Chapter I

Interference and Diffraction

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I.1 - Simple Interference

As an electromagnetic radiation passes some position the electric field varies sinusoidally with time.

$$A \cos(\omega\, t + \phi).$$

where the amplitude A is the peak electric field, $A = E_{\max}$. When we combine waves from two sources, we add the electric fields for each wave; this is known as the principle of superposition. Suppose we combine waves from two sources with the same amplitude A_0 and frequency, but with a different relative phase angle ϕ. To do this we just add the two functions of time

$$A_0 \cos \omega\, t + A_0 \cos(\omega\, t + \phi).$$

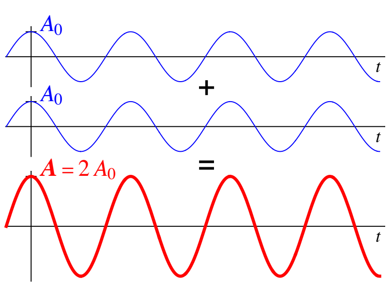
If the two waves are in phase $\phi = 0$ we get *constructive interference* and

$$A_0 \cos \omega\, t + A_0 \cos \omega\, t = 2\, A_0 \cos \omega\, t$$

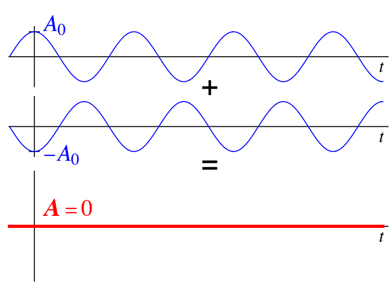
and the resulting amplitude of the combined wave is $A = 2\, A_0$. If the two waves are totally out of phase $\phi = \pi$ we get *destructive interference*. In the destructive case, since $\cos(\omega\, t + \pi) = -\cos \omega\, t$, we get

$$A_0 \cos \omega\, t + A_0 \cos(\omega\, t + \pi) = 0;$$

the resulting combined amplitude is $A = 0$.



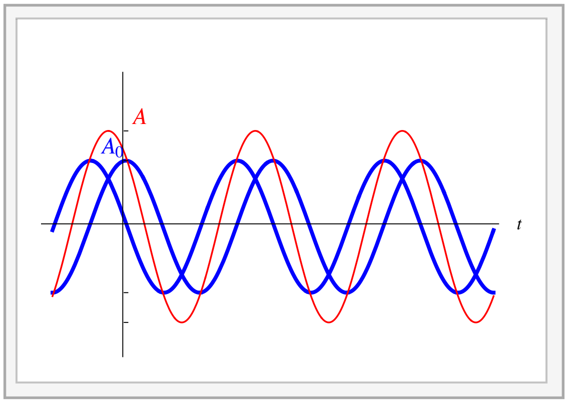
Constructive Interference



Destructive Interference

The more general result is a bit more complicated; we will just state the result and not use it beyond here. The essential fact is that constructive and destructive interference are not the only possibilities; they are the limiting cases of something more general.

$$A_0 \cos \omega t + A_0 \cos(\omega t + \phi) = A \cos\left(\omega t + \frac{\phi}{2}\right) \text{ where } A = 2 A_0 \cos \frac{\phi}{2}$$



Interactive Figure - General Interference

The intensity is proportional to the square of the peak electric field and this is just the amplitude.

$$I = \frac{1}{2} \varepsilon_0 c E_{max}^2 \quad \Longrightarrow \quad I \propto A^2$$

If two flashlights, each with intensity I_0 shine at the same point the resulting intensity is $2 I_0$; this is incoherent mixing, meaning that the phase of light of one source is not related to the phase of the other. The key to interference is coherence; the phases of the two sources are related. The following table summarizes the amplitudes and intensities for the special cases of constructive and destructive interference, for the general case and for incoherent mixing. In all cases the amplitude and intensity of the uncombined waves are A_0 and I_0.

