\mathcal{P}_{\text{out}} = I\,\mathcal{E} = I\,v\,B\,\ell,$$

where we are making the idealizing assumption that all the resistance in the circuit is in the load.

This is a complete circuit, so all the current through the load passes through the rod. A current through a conductor in a magnetic field creates a backward magnetic force, $\vec{F}_{\text{mag}}$.

$$F_{\text{mag}} = I\,\ell\,B_{\perp} \implies F_{\text{mag}} = I\,\ell\,B$$

To keep the rod moving at a constant speed there must be zero net force, so there must be some forward external applied force $\vec{F}_{\text{app}}$. Making another idealizing assumption of no friction we get

$$\vec{F}_{\text{app}} = -\vec{F}_{\text{mag}} \implies F_{\text{app}} = F_{\text{mag}} = I\,\ell\,B$$

The applied force does work and this is the source of the energy. The rate that it does work is the power $\mathcal{P}_{\text{in}}$. Power is related to force by

$$\mathcal{P} = \frac{\Delta\text{Work}}{\Delta t} = \frac{F\,\Delta r_{\parallel}}{\Delta t} = F\,v_{\parallel} \quad (\text{small } \Delta t)$$

so we get in this case

$$\mathcal{P}_{\text{in}} = F_{\text{app}}\,v = I\,\ell\,B\,v.$$

Comparing this with the power output gives

$$\mathcal{P}_{\text{in}} = \mathcal{P}_{\text{out}}.$$

If we relax our idealizing assumptions and include mechanical friction and resistance in the circuit elsewhere than the load then we get

$$\mathcal{P}_{\text{in}} > \mathcal{P}_{\text{out}}.$$

Example E.1 - Motional EMF

A conducting rod slides with a speed of 18 m/s along parallel horizontal conducting rails separated by 1.2 m. Suppose there is a downward magnetic field of 15 mT.

(a) What is the voltage across the rod?

Solution

Using the values $v = 18\text{ m/s}$, $\ell = 1.2\text{ m}$ and $B = 15\times10^{-3}\text{ T}$ we get

$$\mathcal{E} = v\,B\,\ell = 0.324\text{ V}$$

(b) Which side of the rod, left or right (relative to the velocity) of the rod, is at higher potential?

Solution

We need to find the force on a positive charge moving with the rod. With your fingers down and thumb forward, your palm points to the left. If positive charges are pushed to the left then that is the side at higher potential.

(c) Suppose a load resistance placed across the rails to complete the circuit as a DC generator. If there is a 5.5 mA current flowing, then what is the load resistance.

Solution

When you complete the circuit with a resistance then we have, using the EMF as a voltage and using the current as $I = 5.5 \times 10^{-3}$ A.

$$V = \mathcal{E} = I\,R \implies R = \frac{\mathcal{E}}{I} = 58.9\,\Omega$$

(d) What is the backward magnetic force on the rod after the load resistance is added?

Solution

The magnetic force on the rod, now that there is a current, is: $F = I\,\ell\,B_\perp = I\,\ell\,B = 9.9 \times 10^{-5}$ N

(e) What is the rate of power dissipation in this circuit?

Solution

$$\mathcal{P} = \mathcal{P}_{\text{out}} = I\,\mathcal{E} = 1.78 \times 10^{-3}\ \text{W}$$

E.3 - Faraday's Law Examples

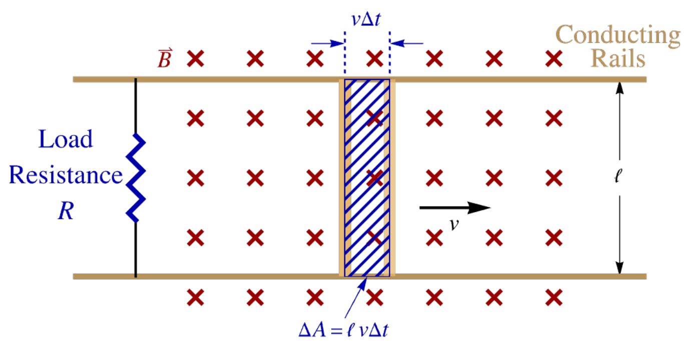
Usually when we use Faraday's law we will only consider the magnitude of the induced EMF or current. We will see how to find the polarity in the Lenz's law discussion that follows. We can neglect polarity information by inserting absolute values into Faraday's law.

