Chapter G

Electromagnetic Waves

Blinn College - Physics 1402 - Terry Honan

G.1 - The Nature of Electromagnetic Radiation

The Discoveries

With Faraday's Law, we saw that a changing magnetic field can produce an EMF, an electromotive force. It turns out that this EMF can be written in terms an induced electric field; that field pushes the charges around a circuit causing the EMF. James Clerk Maxwell then showed that in addition to changing magnetic fields producing electric fields, changing electric fields can produce magnetic fields. This makes electromagnetic radiation possible. Sinusoidally varying electric field induce sinusoidally varying magnetic fields, which in turn, produce sinusoidally varying electric fields. This arrangement then propagates (in a vacuum) at the speed of light.

In 1864, Maxwell showed mathematically that the equations describing electromagnetism give rise to wave solutions and the speed of those waves can be described in terms of our electromagnetic constants ε_0 and μ_0.

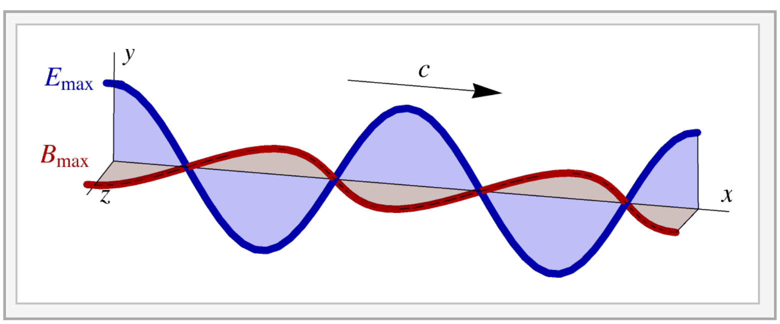
$$c = \frac{1}{\sqrt{\mu_0\,\varepsilon_0}}.$$

Maxwell then realized that the speed of these waves was the previously measured value of the speed of light in a vacuum. In addition to explaining light, Maxwell predicted the rest of the electromagnetic spectrum.

After Maxwell in 1887, Heinrich Hertz was able to show how to produce and receive these electromagnetic waves in a laboratory. Guglielmo Marconi was able to find a practical application of Maxwell's and Hertz's discoveries, using them for communication. In 1901 Marconi sent Morse code as a radio signal across the Atlantic. This became the basis of modern wireless communications.

The Propagating Fields

In an electromagnetic wave the electric and magnetic field are perpendicular and both are perpendicular to the direction of wave propagation. If we choose the direction of the electric fields to be in the y-direction and the magnetic fields to be in the z-direction then the wave propagates at the speed of light in the x-direction.



There is a right-hand rule to get the direction of wave propagation: Put your thumb in the direction of $\vec{E}$ and your fingers in the direction of $\vec{B}$ then your palm is in the direction of wave propagation. Note that if you look at any x position the electric field varies as a sinusoidal function, like what we studied in the previous chapter.

Example G.1 - Directions of Fields

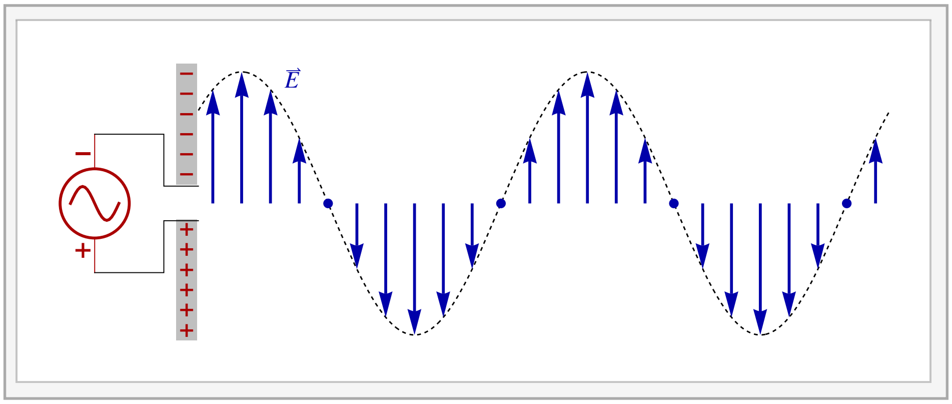
An electromagnetic wave propagates in the z-direction. If the electric field is in the y-direction, then what is the direction of the magnetic field?

Solution

Using the right-hand rule, put your thumb in the direction of the electric field, the y-direction and rotate your hand so that the palm points to the z-direction. Your fingers are then in the negative-x direction; that is then the direction of the magnetic field. (Note that your coordinate system must be right-handed; if your thumb is in the x-direction and fingers in the y-direction then your palm is in the z-direction.)

Producing and Receiving Electromagnetic Waves

Accelerating electric charges are the ultimate source of electromagnetic waves. To produce waves consider an AC voltage source connected across two separated conductors, shown as gray bars in the diagram below. As the polarity of the source varies the charges on the conductors varies and this creates a sinusoidally varying electric field. These fields then propagate as a wave.



This, essentially, is a broadcasting antenna.

