Chapter K

Quantum Physics

Blinn College - Physics 1402 - Terry Honan

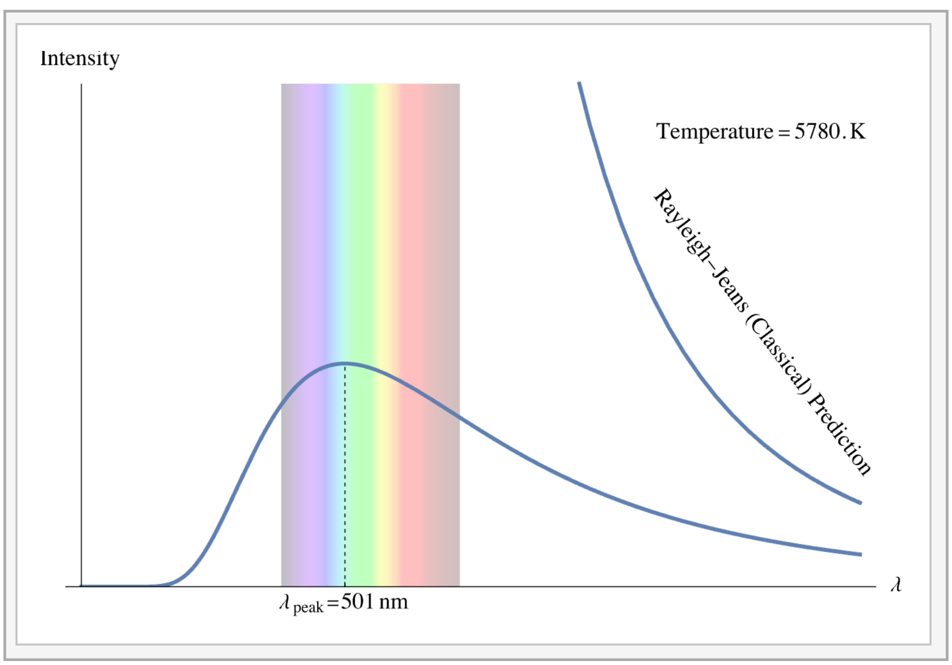
K.1 - Planck's Constant and Black-body Radiation

Black Bodies

At the end of the nineteenth century it looked as if almost all of physics was settled, except for a few loose ends. These loose ends led to two major revolutions in physics in the twentieth century. The first, of course, was relativity. The second was the quantum physics revolution, which much more fundamentally altered the world view of physicists. In addition to the nineteenth century progress in electromagnetism that we have been discussing for most of the semester, there was also tremendous progress in thermodynamics, some of which was covered in Physics I. When the rules of thermodynamics were applied to the electromagnetic field, some bizarre results calculated.

Any hot object, meaning something at a finite temperature, radiates electromagnetic energy. Something near the temperature of the sun radiates energy in and around the visible spectrum. Things around room temperature radiate in the infrared. The spectrum of light a hot object radiates depends on its temperature and how reflective it is. A black body is an idealized object that is perfectly black, meaning that it reflects no light. To approximate such an idealization imagine a hollow container with a small hole; any light that enters the hole reflects multiple times and is absorbed by the inner surface. Most hot objects produce spectra that are close to the idealized black body spectrum after accounting for its reflectivity.