	Amplitude	**Intensity**
Constructive Interference **$\phi = 0$**	$2 A_0$	$4 I_0$
Destructive Interference **$\phi = \pi$**	0	0
General Case **(Any ϕ)**	$0 \leq A \leq 2 A_0$	$0 \leq I \leq 4 I_0$
Incoherent Mixing **(two flashlights)**	–	$2 I_0$

What does this look like? Constructive interference corresponds to bright spots, high intensity. Destructive interference has zero intensity and it this a dark spot. If the phase difference is $\phi = m\, 2\pi$, where m is any integer then the sinusoidal functions are in phase and that is constructive interference. If the phase difference is $\phi = m\, 2\pi + \pi = (m + 1/2)\, 2\pi$ then they destructively interfere as well.

Interference applies to any type of wave. Suppose you have sound from two speakers that leaves each speaker in phase and with the same amplitude. If you are a distance r_1 from one speaker and r_2 from the other then even though the sound starts off in phase, they travel different distances and end up out of phase. If $r_1 = r_2$ then the waves are in phase and that then is a loud spot, where loud is high intensity. If $r_2 - r_1 = \lambda/2$, where λ is the sound wavelength, then the waves destructively interfere and there is no sound there.

Example I.1 - A Middle-C Note from Two Speakers

The musical note middle-C is an important reference for sound. If has a frequency of $\boxed{f = 261.6\ \text{Hz}}$. If we take the speed of sound (for dry air at 20 °C) to be $v_{sound} = 343\ \text{m/s}$, then we can find the wavelength from the general wave expression: $f \lambda = v$.

$$\lambda = \frac{v_{sound}}{f} = 1.31\ \text{m}$$

If the distances from the speakers are r_2 and r_1 then where $r_2 = r_1$ we get constructive interference and a loud spot.

(a) For what values of $r_2 - r_1$ will we get constructive interference, a loud spot?

Solution

When the path difference $r_2 - r_1$ is an integer number of wavelengths we will get constructive interference.

$$r_2 - r_1 = m\,\lambda = m\,1.31\text{ m} \;\text{ for }\; m = 0, \pm 1, \pm 2, \pm 3, \ldots$$
$$\text{or } r_2 - r_1 = 0, \pm 1.31\text{ m}, \pm 2.61\text{ m}, \pm 3.92\text{ m}, \ldots$$

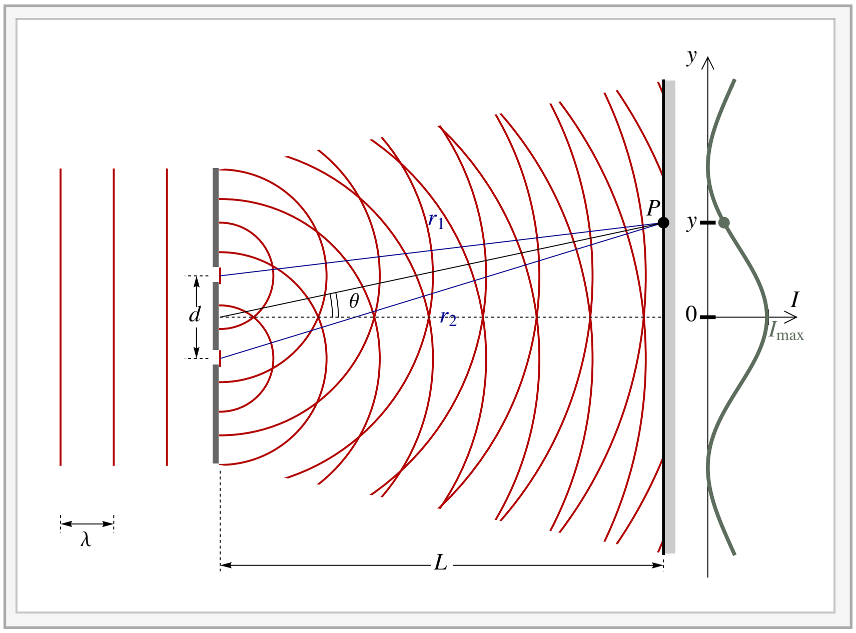
(b) For what values of $r_2 - r_1$ will we get destructive interference, a quiet spot?