A receiving antenna is basically the same thing in reverse. An incoming electromagnetic wave creates a very small sinusoidal voltage across an antenna and that small voltage can be amplified into a signal. Radio waves hit an antenna at all frequencies. A simple resonance circuit selects for the correct frequency. The LC Resonance frequency is $f = 1/\left(2\pi\,\sqrt{LC}\right)$. Although modern radio tuners use complexe solid state circuitry, the knob to adjust frequency on an old radio tuner is simply a variable capacitor that tunes for the correct f in an LC circuit.

G.2 - The Speed of Light and the Electromagnetic Spectrum

The Speed of Light

The speed of light in a vacuum is to three digits

$$c = 3.00\times 10^{8}\ \mathrm{m/s}.$$

Because of its most fundamental nature we can choose our units so that c have the exact value

$$c = 2.99792458\times 10^{8}\ \mathrm{m/s}.$$

The definition of a second is given in terms of our most accurate method of measuring time, with atomic clocks, and the above definition of c then gives a definition of the meter.

The first documented attempt to measure the speed of light was due to Galileo. He and an assistant both had lanterns they could cover and they were separated by a large distance. When the distant assistant saw Galileo cover his lantern, he covered his. Galileo then looked for a time delay. Of course, light is much too fast for that, but that is only obvious to us because we know light is very fast. A similar set-up could measure the speed of sound. By viewing light as something physical that traveled at a finite speed, Galileo showed great insight.

Historically, the first reasonable estimate of the speed of light was made by Roemer. By plotting the period of Io, one of Jupiter's moons, Roemer was able to explain the lack of periodicity in Io's orbit in terms of the time difference for light from Io to reach the Earth as the relative distance between Jupiter and Earth varies as both are in different positon relative to the sun in their orbits.

An accurate measure of c was first found by Fizeau. By shining a light through a rapidly rotating toothed wheel and reflecting it off a distant mirror, he could essentially redo Galileo’s experiment with very short time intervals and large distances.

The speed of light is a fundamental speed limit. It is impossible to send any information faster than c.

Example G.2 - Time of Travel for Light

(a) The earth-sun distance is

$$R_{ES} = 1.50 \times 10^{11} \text{ m}$$

How long does it take for a light signal to travel from the sun to the earth?

Solution

Since $d = v\,t = c\,t$ we can simply solve for t.

$$t = \frac{d}{c} = \frac{R_{ES}}{c} = 500 \text{ s} = 8.33 \text{ min}$$

With the speed of light as the ultimate speed limit this means that we cannot know what is happening on the sun now, only what happened over eight minutes ago.

(b) The earth-moon distance is

$$R_{EM} = 3.84 \times 10^{8} \text{ m}$$

How long does it take for a light signal to travel from the moon to the earth?

Solution

$$t = \frac{d}{c} = \frac{R_{EM}}{c} = 1.28 \text{ s}$$

There was a very noticeable time delay when astronauts were on the moon. It took over two and a half seconds for a radio signal to get from the earth to the moon and back.

(c) How far does light travel in a nano-second?

Solution

$$t = 10^{-9} \text{ s} \implies d = c\,t = 0.3 \text{ m} \simeq 1 \text{foot}$$

We think of the speed of light as a significant communication obstacle when dealing with large distances, but with computers time scales are on the order of nano-seconds so the speed of light is also significant. As computers get faster, they must necessarily get smaller.

(d) A light-year ly is the distance light travels in one year. What is this in meters?

Solution

$$1 \text{ ly} = c \times (1 \text{ yr}) = 3.00 \times 10^{8} \frac{\text{m}}{\text{s}} \times 365.24 \text{ days} \times \frac{24 \text{ h}}{\text{day}} \times \frac{3600 \text{ s}}{\text{h}} = 9.47 \times 10^{15} \text{ m}$$

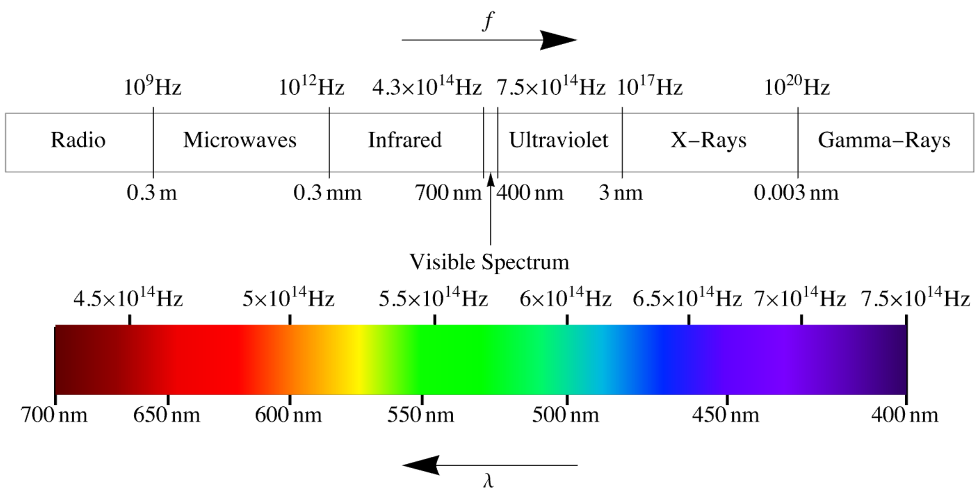
The Electromagnetic Spectrum

The Full Spectrum

Electromagnetic radiation can be described by its wavelength and frequency. Since frequency and wavelength are related by