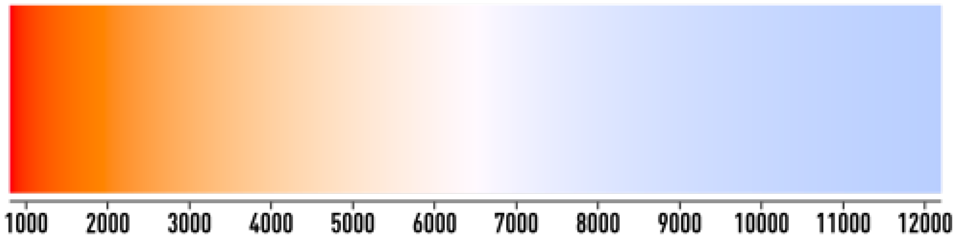
When the laws of thermodynamics were applied to electromagnetic waves from a hot object, it was calculated that theoretically, it should radiate away all its energy almost instantaneously. The theory. known as the Rayleigh-Jeans theory, predicted the experimental results well for small frequencies, but at higher frequencies the theory gave results that approached infinity as frequencies increased. The problem was that in classical physics there was no limit to the number of internal degrees of freedom in matter and that that led to unlimited energy being radiated. Each oscillating degree of freedom in matter would get an equal share of the total energy and there were unlimited oscillating degrees of freedom at high frequencies. This anomalous result was called, rather dramatically, the *ultraviolet catastrophe*; since higher frequency radiation is ultraviolet.



The black body spectrum and the Rayleigh-Jeans anomalous result.

The black body spectrum can be written in terms of the frequency or wavelength. The peak wavelength λ_{peak} varies with the temperature (in kelvin) by the Wien displacement law. (Note that the Walker text discusses black body radiation in terms of frequency and not wavelength. Moreover, $f_{\text{peak}} \neq c / \lambda_{\text{peak}}$ but the explanation is too complicated to get into.)

$$\lambda_{\text{peak}} = (0.00290\ \text{m}\cdot\text{K})/T$$



This shows the apparent colors of the spectra from different temperatures in Kelvin.

By Bhutajata - Own work, CC BY-SA 4.0, https://commons.wikimedia.org/w/index.php?curid=44144928

Example K.1 - Light Bulb Temperatures

Light bulb temperatures for incandescent or florescent bulbs reflect the actual temperature of the hot filament or hot gas. For modern LED bulbs it is more complicated but the listed temperature represents the best approximation to the black body spectrum that the bulb produces. What are the peak wavelengths for a “warm white” bulb at 2500 K, a “cool white” bulb at 3800 K and a “daylight” bulb at 5800 K.

Solution

Use the Wien displacement law.

$$\lambda_{\text{peak}} = (0.00290\ \text{m}\cdot\text{K})/T$$

$$T = 2500\ \text{K} \implies \lambda_{\max} = 1160\ \text{nm}$$

$$T = 3800\ \text{K} \implies \lambda_{\max} = 763\ \text{nm}$$

$$T = 5800\ \text{K} \implies \lambda_{\max} = 500\ \text{nm}$$

Note that with lighting colors people use terms like warm for reddish hues and cold for bluish hues. This usage is reverse from the Kelvin temperatures.

Example K.2 - Spectra of Stars

The color of stars is based on the temperature of their surface. Find the peak wavelengths for the sun at 5780 K, Betelgeuse at 3500 K, Polaris (the north star) at 6000 K and Sirius at 10 000 K.

Solution

Again use the Wien displacement law.

$$\lambda_{\text{peak}} = (0.00290\ \text{m}\cdot\text{K})/T$$

$$\text{Sun} \implies T = 5780\ \text{K} \implies \lambda_{\max} = 502\ \text{nm}$$

$$\text{Betelgeuse} \implies T = 3500\ \text{K} \implies \lambda_{\max} = 829\ \text{nm}$$

$$\text{Polaris} \implies T = 6000\ \text{K} \implies \lambda_{\max} = 483\ \text{nm}$$

$$\text{Sirius} \implies T = 10\,000\ \text{K} \implies \lambda_{\max} = 290\ \text{nm}$$

Planck’s Constant

To find a match for the experimentally observed black-body spectrum, Planck, in 1900, made an assumption that the atoms inside the cavity were not oscillating at all possible energies but at some frequency the energies were quantized as multiples of the frequency.