$$|\mathcal{E}_{\text{ave}}| = N\left|\frac{\Delta\Phi}{\Delta t}\right| \quad \text{and} \quad |\mathcal{E}| = |\mathcal{E}_{\text{ave}}| \quad (\text{small } \Delta t)$$

If one is given some problem involving induced currents then this is related to the induced EMF by Ohm's law

$$\mathcal{E} = I\,R$$

The Translating Rod



There is an equivalence between motional EMF and Faraday's law. For the case of the translating rod that we analyzed above using motional EMF, we ought to be able to derive the same results using Faraday's law directly. If a load resistor is placed in the circuit then the complete circuit consists of the rod, rails and load. As the rod moves the area inside the circuit loop is increasing and so is the flux. In the time Δt the rod moves by $v\,\Delta t$ and the area increases by

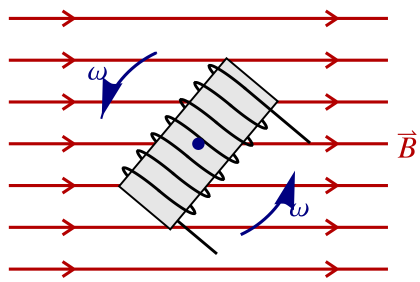
$$\Delta A = \ell\,v\,\Delta t$$

The flux, since the field is uniform and perpendicular to the surface, is $\Phi = B\,A$ and since the field is constant we get $\frac{\Delta\Phi}{\Delta t} = B\,\frac{\Delta A}{\Delta t}$. The induced EMF becomes

$$|\mathcal{E}| = \left|\frac{\Delta\Phi}{\Delta t}\right| = B\,\frac{\Delta A}{\Delta t} = B\,\ell\,v$$

which is the same as the expression derived using motional EMF considerations.

The AC Generator



Consider a solenoid or coil with its central axis rotating with angular velocity ω in a uniform magnetic field of magnitude B. Take the central axis to be perpendicular to the rotational axis and the rotational axis to be perpendicular to the field. The coil has N turns and each loop has a cross-sectional area A. Take the angle between the axis of the coil and the field to be θ. This varies by

$$\theta = \omega\,t$$

The flux through a loop is

$$\Phi = B\,A\cos\theta = B\,A\cos\omega\,t.$$

By Faraday's law $\mathcal{E} = -N\,\frac{\Delta\Phi}{\Delta t}$ (for small Δt) we get the induced EMF as a function of time to be

$$\mathcal{E}(t) = N\,B\,A\,\omega\sin\omega\,t.$$

This is an AC voltage. We will discuss AC in detail in the AC circuit chapter. We will state for future reference that the peak EMF is

$$\mathcal{E}_{\max} = N\,B\,A\,\omega \;\text{ where }\; \omega = 2\,\pi\,f$$

and the frequency of AC is the same as the rotational frequency. We will see in the next chapter that the angular frequency ω is related to the frequency f by $\omega = 2\,\pi\,f$.

The AC Motor

With a generator one does work to turn the coil and that energy is converted to electrical energy. A generator run in reverse is a motor and electrical energy is converted to work.

Example E.2 - Faraday's Law

A flat circular coil has 20 turns, a radius of 12 cm and sits in a vertical magnetic field of magnitude 30 mT. The coil is initially in a horizontal plane and is rotated to a vertical plane in 2.3 s.

(a) What is the average induced EMF in the coil during the rotation.

(b) If the two ends of the coil are shorted (connected) and the coil has a total resistance of 85 mΩ, then what is the average induced current during the rotation.