$$f\,\lambda = c$$

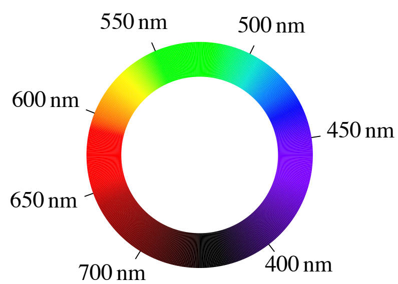
for radiation in a vacuum, it follows that all electromagnetic radiation can be written along a line of increasing frequency and decreasing wavelength.



Long wavelength waves are radio waves. Wavelengths shorter than around 0.3 meter are usually labeled microwaves. Smaller than about 1 mm start the infrared (IR) waves. At 700nm we get into the visible spectrum which is a narrow band of wavelengths down to 400 nm. Below 400 nm to around 3 nm are the ultraviolet (UV) waves. Wavelengths smaller than that are called X-rays and the small wavelength limit, beyond around 0.003 nm, are called gamma rays.

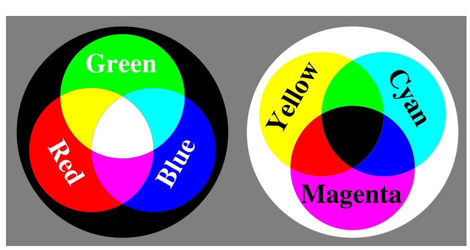
The Visible Spectrum and Primary Colors

The visible spectrum consists of the narrow band of wavelengths between 400 and 700 nm. In order of decreasing wavelength (increasing frequency) we have ROYGBIV: Red, Orange, Yellow, Green, Blue, Indigo and Violet. There is approximately a factor of two of wavelengths (and frequencies) we can see; this is crucial to our perception of light. With sound we can hear many octaves, where going up an octave is doubling a frequency. As you go through the "do-re-mi" scale from the low "do" to the high "do", you jump up an octave, doubling the frequency. With light we can see the equivalent of only one octave of light. Our brain matches the ends of the visible spectrum into a circle; violet appears as a reddish blue. If we perceived sound similarly, the low "do" and high "do" would sound the same.



Notice that the visible spectrum is not quite a factor of two of wavelengths and frequencies. There is a missing color, magenta. Although we may perceive magenta as a combinations of other colors, there is no pure color, meaning color of a single wavelength, corresponding to magenta.

Matching the ends of the visible spectrum together makes it possible to represent colors as combinations of three primary colors. There are two notions of primary colors: additive mixing and subtractive mixing. Additive mixing is used with computer monitors and television screens. We begin with black and add colors. The additive primary colors are Red, Green and Blue. Combining all three we can get white as shown. Subtractive mixing is where we begin with white and remove colors. This is used when mixing paints or for color printers. Here the primary colors are Cyan (a blue-green color), Magenta (a reddish violet) and Yellow. Removing all three subtractive colors gives black.



Additive Mixing Subtractive Mixing

Example G.3 - Frequencies and Wavelengths

(a) Modern microwave ovens operate at a frequency of 2450 MHz. What is the wavelength of the radiation?

Solution

$$c = 3.00 \times 10^8 \ \frac{\text{m}}{\text{s}} \ \text{ and } \ f = 2450 \text{ MHz} = 2450 \times 10^6 \text{ Hz} \quad \Longrightarrow \quad \lambda = \frac{c}{f} = 0.122 \text{ m} = 12.2 \text{ cm}$$

There is a mesh on the door of all microwaves to keep the microwaves inside. But we can see inside, so clearly some electromagnetic radiation passes through. Why can light pass through and microwaves not? This has to do with wavelengths. The wavelength of the microwaves is much larger than the spacing on the mesh, so the waves are reflected back as if the mesh were a uniform conducting sheet. The spacing on the mesh is much larger than the wavelength of visible light so that passes through easily.

(b) Radio stations identify by their frequency, FM stations give their frequency in MHz, ranging from 87.5 to 108 MHz. AM stations are in kHz, between 525 and 1705 kHz. What are the wavelengths of an FM station at 90.9 MHz and an AM station at 1620 kHz.

Solution

$$f = 90.9 \times 10^6 \text{ Hz} \quad \Longrightarrow \quad \lambda = \frac{c}{f} = 3.30 \text{ m} \ \text{ and } \ f = 1705 \times 10^3 \text{ Hz} \quad \Longrightarrow \quad \lambda = \frac{c}{f} = 176 \text{ m}$$

(c) What is the frequency of an X-ray with a wavelength of 0.030 nm?