Solution

When the path difference $r_2 - r_1$ is an integer number of wavelengths we will get constructive interference.

$$r_2 - r_1 = \left(m + \frac{1}{2}\right)\lambda = \left(m + \frac{1}{2}\right)1.31\text{ m} \;\text{ for }\; \left(m + \frac{1}{2}\right) = \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \ldots$$
$$\text{or } r_2 - r_1 = \pm 1.96\text{ m}, \pm 3.27\text{ m}, \pm 4.57\text{ m}, \ldots$$

I.2 - Young's Double-slit Experiment



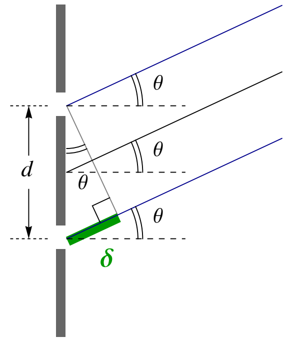
Interactive Figure

Consider monochromatic light of wavelength λ normally incident on a pair of narrow vertical slits each of width W and separated by d, the distance between the centers of the slits. We will make the simplifying assumption of narrow slits, or $W \ll d$. We will take L to be the distance from the slits to some distant screen; by distant we mean that L is much larger than both d and λ. A point P on the screen can be labeled by θ, the angle of deflection or by y, the distance along the screen, where $y = 0$ and $\theta = 0$ describe undeflected rays. θ and y are related by

$$\tan\theta = \frac{y}{L}.$$

The distance from one slit to P is r_1 and r_2 is the distance from the other slit to P. Each r is much larger than d but their difference δ is on the same order. Using trig and the approximation that $d \ll r$ we get $\delta = d\sin\theta$

$$\delta = r_2 - r_1 = d\sin\theta.$$



Constructive and Destructive Interference

Huygen's principle states that every point on a wave front behaves as a separate point source. When a plane wave hits a pair of narrow slits then each slit represents a different source. The key point is that each source is in phase with the other; this coherence is the key to interference. If the two distances r_1 and r_2 are equal (so $\delta = 0$) then the light rays hitting P are in phase and there is constructive interference. Moreover, if the path difference δ is an integer number of wavelengths there is constructive interference. If the path difference is half a wavelength then the rays are out of phase and there is destructive interference.

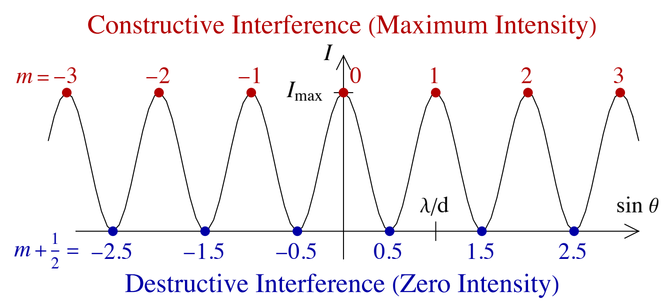
Using that the path difference is $d \sin\theta$ and taking m to be any integer we can write the conditions for constructive and destructive interference.

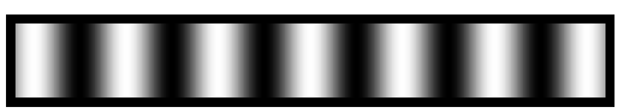
$$d \sin\theta = m\lambda \qquad \text{(constructive interference)}$$

$$d \sin\theta = \left(m + \frac{1}{2}\right)\lambda \qquad \text{(destructive interference)}$$

where $m = 0, \pm 1, \pm 2, \ldots$ and $m + \frac{1}{2} = \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \ldots$.