(c) Suppose instead the coil stays in the horizontal plane but the magnetic field decreases from 85 mT to 25 mT in 1.8 s. What is the induced EMF in the coil and, if the ends of the coil are shorted as in part (b), what is the induced current in the coil?

E.4 - Lenz's Law

The minus sign in Faraday's law is known as Lenz's law. It gives the polarity of the induced EMF. If the circuit is completed with a load resistance the induced EMF causes an induced current. The induced current induces a magnetic field $\vec{B}_{\text{ind}}$ which in turn induces a flux Φ_{ind} through the loop.

The induced flux tends to cancel the change in the flux.

This cancellation is exact for a superconductor.

The magnetic flux through a superconducting loop is a constant.

Using Lenz's Law

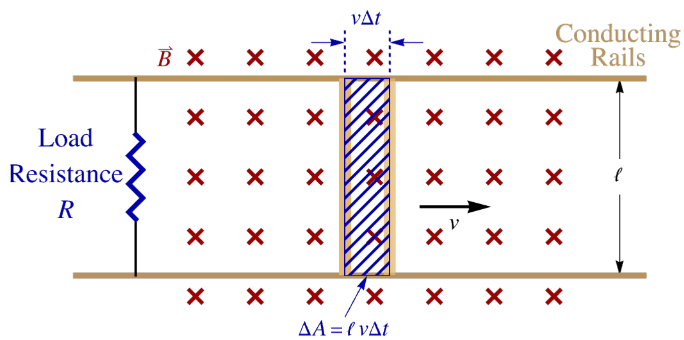
A simple procedure for applying Lenz's law follows. Identify a sign convention and then make a table consisting of the appropriate signs for Φ, $\frac{\Delta\Phi}{\Delta t}$, Φ_{ind} and give the sense of circulation for $\mathcal{E}$. Flux is a scalar but it has a sign; we can associate the signs of our flux variables with the directions of the normals to the loop. The first step is to identify the circuit loop and choose a positive direction relative to the normal to the loop. Make a table with the first three columns having signs and the fourth having a sense of rotation.

- The sign of Φ is the direction of the field through the loop. If the field is in the same direction as the normal it is positive and negative otherwise.
- The sign of $\Delta\Phi/\Delta t$ is the same as Φ when the flux is increasing, and opposite to Φ when the flux is decreasing.
- By Lenz's law, the sign of Φ_{ind} is always opposite to that of $\frac{\Delta\Phi}{\Delta t}$.
- To get the direction of the induced emf and current, $\mathcal{E}$ and I, use the right-hand rule to determine which sense current gives the induced flux.

Example E.3 - Polarity of Motional EMF

When we used Faraday's law to find the EMF of the translating rod, we only found its magnitude. In the motional EMF analysis of the same problem we saw we induced a counterclockwise voltage and current. Use Lenz's law to get this same result.

Solution

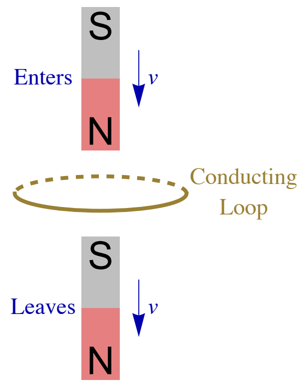


Follow the procedure outlined above. Here we will choose the positive direction to be out of the page and this corresponds to a counterclockwise sense of rotation. Since the field is opposite the normal we have the flux as negative. The flux is increasing (more field lines pass through with time) so the sign of the derivative of the flux is the same as the flux, negative. (it is becoming increasingly negative.) The induced flux is always opposite the change in the flux, so it is positive. That corresponds to a counterclockwise current and EMF.

Positive normal	Sign of Φ	Sign of $d\Phi/dt$	Sign of Φ_{ind}	Sense of $\mathcal{E}$ or I
•	−	−	+	↺

Example E.4 - Magnets Falling through Conducting Loops

(a) A magnet with its north pole at the bottom is dropped through a horizontal conducting loop. What is the sense of the induced current as the magnet enters and then leaves? Answer clockwise or counterclockwise as viewed from above.