$$E_n = n\,h\,f \ \text{ where } \ n = 0,\ 1,\ 2,\ \ldots$$

In this expression, f is the frequency, n is a non-negative integer and h is a new constant that Planck introduced. Planck’s constant has the numerical value, in SI units of

$$h = 6.63 \times 10^{-34}\ \text{J}\cdot\text{s}$$

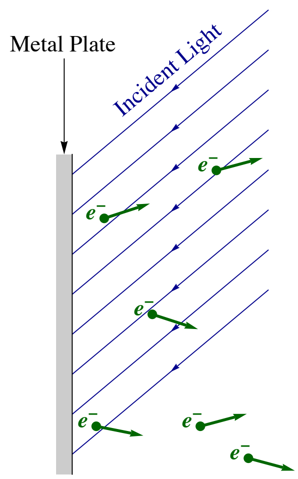
By controlling the number of high frequency degrees of freedom, he was able to high frequency behavior of the radiation spectrum.

Essentially, Planck added a fudge factor to fit the curve. But his fudge factor had physical motivation and it got physicists asking the right questions. How are matter and radiation quantized?

K.2 - The Photon Hypothesis

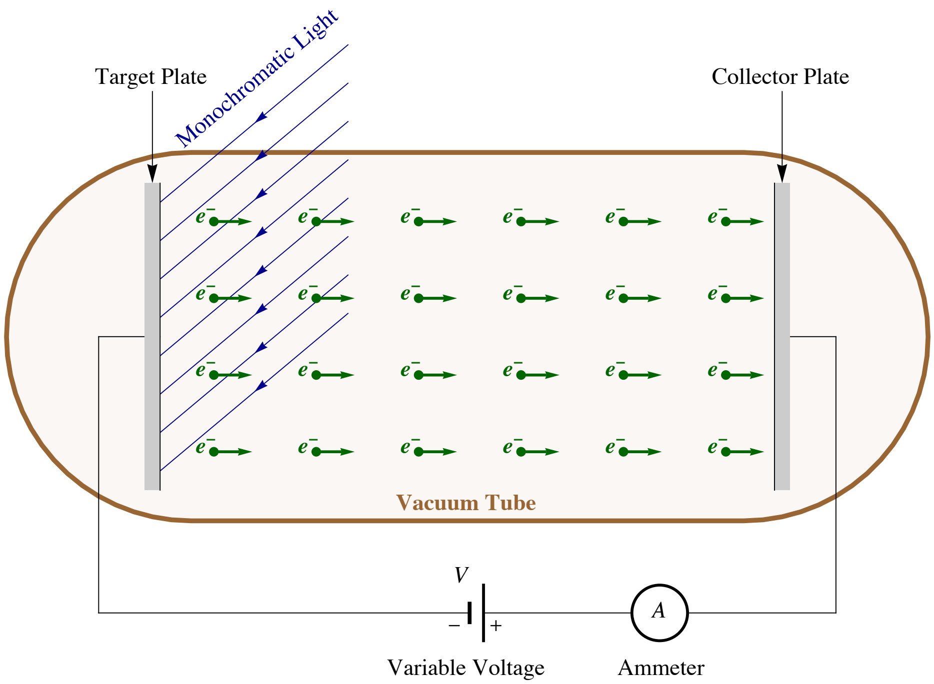
The Photoelectric Effect

In 1905 Einstein was working at a Swiss patent office. 1905 is referred to as Einstein's *annus mirabilis*, his miracle year. In addition to his two revolutionary papers on relativity, Einstein had another paper on Brownian motion, where he described the random behavior of small particles in a fluid caused by the collisions between the particles and the atoms and molecules; this paper is credited as having ended the debate on the existence of atoms. His most controversial paper that year introduced the idea that light was quantized; it was absorbed and emitted in discrete units which we now call photons.



Light hitting a metal surface and ejecting electrons e^-.

When electromagnetic radiation hits the surface of a metal the energy of the light can energize electrons in the metal and they can escape from the surface. This is known as the photoelectric effect; it is how a photocell or "electric eye" works. To study this we will imagine this plate, which we will call the *target plate* inside a vacuum tube with another metal plate, the *collector plate*. A variable voltage is placed across the plates and the current due to the ejected electrons is measured.