Graph of Intensity





Example I.2 - Young's Double-slit Experiment

Suppose light shines through two narrow vertical slits separated by 0.0165 mm and onto a screen 3.5 m away.

(a) If light from a He-Ne laser shines through these slits, then at what distance from the center of the central maximum are the first and second dark fringes and the first and second bright fringes? Laser light is monochromatic, meaning that is has only one color or wavelength and the most common red laser light is from a He-Ne laser which has the wavelength 632.8 nm.

Solution

First, list what we are given. The slit separation is d and the distance to the screen is L.

$$d = 0.0165 \times 10^{-3}\ \text{m}\,, \quad L = 3.5\ \text{m} \ \text{ and } \ \lambda = 632.8 \times 10^{-9}\ \text{m}$$

The dark fringes are at $d \sin\theta = (m + 1/2)\,\lambda$ and the first two dark fringes correspond to $m + 1/2$ having the values $1/2$ and $3/2$. We then find the distance from the center y from the angle using $\tan\theta = y/L$.

$$1^{\text{st}}\text{ dark fringe}: \quad \theta = \sin^{-1}\left(\frac{1}{2}\frac{\lambda}{d}\right) = 1.10\,° \implies y = L\tan\theta = 0.067\text{ m}$$

$$2^{\text{nd}}\text{ dark fringe}: \quad \theta = \sin^{-1}\left(\frac{3}{2}\frac{\lambda}{d}\right) = 3.30\,° \implies y = L\tan\theta = 0.202\text{ m}$$

The bright fringes are at $d\sin\theta = m\,\lambda$ and the first two correspond to $m = 1$ and $m = 2$. Repeating the previous procedure gives.

$$1^{\text{st}}\text{ bright fringe}: \quad \theta = \sin^{-1}\left(1\,\frac{\lambda}{d}\right) = 2.20\,° \implies y = L\tan\theta = 0.134\text{ m}$$

$$2^{\text{nd}}\text{ bright fringe}: \quad \theta = \sin^{-1}\left(2\,\frac{\lambda}{d}\right) = 4.40\,° \implies y = L\tan\theta = 0.269\text{ m}$$

(b) If white light shines through this double-slit then what is the angular separation $\Delta\theta$ between the first bright fringes of red light with $\lambda = 700$ nm and of violet light with $\lambda = 400$ nm.

Solution

The d value is the same. Since we are only concerned with angles here we do not need L.

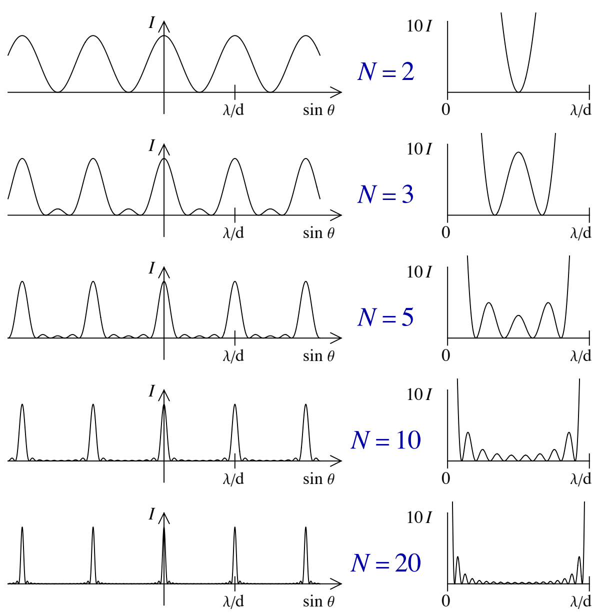
$$\lambda_{\text{red}} = 700\times10^{-9}\text{ m} \text{ and } \lambda_{\text{violet}} = 400\times10^{-9}\text{ m}$$

The first bright fringe is at $m = 1$ so $d\sin\theta = m\,\lambda$ gives $\theta = \sin^{-1}(\lambda/d)$.