Solution

The field of the magnet points away from the north pole and toward the south pole. Choose the positive normal to be upwards.

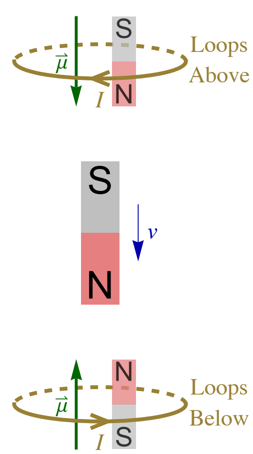
As the magnet enters the field is downward (away from the north pole) so the flux will have a negative sign. The flux is increasing as the magnet enters, since more lines pass through with time; this means that the sign of $d\Phi/dt$ is the same and negative. (The flux is becoming increasingly negative.) The induced flux, by Lenz's law, is always opposite $d\Phi/dt$, so it is positive and this corresponds to a counterclockwise induced current.

As the magnet leaves the field is downward (toward the south pole) as well, pointing toward the south pole. Here the number of field lines is decreasing so $d\Phi/dt$ is opposite Φ and positive. This gives a negative induced flux and a clockwise induced current.

Positive normal		Sign of Φ	Sign of $\Delta\Phi/\Delta t$	Sign of Φ_{ind}	Sense of $\mathcal{E}$ or I
$\uparrow$	Enters	−	−	+	counterclockwise
	Leaves	−	+	−	clockwise

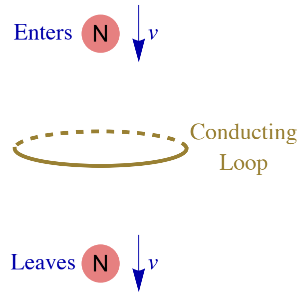
(b) A magnet with its north pole at the bottom is dropped through a long vertical aluminum (conducting but not magnetic) pipe. The falling magnets induce circular eddy currents in the pipe and energy is lost to the Joule-heating of the currents, slowing the magnet's descent through the aluminum tube. Identify the induced magnetic moments from the loops above and below the falling magnetic when inside the tube.

Solution



From the analysis in part (a) we can identify the induced currents and then the induced magnetic moments due to the falling magnet. Loops below the magnet correspond to the case where the magnet enters the loop in part (a). The induced current is counterclockwise and the corresponding magnetic moment is, by the right-hand rule, upwards; since an upward magnetic moment is equivalent to a magnet with the north pole on the top, we can see that induced magnetic moments of the loops below push upward on the falling magnet. For the loops above, this now corresponds to the case where the magnet leaves the loop. Now the induced current is clockwise giving a downward magnetic moment which is equivalent to a bar magnet with the north pole on the bottom; this induced magnetic moment will then pull upwards on the falling magnet below. It is now clear that all the induced currents slow the falling magnet. This must be the case; energy is being lost to Joule heating.

(c) Suppose instead that an isolated north pole were discovered. How would the results in part (a) change is an isolated north pole passed through.



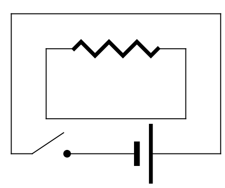
Solution

The case of the pole entering the loop is the same as when the bar magnet enters. As the pole leaves the field is now upwards and decreasing; this gives a positive induced flux and a counterclockwise induced current.

Positive normal		Sign of Φ	Sign of $\Delta\Phi/\Delta\ t$	Sign of Φ_{ind}	Sense of $\mathcal{E}$ or I
↑	Enters	−	−	+	counterclockwise
	Leaves	+	−	+	counterclockwise

This shows an experiment to look for isolated magnetic poles. A superconducting loop is monitored for currents. If a spontaneous current is set up in the loop then that is evidence that an isolate poles passed through the loop. When a dipole passes through the induced currents cancel giving no net effect.