Solution

$$\lambda = 0.020 \times 10^{-9} \text{ m} \quad \Longrightarrow \quad f = \frac{c}{\lambda} = 1.5 \times 10^{19} \text{ Hz}$$

The Doppler Effect

We have all observed the Doppler effect; when a train is blowing its horn or a police car blows its siren while moving toward you, the frequency is increased and when moving away it is decreased. This is is called the Doppler effect and it occurs for all waves. When the relative motion between you and some wave source is toward each other, either the source is moving toward you or you toward the source, the observed frequency of the wave increases. When the the relative motion is away, the frequency decreases.

For electromagnetic waves, as long as the relative speed u is mush less that the speed of light c (which we write $v << \text{c}$) we get

$$f' = f\left(1 \pm \frac{u}{c}\right) \text{ or } \Delta f = f' - f = \pm f \ \frac{u}{c} \quad \text{for } u << \text{c}$$

where f is the original frequency, f' is the shifted frequency and $\Delta f = f' - f$ is the frequency shift. We will see a corrected formula for this in the Relativity chapter that will apply as the relative speed is larger, approaching c.

When a wave reflects off a moving target, there is a frequency shift Δf between the source and the target and another Δf shift in the returning waves giving a double shift.

$$\Delta f_{\text{tot}} = 2\,\Delta f = \pm 2\, f \ \frac{u}{c} \quad \text{(reflection from a moving target)} \ \text{ for } u << \text{c}$$

Example G.4 - Doppler Effect for a Jet

A jet moves at 325 m/s relative to a radio source broadcasting with a frequency of 475 MHz. What is the observed frequency shift to the jet when it is moving toward the source? What if when moving away?

Solution

$$f = 475 \times 10^6 \text{ Hz}\,, \quad u = 325 \ \frac{\text{m}}{\text{s}} \ \text{ and } \ c = 3.00 \times 10^8 \ \frac{\text{m}}{\text{s}} \quad \Longrightarrow \quad \Delta f = f' - f = \pm f \ \frac{u}{c} = \pm 515 \text{ Hz}$$

When moving toward the source, the Δf is positive, an increase, and when moving away it is negative and thus a decrease.

Example G.5 - A Police Radar Gun

Police radar uses the Doppler shift on reflected radio waves to measure the speed of a car. If the frequency of the source, the radar gun, is 10.5 GHz then what is the frequency shift when reflected off a car moving at 90 mi/h”

Solution

$$f = 10.5\times 10^9\ \mathrm{Hz}\ ,\quad u = 90\ \frac{\mathrm{mi}}{\mathrm{h}} \times \frac{1609\ \mathrm{m}}{\mathrm{mi}} \times \frac{\mathrm{h}}{3600\ \mathrm{s}} = 40.225\ \frac{\mathrm{m}}{\mathrm{s}} \implies \Delta f_{\mathrm{tot}} = 2\,\Delta f = \pm 2\, f\ \frac{u}{c} = \pm 2820\ \mathrm{Hz}$$

When the car is moving toward the radar gun, Δf_{tot} is positive and when moving away it is negative.

G.3 - Energy and Momentum in Electromagnetic Radiation

Energy and Intensity

Energy Density

The energy density u in an electromagnetic field can be written as a sum over electric and magnetic contributions

$$u = u_E + u_B = \frac{1}{2}\ \varepsilon_0\, E^2 + \frac{1}{2\,\mu_0}\ B^2$$

We can find the average energy density u_{ave} using the fact that the fields are sinusoidal and that the average of $\sin^2$ is $\frac{1}{2}$.

$$u_{\mathrm{ave}} = u_{E,\mathrm{ave}} + u_{B,\mathrm{ave}} = \frac{1}{2}\ \varepsilon_0\ \frac{1}{2}\ E_{\max}^2 + \frac{1}{2\,\mu_0}\ \frac{1}{2}\ B_{\max}^2$$

Because of the symmetry between electric and magnetic fields in electromagnetism, the electric and magnetic contributions are equal, $u_{E,\mathrm{ave}} = u_{B,\mathrm{ave}}$.

It follows that

$$u_{\mathrm{ave}} = \frac{\varepsilon_0}{2}\ E_{\max}^2 = \frac{B_{\max}^2}{2\,\mu_0} = \varepsilon_0\, E_{\mathrm{rms}}^2 = \frac{B_{\mathrm{rms}}^2}{\mu_0}$$

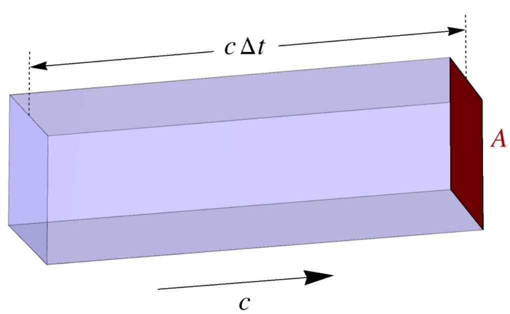
where we used the relationship between rms and peak quantities for any sinusoidal function: $E_{\mathrm{rms}} = E_{\max} / \sqrt{2}$ and $B_{\mathrm{rms}} = B_{\max} / \sqrt{2}$

$$c = \frac{1}{\sqrt{\mu_0\,\varepsilon_0}} = \frac{E_{\max}}{B_{\max}} = \frac{E_{\mathrm{rms}}}{B_{\mathrm{rms}}} = \frac{E}{B}$$