$$\theta_{\text{red}} = \sin^{-1}\left(\frac{\lambda_{\text{red}}}{d}\right) = 2.43\,° \text{ and } \theta_{\text{violet}} = \sin^{-1}\left(\frac{\lambda_{\text{violet}}}{d}\right) = 1.39\,° \implies \theta_{\text{red}} - \theta_{\text{violet}} = 1.04\,°$$

I.3 - Many Slits and Diffraction Gratings

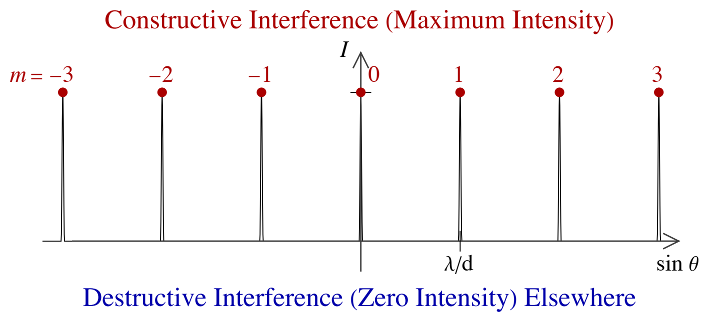
Now consider the case of N narrow slits. The following graphs show the intensity graphs. The graphs on the right is an enlargement of one period of the graph where the intensity is multiplied by 10. Note that the condition for constructive interference is the same but as the number of slits increases the intensity goes to zero at all positions in between.

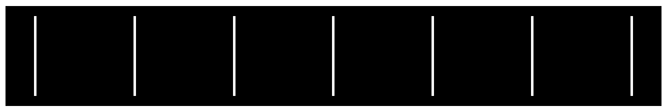


A diffraction grating is the limit as N becomes large. The interference conditions for a diffraction grating is the same as for the double slit for constructive interference but everywhere else there is destructive interference. There is a crisp pattern of lines representing the bright fringes.

$$d\sin\theta = m\,\lambda \text{ for constructive interference}$$

and destructive interference elsewhere.





When a laser, which has monochromatic light, is shot through a diffraction with vertical slits, you observe a series of horizontally-spaced dots representing the constructive interference. The difference from the double slit is that it is dark everywhere else.

Note that the position of the m^{th} maximum varies with wavelength. If white light passes through a diffraction grating then the central fringe ($m = 0$) is the same for all wavelengths and thus is white, but the higher order fringes will break light into its spectrum. Diffraction gratings are better for spectroscopy (resolving light into its constituent wavelengths) than prisms. With a prism, red is the color that refracts the least. This is reversed with a diffraction grating; since red has the longest wavelength, it diffracts the most.

Example I.3 - The Visible Spectrum and the *m* = 1 Fringe

Show that for any slit spacing d, the visible spectrum (from 400 nm to 700 nm) for the $m = 1$ fringe does not overlap with the spectrum for the $m = 2$ fringe. Show also that far any d the second-order spectrum always overlaps the third-order spectrum.

Solution

The wavelengths satisfy $400\text{ nm} \le \lambda \le 700\text{ nm}$. With this we can find an expression for the range of possible angles for the m^{th} fringe.

$$d\sin\theta = m\lambda \implies \sin\theta = \frac{m\lambda}{d} \implies \frac{m\,400\text{ nm}}{d} \le \sin\theta \le \frac{m\,700\text{ nm}}{d}$$

The largest value of $\sin\theta$ for the $m = 1$ fringe is $1\times 700\text{ nm}/d$ and the smallest value of $\sin\theta$ for $m = 2$ fringe is $2\times 400\text{ nm}/d$. Because $700 < 2\times 400$ we can see that for any d there is no overlap.

For the $m = 2$ fringe, the largest value of $\sin\theta$ is $2\times 700\text{ nm}/d$ and the smallest value of $\sin\theta$ for $m = 3$ fringe is $3\times 400\text{ nm}/d$. Because $2\times 700 > 3\times 400$ we can see that for any d there is overlap.