Example E.5 - Induced Current around an Inner Loop Due to a Changing Current in an Outer Loop



While the switch is closed there is a steady counterclockwise current in the outer loop and this creates a magnetic field, by the right-hand rule, that is out of the page. This steady current creates a steady flux through the inner loop and thus, doesn't induce a current in the inner loop. But when the switch is closed, at that instant, there is an abrupt increase in the current and when the switch is opened there is an abrupt drop in the current at that instant.

What is the direction of the induced current through the resistor when the switch is closed and then when the switch is opened?

Solution

The clockwise current in the outer loop creates an outward field in the inner loop, so choosing the positive normal to be outward then the flux is positive in both cases. When the switch is closed, the current increases so the flux increases and when the switch is opened the current decreases and the flux decreases.

Positive normal		Sign of Φ	Sign of $\Delta\Phi/\Delta\ t$	Sign of Φ_{ind}	Sense of $\mathcal{E}$ or I
•	Closed	+	+	−	clockwise $\Longrightarrow$ Current to the right through R
	Opened	+	−	+	counterclockwise $\Longrightarrow$ Current to the left through R

E.5 - Inductance

(Self) Inductance

As a consequence of Faraday's law a changing current through one coil induces an EMF in another coil; this is known as mutual inductance. Similarly, a changing flux in a coil induces an EMF in the same coil; this is self inductance and a circuit component with inductance is called an inductor.

A changing current through a coil can induce an EMF (voltage) across the coil. This is called self inductance. When we use the term inductance by itself self inductance is implied. A current through a coil creates a field and that causes a flux through the coil itself.

$$I \implies \vec{B} \implies \Phi$$

A changing current creates a changing flux with induces an EMF in the coil.

$$\frac{\Delta I}{\Delta t} \implies \frac{\Delta \Phi}{\Delta t} \implies \mathcal{E}$$

The above relationships are proportionalities. The constant of proportionality is defined as the inductance L.

$$\mathcal{E} = -L\,\frac{\Delta I}{\Delta t} \quad (\text{small } \Delta t)$$

The sign in the above expression is due to Lenz's law. Take ΔV to be the change in the voltage when moving through the inductor in the direction of the current. A simple Lenz's law analysis shows that if the current is increasing the voltage change is negative. If we write V as the voltage drop we get

$$\Delta V = -L\,\frac{\Delta I}{\Delta t} \;\text{ and }\; V = L\,\frac{\Delta I}{\Delta t}.$$

The sign conventions for inductors is the same as that for resistors and capacitors, V is the voltage drop when moving with the current.

$$\Delta V = -I\,R \;\text{ and }\; V = I\,R$$

$$\Delta V = -\frac{Q}{C} \;\text{ and }\; V = \frac{Q}{C} \;\text{ where }\; I = \frac{\Delta Q}{\Delta t} \;(\text{for small } \Delta t)$$

Units: The SI unit for Inductance is: henry = H

Inductance of a Long Solenoid

Consider a long solenoid of length ℓ, with N turns and a cross-sectional area A. As before we define n as the number of turns per length $n = N/\ell$. Passing from the current to the field to the flux gives

$$I \implies B = \mu_0\, n\, I \implies \Phi = B\,A = \mu_0\, n\, I\, A$$

and using Faraday's law we get

$$\mathcal{E} = -N\,\frac{\Delta \Phi}{\Delta t} = -N\,\mu_0\, n\, A\,\frac{\Delta I}{\Delta t} \quad (\text{for small } \Delta t).$$

Using $\mathcal{E} = -L\,\Delta I/\Delta t$ (for small Δt), we can read the inductance from this expression

$$L = \mu_0\,\frac{N^2}{\ell}\,A,$$

where we used $n = N/\ell$.

Example E.6 - A Long Solenoid as an Inductor

Consider a solenoid with a circular cross-section with a radius of 3 cm, a length of 75 cm and 200 turns. You may consider the length long compared to the radius.