Intensity

For light, intensity is a measure of its brightness. We will define intensity for all electromagnetic radiation. Intensity I is a measure of the power (energy/time) per area. If A is the area of a surface normal to the radiation, Δt is a time, U is the energy passing through the surface and $\mathcal{P}$ is the power $(U/\Delta t)$ through the surface, then these are related by

$$I = \frac{\mathcal{P}}{A} = \frac{U}{A\,\Delta t}.$$



Consider all the radiation in the right cylinder (with any shape cross-section, even rectangular) and length $c\Delta t$. Since all the radiation, and thus all the energy, is flowing at speed c, it follows that all the energy in the cylinder passes A in Δt. The volume of the cylinder is $Ac\Delta t$, so

$$U = u_{\mathrm{ave}}\ A\ c\ \Delta t \ \text{ and } \ I = \frac{\mathcal{P}}{A} = \frac{U}{A\,\Delta t} = u_{\mathrm{ave}}\ c$$

This gives several equivalent expressions for the intensity

$$I = u_{\text{ave}}\; c = \frac{\varepsilon_0\, c}{2}\, E_{\max}^2 = \varepsilon_0\, c\, E_{\text{rms}}^2$$

Example G.6 - The Intensity of Sunlight

The intensity of sunlight drops off with distance. At the earth's distance from the sun of $R_{\text{ES}} = 1.50 \times 10^{11}$ m the intensity of sunlight is $1370\ \text{W}/\text{m}^2$.

(a) What are the rms electric and magnetic fields (E_{rms} and B_{rms}) in this radiation?

Solution

The intensity is given. The constants we need are ε_0 and c.

$$\varepsilon_0 = 8.85 \times 10^{-12}\ \frac{\text{C}}{\text{N}\cdot\text{m}^2}\ ,\ c = 3.00 \times 10^8\ \text{m}/\text{s}\ \text{ and }\ I = 1370\ \frac{\text{W}}{\text{m}^2}$$

Using the formula for the intensity in terms of the rms electric field we can solve for E_{rms}.

$$I = \varepsilon_0\, c\, E_{\text{rms}}^2 \implies E_{\text{rms}} = \sqrt{\frac{I}{\varepsilon_0\, c}} = 718\ \frac{\text{V}}{\text{m}}$$

To find the rms magnetic field we know that the electric and magnetic fields are proportional.

$$\frac{E_{\text{rms}}}{B_{\text{rms}}} = c \implies B_{\text{rms}} = \frac{E_{\text{rms}}}{c} = 2.39 \times 10^{-6}\ \text{T}$$

(b) What is the total power (energy/time) radiated by the sun?

Solution

Intensity is power per area, $I = \mathcal{P}/A$. To find the correct area you must visualize how the energy is distributed. For the sun, the light that leaves at some instant is, at a later time, spread over the surface of a sphere. The relevant area here is the surface area of a sphere, $A = 4\,\pi\, r^2$.

$$r = R_{\text{ES}} = 1.50 \times 10^{11}\ \text{m}$$

We can now solve for the total power.

$$I = \frac{\mathcal{P}}{A} = \frac{\mathcal{P}}{4\,\pi\, r^2} = \frac{\mathcal{P}}{4\,\pi\, R_{\text{ES}}^2} \implies \mathcal{P} = 4\,\pi\, R_{\text{ES}}^2\, I = 3.87 \times 10^{26}\ \text{W}$$

Example G.7 - Laser Light

A 35-mW laser has a beam with a diameter of 2.5 mm. What is the intensity of the laser light and what is the rms electric field in the beam? (The power rating of the laser is the total power (energy/time) of light produced, $\mathcal{P} = 35$ mW.

Solution

The diameter will give us the area and the that with the power will give the intensity. We can then find E_{rms}.

$$A = \pi\, r^2 = \pi \left(\frac{0.0025\ \text{m}}{2}\right)^2 = 4.9087 \times 10^{-6}\ \text{m}^2$$

$$\mathcal{P} = 35\ \text{mW}\ \text{ and }\ I = \frac{\mathcal{P}}{A} = 7130\ \frac{\text{W}}{\text{m}^2}$$

Why are we using $\pi\, r^2$ here and $4\,\pi\, r^2$ in the previous problem? To get the correct area you must consider how the energy is distributed. The energy that leaves the laser at one instant is spread over the circular cross-section of the beam at a later instant, so here the area of a circle is correct.

From the intensity we can then find E_{rms} as we did in the previous problem.

$$I = \varepsilon_0\, c\, E_{\text{rms}}^2 \implies E_{\text{rms}} = \sqrt{\frac{I}{\varepsilon_0\, c}} = 1640\ \frac{\text{V}}{\text{m}}$$

Momentum and Pressure

Momentum Carried by Radiation

Electromagnetic radiation carries energy. Since there is moving energy there is also momentum. To see this consider Einstein's famous formula $E = m c^2$. This was shown to be generally true in 1905, but in the case of electromagnetism an analogous expression had been derived previously. We are using U for energy, so let us write this as $U = m c^2$. Since momentum is $p = m v$ and the radiation is moving at c we can write $p = m c$. Combining these expressions we get

$$p = \frac{U}{c}.$$

This is the momentum carried by electromagnetic radiation.