Example I.4 - Diffraction Grating

Light from a He-Ne laser (wavelength 632.8 nm) shines through a diffraction grating with 230 lines/mm.

(a) If the light passes onto a screen 5.8-m away from the grating then what is the distance from the central bright fringe to the first, second and third order fringes on the screen?

Solution

First we need to find the slit spacing from the lines per length; since d is the distance between lines we take the reciprocal.

$$d = \frac{1}{230\text{ lines/mm}} = \frac{1\text{ mm}}{230} = \frac{10^{-3}\text{ m}}{230} = 4.348\times 10^{-6}\text{ m}$$

We are also given the wavelength and L.

$$\lambda = 632.8\text{ nm} \quad\text{and}\quad L = 5.8\text{ m}$$

The angle for the m^{th} fringe is found from $d\sin\theta = m\lambda$. From θ we can then find the distance from the center on the screen y using $\tan\theta = y/L$.

$$\text{for } m = 1,\ \ \theta = \sin^{-1}\left(1\,\frac{\lambda}{d}\right) = 8.37^\circ \implies y = L\tan\theta = 0.853\text{ m}$$

$$\text{for } m = 2,\ \theta = \sin^{-1}\left(2\,\frac{\lambda}{d}\right) = 16.9\,° \implies y = L\tan\theta = 1.76 \text{ m}$$

$$\text{for } m = 3,\ \theta = \sin^{-1}\left(3\,\frac{\lambda}{d}\right) = 25.9\,° \implies y = L\tan\theta = 2.81 \text{ m}$$

(b) From Part (a) of this problem we can see that as the order m increases, the angle increases. There is only a finite number of fringes since the angle must be less than $90\,°$. Using this laser and grating, what is the highest order bright fringe and how many fringes are present in total?

Solution

Since we have the same laser and grating, d and λ are the same as in Part (a). Because the angle cannot be larger than $90\,°$ the sine function must be less than 1.

$$d\sin\theta = m\lambda \implies m\,\frac{\lambda}{d} = \sin\theta < 1 \implies m < \frac{d}{\lambda} = 6.87$$

m is an integer, so it follows that the largest such m is

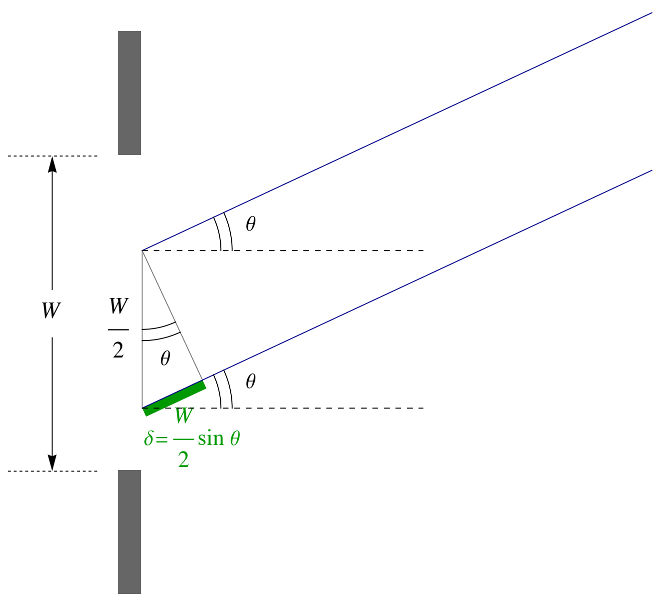
$$m_{\max} = 6$$

This is the highest order fringe. For the total number of bright fringes, remember that there are fringes on either side of the center and one in the center.

$$\text{total number of fringes} = 2\,m_{\max} + 1 = 13$$

I.4 - Single-slit Diffraction

When light passes through a single narrow slit of width W one observes a diffraction pattern. Here each point in the slit is a separate source. It is simple to find the condition for destructive interference.