(a) If the current through the solenoid varies from 25 A to 11 A in 3.5 ms, then what is the magnitude of the average voltage across the solenoid?

Solution

We have the given values.

$$\ell = 0.75\ \mathrm{m},\ N = 200,\ \mu_0 = 4\,\pi \times 10^{-7}\ \frac{\mathrm{N}}{\mathrm{A}^2}\ \text{ and }\ r = 0.030\ \mathrm{m}\ \Longrightarrow\ A = \pi\, r^2 = 0.0028274\ \mathrm{m}^2$$

We can now solve for the inductance.

$$L = \mu_0\ \frac{N^2}{\ell}\ A = 1.8950 \times 10^{-4}\ \mathrm{H}$$

The average EMF is

$$\mathcal{E} = -L\ \frac{\Delta I}{\Delta t}\ \ (\text{small}\ \Delta t)\ \Longrightarrow\ \mathcal{E}_{\text{ave}} = -L\ \frac{\Delta I}{\Delta t}$$

We also have

$$\Delta I = 11\ \mathrm{A} - 25\ \mathrm{A} = -14\ \mathrm{A}\quad \text{and}\quad \Delta\, t = 3.5\ \mathrm{ms} = 3.5 \times 10^{-3}\ \mathrm{s}$$

Taking absolute values for the magnitude we get our answer.

$$|\mathcal{E}_{\text{ave}}| = L \left| \frac{\Delta I}{\Delta t} \right| = 0.758\ \mathrm{V}$$

E.6 - Energy Considerations

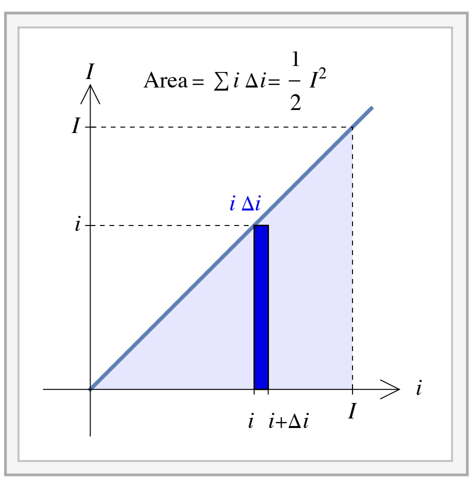
Energy in an Inductor

Inductors, like capacitors, store energy, while resistors dissipate energy. Use U to denote the energy in an inductor. The rate that energy is being stored in an inductor is (for small Δt)

$$\frac{\Delta U}{\Delta t} = \mathcal{P} = I\ V = I\ L\ \frac{\Delta I}{\Delta t}\ \ \Longrightarrow\ \ \Delta U = I\ L\ \Delta\, I$$

Increase the current from 0 to I; call the intermediate current i and then we have $0 \leq i \leq I$ and $\Delta U = L\ i\ \Delta\, i$. Summing over these small ΔU values as i varies from 0 to I gives the total energy.

$$U = L \sum i\ \Delta\ i$$



The diagram above shows that $\sum i\,\Delta i = \frac{1}{2}\,I^2$. We then get the expression for the energy in an inductor.

$$U = \frac{1}{2}\ L\, I^2$$

Example E.7 - A Long Solenoid as an Inductor (Continued)

Using the same solenoid as in the previous problem, with a circular cross-section with a radius of 3 cm, a length of 75 cm and 200 turns. You may consider the length long compared to the radius.

(b) If at some instant, the current through the solenoid is 18 A, then what is the total energy stored in the solenoid?

Solution

Use the same inductance $L = 1.8950 \times 10^{-4}$ H from the previous example and $I = 18$ A we get.

$$U = \frac{1}{2} L I^2 = 0.0307 \text{ J}$$

Energy in a Magnetic Field

In the capacitance chapter we derived an expression for the energy density (Energy/Volume) in an electric field.

$$u = \frac{1}{2} \varepsilon_0 E^2$$

To derive this we used the fact that the electric field is uniform inside a parallel plate capacitor. Combining expressions for the energy in a capacitor and for the capacitance gave the above expression for u. A similar analysis will give the energy density in a magnetic field.