Some comments should be made on the mass referred to above. The mass in $E = m c^2$ is known as the *relativistic mass*. This is to be distinguished from the *rest mass*. The tabulated values of the masses of particles are their rest masses. In relativity, a particle with a rest mass can never be accelerated to the speed of light, but it can reach a speed arbitrarily close to that of light. A particle of light is known as a photon; this is called a massless particle meaning that it has no rest mass. Massless particles must always move at c.

Momentum from Radiation Normally Incident on a Surface

If the radiation is normally incident on a surface we can derive simple expressions for the momentum given to the surface. First consider the case of a surface that is a perfect absorber. All of the momentum of the radiation is given to the surface, giving

$$p = \frac{U}{c} \quad \text{(perfect absorber)}.$$

If the surface is a perfect reflector then the change in the momentum of the radiation is twice the value of the incident radiation. Recall that momentum is a vector and here we are subtracting two vectors in the opposite direction. Since momentum must be conserved, the change in the momentum of the radiation is equal (in magnitude) to the momentum given to the surface.

$$p = 2\,\frac{U}{c} \quad \text{(perfect reflector)}$$

Pressure and Force on a Surface from Normally Incident Radiation

Newton's second law $\vec{F}_{\text{net}} = \frac{\Delta}{\Delta t}\,\vec{p}$ relates force to momentum. The force can be related to the momentum in the case of normally incident radiation by the expression

$$F = \frac{p}{\Delta t}.$$

Pressure is defined as force per area.

$$\text{Pressure} = \frac{F}{A}.$$

We can then write the pressure Pressure in terms of the momentum p.

$$\text{Pressure} = \frac{p}{A\,\Delta t}$$

Using the definition of intensity

$$I = \frac{\mathcal{P}}{A} = \frac{U}{A\,\Delta t}.$$

We can turn the momentum expressions, which involve the energy U into expressions for the pressure involving the intensity I by dividing both sides of the momentum expressions by $A\Delta$t.

$$\text{Pressure} = \frac{I}{c} \quad \text{(perfect absorber)}$$

$$\text{Pressure} = 2\,\frac{I}{c} \quad \text{(perfect reflector)}$$

In the cases of both pressure on a surface and momentum given to a surface, the effect is very small. to get both pressure and momentum we divide by the speed of light and this will typically give a small value. Both effects are difficult to notice and it is hard to find a practical application for them.

Example G.8 - Pushing with a Laser

In space, a laser is used to push a small rocket. If a 200-mW laser targets this rocket and hits it perpendicularly to the a perfectly reflecting surface, then after one year, how much momentum has the small rocket acquired?

Solution

Power is energy/time so from the power of the laser we can find the total energy in one year.

$$\mathcal{P} = 200\text{ mW} \implies U = \mathcal{P}\,t \;= \mathcal{P}\times 1\text{ yr} = \mathcal{P}\times 365.24\text{ days}\times\frac{24\text{ h}}{\text{day}}\times\frac{3600\text{ s}}{\text{h}} = 262\,970\text{ J}$$

Although this seems like a lot of energy, to get the momentum we divide by the speed of light and get a tiny, tiny momentum. Here we have a perfect reflector.

$$p = 2\,\frac{U}{c} = 0.00175\text{ kg}\cdot\text{m/s}$$

Example G.9 - Radiation Force on the Earth

The radius of the earth is 6.37×10^6 m and the intensity of sunlight at the earth is $1370\text{ W}/\text{m}^2$. Assuming the earth is a perfect absorber, then what is the force of the sun's radiation on the earth?

Solution

Here the intensity will allow us to calculate the pressure. The pressure multiplied by the area will give us the force. First let us find the pressure. For a perfect absorber we have

$$\text{Pressure} = \frac{I}{c} = 4.5667\times 10^{-6}\text{ Pa}$$

Compare this number with atmospheric pressure which is around 10^5 Pa = 10^5 pascals.

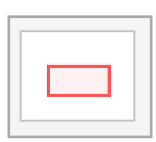
To find the force we need the area. Although the sunlight is hitting half the spherical surface of the earth, we do not want half the area of a sphere here. Our formulas are for normally incident radiation and the area of the earth perpendicular to the sunlight is πR_E^2. What matters here is how much sunlight the earth absorbs and πR_E^2 is the area of the shadow behind the earth.

$$R_E = 6.37\times 10^6\text{ m} \implies F = \text{Pressure}\times A = \text{Pressure}\times\pi R_E^2 = 5.82\times 10^8\text{ N}$$