Consider two rays from points separated by $W/2$ in the slit. The difference of the path lengths δ is given by

$$\delta = r_2 - r_1 = \frac{W}{2}\sin\theta$$

If this distance is half a wavelength.

$$\delta = \frac{W}{2}\sin\theta = \frac{\lambda}{2}.$$

then those two rays destructively interfere. But if this condition is met then the ray leaving *any* point in the slit will be exactly canceled by the ray from $W/2$ away and this is the condition for destructive interference for all the rays.

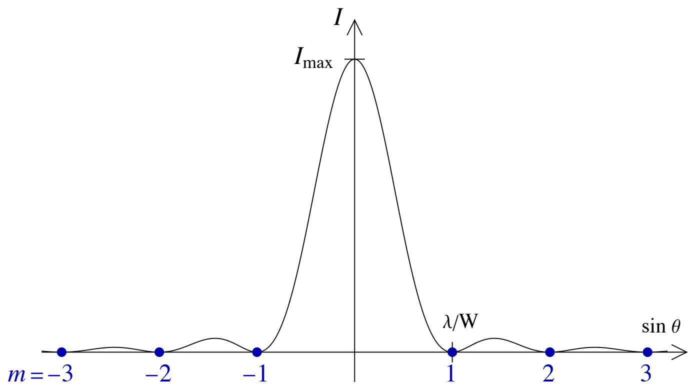
This argument fails if we break the slit into thirds, because there would always be one ray that is not canceled. However if we break the slit into any even number of pieces $2\,m$, then light from a point $\frac{W}{2\,m}$ away would destructively interfere and all the rays could be canceled

$$\delta = \frac{W}{2\,m}\sin\theta = \frac{\lambda}{2}.$$

The condition for destructive interference is then

$$W\sin\theta = m\,\lambda, \quad \text{where } m = \pm 1,\ \pm 2,\ \pm 3,\ \ldots \text{(destructive interference)}$$

Below is the graph of the intensity pattern for a single slit. Note that the $m = 0$ position is not destructive interference; it is the center of the central bright fringe.



Destructive Interference (Zero Intensity)

The central bright fringe is from $m = -1$ to $m = 1$ and has twice the width of the other bright fringes. It is also much brighter.

As an aside, the intensity formula for the above graph is

$$I = I_{\max}\left[\frac{\sin[(\pi\,W/\lambda)\sin\theta]}{(\pi\,W/\lambda)\sin\theta}\right]^2.$$

Example I.5 - SIngle-slit Diffraction

Light from a He-Ne laser (wavelength 632.8 nm) shines through a narrow vertical slit with a width of 0.075mm and onto a screen 3.5 m away.

(a) What is the width of the central maximum on the screen?

Solution

We are given W, L and λ.

$$W = 0.075\times 10^{-3}\ \text{m}\,,\quad L = 3.5\ \text{m}\ \text{ and }\ \lambda = 632.8\ \text{nm}$$

The central maximum is from $m = -1$ to $m = 1$ but it will just be twice the distance from the center to the $m = 1$ dark fringe. If this distance is labeled y_1 and the angle θ_1, then we are looking for $2\,y_1$

$$W\sin\theta = m\,\lambda \implies W\sin\theta_1 = 1\,\lambda \implies \theta_1 = \sin^{-1}\left(\frac{\lambda}{W}\right) = 0.483\,^\circ \implies y_1 = L\tan\theta_1 = 0.0295\ \text{m}$$
$$\implies 2\,y_1 = 0.0581\ \text{m}$$

(b) What is the distance from the center to the fifth dark fringe.

Solution

Since we start counting the dark fringes at $m = 1$, the fifth fringe is must $m = 5$. We want the y value for $m = 5$.

$$W\sin\theta = m\,\lambda \implies W\sin\theta_5 = 5\,\lambda \implies \theta_5 = \sin^{-1}\left(\frac{5\,\lambda}{W}\right) = 2.42\,^\circ \implies y_5 = L\tan\theta_5 = 0.148\ \text{m}$$