The magnetic field is uniform inside a long solenoid. Combining the expression for the inductance of a long solenoid with the energy in an inductor gives

$$U = \frac{1}{2} L I^2 \text{ and } L = \mu_0 n^2 A \ell \implies U = \frac{1}{2} \mu_0 n^2 A \ell I^2$$

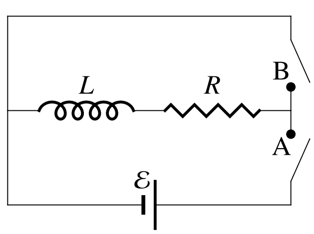
Using $B = \mu_0 n I$ and $u = U / \text{Volume} = U / (A \ell)$ gives the energy density in a magnetic field.

$$u = \frac{1}{2 \mu_0} B^2$$

Example E.8 - Energy Density

What is the energy density in the earth's magnetic field in Bryan Texas, where the magnitude of the field is 47.3 μT.

E.7 - RL Circuits



Decaying Current

Begin with switch **A** closed and **B** opened. This creates a current through the inductor and resistor. Close switch **B** and then open **A**. This causes the current to flow through the top branch of the above circuit. Applying the loop rule around the circuit gives

$$0 = L \frac{\Delta I}{\Delta t} + R I.$$

Rewrite this as

$$\frac{\Delta I}{\Delta t} = -\frac{1}{\tau} I,$$

where the time constant τ is defined by

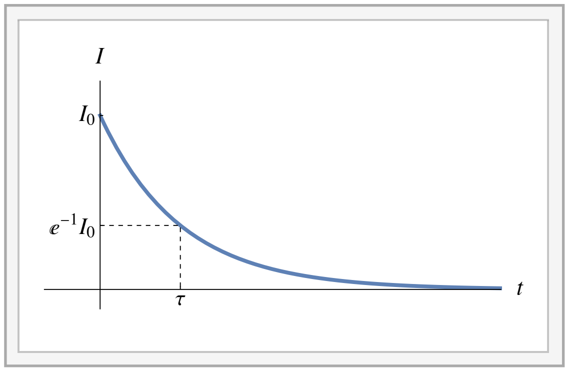
$$\tau = \frac{L}{R}.$$

Take the initial current to be I_0. Solving the equation for t is a calculus problem. We will just write down the solution.

$$I(t) = I_0 \, e^{-t/\tau},$$

This is a simple exponential decay, analogous to the decay of the charge for a discharging capacitor.

The initial energy in the inductor $U = \frac{1}{2} L \, I_0^2$ is converted to heat in the resistor.



Interactive Figure - Current Decay in an RL Circuit

Growing Current

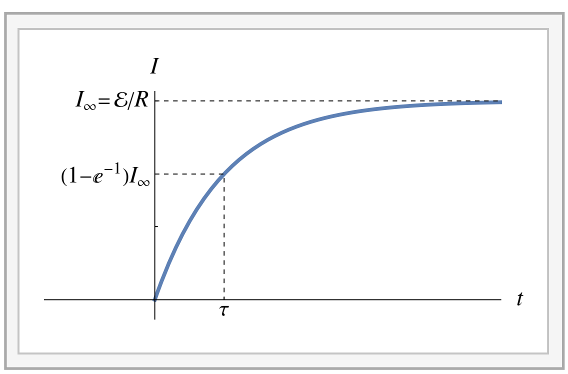
With both switches opened giving zero current, close switch **A** at $t = 0$. This causes the current to gradually build up to a steady-state value. Apply the loop rule to the circuit gives the first order ODE.

$$\mathcal{E} = L \frac{\Delta I}{\Delta t} + R\, I.$$

Using the same time constant τ we can write down the solution where we take the initial current, at time zero, to be zero.

$$I(t) = I_\infty \left(1 - e^{-t/\tau}\right) \text{ where } I_\infty = \frac{\mathcal{E}}{R}$$

is the steady-state current.