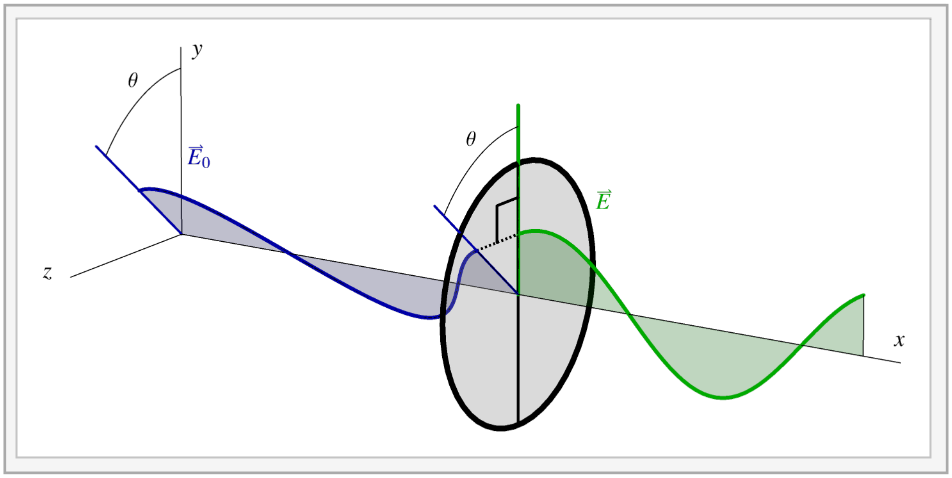
Although this may seem like a large number, remember that it is acting on a planet. Comparing this with the gravitational force of the sun on the earth, it would then be totally negligible.

G.4 - Polarization

Recall from section 25.1 that the electric field and magnetic field were perpendicular and both were perpendicular to the direction of propagation. There we took x to be the direction of propagation and the electric field was in the y-direction and the magnetic field was in the z-direction. However, there is a plane of possible direction perpendicular to the direction of propagation, which we will keep as x. Rotating the fields in this plane gives different polarizations. We choose the direction of polarization to be the direction of the electric field.



Polarized Light through a Filter



A polarizing filter allows only the component of the electric field along the axis of the filter. The wave that exits the filter is then polarized along the axis of the filter with a smaller amplitude. The amplitude is the peak electric field.

$$A = E_{\text{max}}$$

If A_0 is the amplitude before the filter and A is the amplitude after the filter then

$$A = A_0 \cos\theta$$

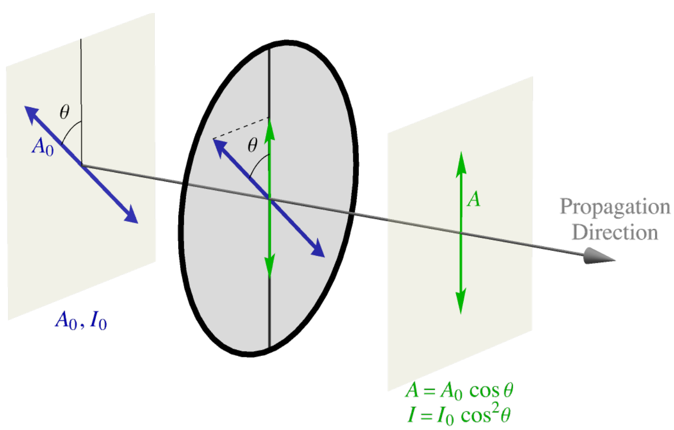
The intensity is proportional to the square of the amplitude $I \propto A^2$, since $I = \frac{1}{2}\,\varepsilon_0\,c\,E_{\text{max}}^2$. Using this we can write the intensity of light after the filter to the intensity before.

$$A = A_0 \cos\theta \text{ and } I \propto A^2 \implies I = I_0 \cos^2\theta$$

Summarizing, when polarized light with intensity I_0 passes through a polarizing filter then it leaves with an intensity of I given by.

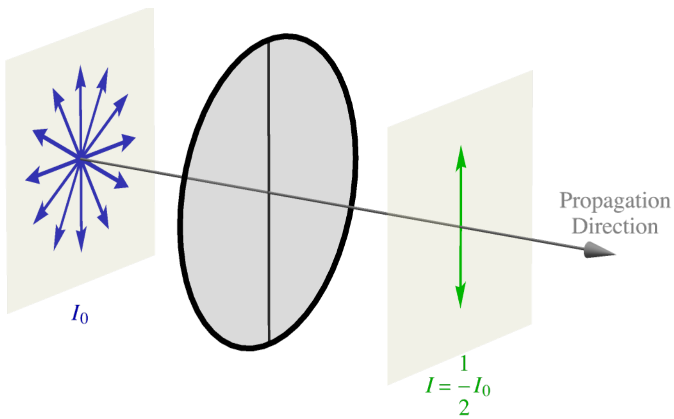
$$I = I_0 \cos^2\theta$$

where θ is the angle between the angle of polarization of the light and the polarizing axis of the filter. This relation is known as Malus's law.



Polarized light of intensity I_0 passing through a polarizing filter. It leaves polarized along the axis of the filter with the intensity $I = I_0 \cos^2\theta$.

Unpolarized Light through a Filter



Unpolarized light passing through a polarizing filter. It leaves polarized along the axis of the filter with half the intensity.

Viewing normal ambient light through a polarizing filter usually shows no effect when the filter is rotated. This is because the light is unpolarized; this means that it is a random mixture of all polarizations. Given this random mixture, we average over all polarizations. Since the average value of $\cos^2$ is 1/2 we get.

$$I = \frac{1}{2} I_0$$

relating the intensities before and after the filter. The light then leaves the filter polarized along the axis of the filter.

Polarization by Reflection and Scattering

Reflected light tends to be polarized. When light reflects of a surface at some angle (not normally incident) there is one possible polarization direction that is parallel to the surface and the reflected light tends to be polarized in this parallel direction. For instance, light reflecting off a horizontal surface tends to be horizontally polarized. Polarizing sunglasses have filters with vertical axes to remove this reflected light or glare. By saying “tends to be polarized” I mean that the light has more of that polarization than the other.

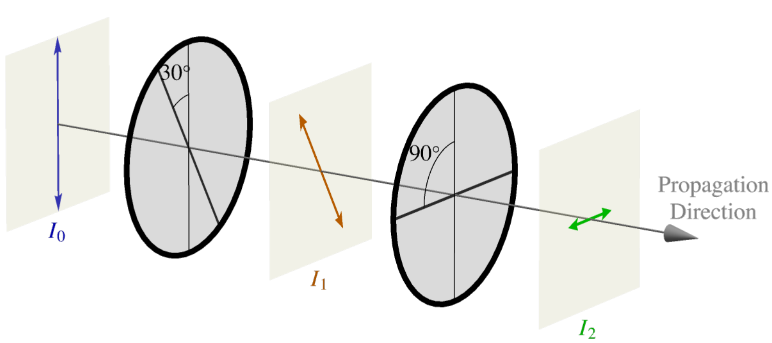
We see the sky because light is scattered. You can view this as light bouncing off air molecules. Light from the sun scatters off an air molecule toward you. The light rays from the sun to the air molecules and from the air molecules to you for a plane and the light tends to be polarized perpendicular to that plane. When you look at the air with the sun behind you, then the light from the sun to the molecules define a vertical plane, and the scattered light then tends to be horizontally polarized. While discussing scattering by air, it should be mentioned that blue light scatters more than the other colors and that is why the sky is blue.

Example G.10 - Light through Two Polarizing Filters

Light with an intensity of $2000\ \mathrm{W/m^2}$ passes through two polarizing filters, the first has an axis at an angle of $30°$ from vertical and the second has a horizontal ($90°$ from vertical) axis.

(a) If the initial light is vertically polarized, then what is the intensity of the light between the filters and after the filters.

Solution



Take $I_0 = 2000\ \mathrm{W/m^2}$ to be the intensity of the light before the filters.

$$I = I_0 \cos^2 \theta$$

Label the intensity between to be I_1 and after to be I_2. The angle between the polarization of the incoming light and the first filter is $\theta_1 = 30°$, so we get

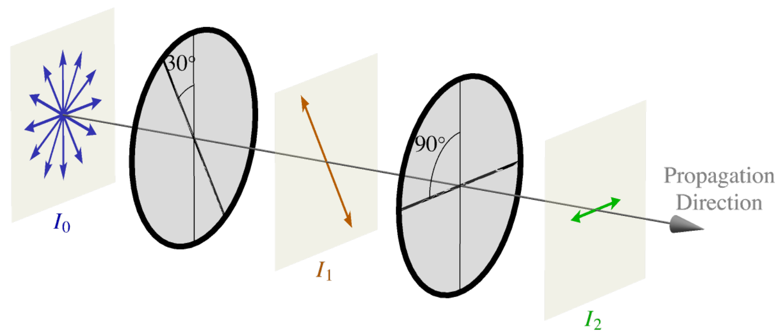
$$I_1 = I_0 \cos^2 \theta_1 = 1500 \text{ W}/\text{m}^2$$

The light between the filters is polarized at θ_1. Take the polarizing angle of the second filter to be $\theta_2 = 90°$. It follows that the angle between the polarization angle of the light hitting the second filter and the axis of the second filter is $\theta_2 - \theta_1 = 60°$. We can then find the intensity after the second filter.

$$I_2 = I_1 \cos^2(\theta_2 - \theta_1) = 375 \text{ W}/\text{m}^2$$

(b) If the initial light is unpolarized, then what is the intensity of the light between the filters and after the filters.

Solution



Now we have the same I_0 before the filters but when unpolarized light passes through a filter it leave with half the intensity and polarized along the axis of the filter.

$$I = \frac{1}{2} I_0$$

So now between the filters we have

$$I_1 = \frac{1}{2} I_0 = 1000 \text{ W}/\text{m}^2$$

The light between the filters is polarized at θ_1. Take the polarizing angle of the second filter to be $\theta_2 = 90°$. It follows that the angle between the polarization angle of the light hitting the second filter and the axis of the second filter is $\theta_2 - \theta_1 = 60°$. We can then find the intensity after the second filter.

$$I_2 = I_1 \cos^2(\theta_2 - \theta_1) = 250 \text{ W}/\text{m}^2$$