Interactive Figure - Current Growth in an RL Circuit

Example E.9 - A Long Solenoid as an Inductor (Continued... More)

Using the same solenoid as in the previous problems, with a circular cross-section with a radius of 3 cm, a length of 75 cm and 200 turns. You may consider the length long compared to the radius. Now suppose the solenoid has a total resistance of 35 mΩ.

(b) If the solenoid is connected across a 12-V battery, then what is the current after a long time:

Solution

Use the same inductance for this solenoid as before. That and the new values are:

$$L = 1.8950 \times 10^{-4}\ \mathrm{H},\ \ \mathcal{E} = 12\ \mathrm{V}\ \ \text{and}\ \ R = 0.85\ \Omega$$

After a long time the current approaches I_∞.

$$I_\infty = \mathcal{E}/R = 14.1\ \mathrm{A}$$

(c) What is the current 0.3 ms after the circuit in part (b) is connected?

Solution

The time constant is

$$\tau = L/R = 0.00022294\ \mathrm{s} = 0.22294\ \mathrm{ms}$$

Using our time of $t = 0.3$ ms and using the formula for current growth we get the current at time t.

$$I(t) = I_\infty \left(1 - e^{-t/\tau}\right) = 10.4\ \mathrm{A}$$

(d) Suppose after a long time when the current has reached the value in part (b), the terminals of the solenoid are connected (shorted) removing the battery from the circuit. What is the current 0.6 ms after this?

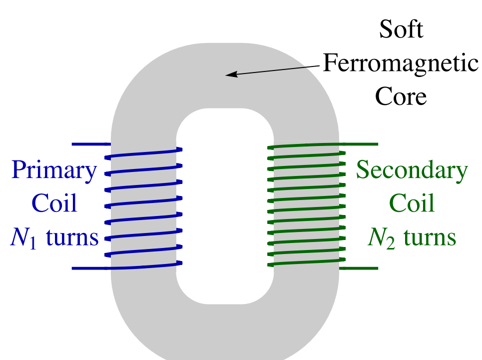
Solution

The time constant is the same as in part (c). Now we have a current decay with $t = 0.6$ ms and $I_0 = I_\infty$ from part (b). Use the current decay formula to get the current.

$$I(t) = I_0\, e^{-t/\tau} = 3.68\ \mathrm{A}$$

E.8 - The Transformer

In addition to self inductance where a changing current through a coil induces an EMF across its terminals, there is mutual inductance where a changing current in one coil induces an EMF in another coil. We will consider only one special case of mutual inductance, the transformer.



A transformer is a case of mutual inductance where, ideally, all of the flux from one coil passes through the other. Take the primary coil to have N_1 turns and the secondary to have N_2. This can be achieved to a good approximation by wrapping both coils around the same soft ferromagnetic core. The core amplifies the field due to the current and directs the field lines around the loop of the core

The fluxes are assumed to be equal.

$$\Phi = \Phi_1 = \Phi_2$$

Then by Faraday's law

$$V_1 = -N_1 \frac{\Delta\Phi}{\Delta t} \text{ and } V_2 = -N_2 \frac{\Delta\Phi}{\Delta t}$$

we get a proportionality between the voltage and number of turns.

$$\frac{V_2}{V_1} = \frac{N_2}{N_1}$$

Take the voltages to be the rms voltages then we see that the effect of the transformer is to vary the voltage. A step-up transformer increases the voltage $V_2 > V_1$ and a step-down transformer decreases the voltage $V_2 < V_1$. If we consider the power in the circuit then the instantaneous power is $\mathcal{P} = V\, I$, so it follows that there is an inverse proportionality between the voltage and current.

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2}$$

Transformers are very common. Many transformers have standard household voltage as its input and DC output at a different voltage. After the transformer a *rectifier circuit* is used to convert the AC output of the transformer to DC.