The photoelectric effect uses a simple photocell that can detect light.

Reversing the polarity of the dc source, allows a measurement of the maximum kinetic energy of the ejected electrons K_{max}. There is a stopping potential V_0 for this reversed polarity $V = -V_0$, and any voltage magnitude above this would give no current. The ejected electrons need to have enough energy to make it across the plates. The maximum kinetic energy of the ejected electrons is related to this stopping voltage.

$$K_{max} = e V_0$$

The observed results of this photoelectric effect experiment included anomalous results.

- As the intensity of the incident light increased, the current increased.

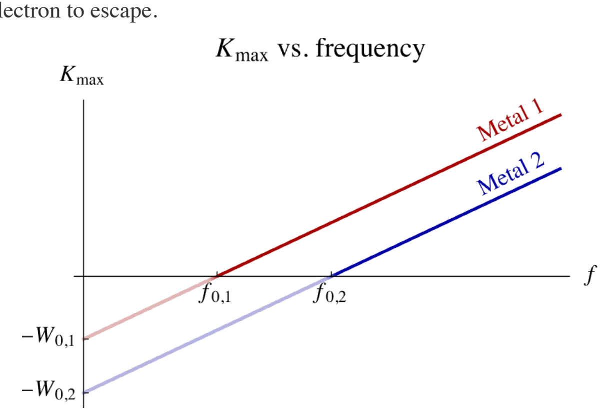
  This was expected.

- There was no time lag for very dim incident light.

  It was expected that if the intensity of the incident light was very low it would take time for electrons to build up enough energy to escape the surface. Instead, although very dim light gave very low currents there was no observable time lag. This was not a cumulative effect.

- There was an unexplainable frequency dependence.

  The maximum kinetic energy K_{max} depended on the metal and on the frequency of the incident light. Moreover, it was totally independent of the intensity. For a given metal there was a cutoff frequency f_0, below which no electrons were ejected. The work function W_0 was the minimum energy needed for an electron to escape.



The figure above shows the maximum kinetic energy versus the frequency for different metals. The slope of the graphs for all metals was Planck's constant. When the graphs were extrapolated to negative values, the intercept had the interpretation as the negative work function

W_0 of that metal and the cutoff frequency was the frequency axis intercept $f_0 = W_0/h$.

$$K_{max} = h\,f - W_0$$

When we write expressions like the one above, it makes the resolution of the anomalies seem obvious, in retrospect. But the resolution was very revolutionary. Einstein said that despite the irrefutable evidence that light was a wave, it also had a particle nature. It is always absorbed or emitted in discrete packets of energy which we now call photons. The energy of a photon is related to its frequency.

$$E = h\,f$$

Light is absorbed and emitted one photon at a time. The consequences of light being both a wave and a particle were huge and revolutionary.

Example K.3 - The Photoelectric Effect

The work function for calcium is 2.90 eV and for copper is 4.70 eV.

(a) What are the cutoff frequencies and corresponding wavelength for each?

Solution

The constants and conversions needed are.

$$1\text{ eV} = 1.60\times10^{-19}\text{ J} \quad\text{and}\quad h = 6.63\times10^{-34}\text{ J}\cdot\text{s}$$

$$W_0 = 2.90\text{ eV} = 4.64\times10^{-19}\text{ J} \implies f_0 = W_0/h = 7.00\times10^{14}\text{ Hz} \implies \lambda = c/f_0 = 429\text{ nm}$$

$$W_0 = 4.70\text{ eV} = 7.52\times10^{-19}\text{ J} \implies f_0 = W_0/h = 1.13\times10^{15}\text{ Hz} \implies \lambda = c/f_0 = 264\text{ nm}$$

(b) For both calcium and copper, what are the maximum kinetic energies of ejected electrons from light with a frequency of 1.50×10^{15} Hz?

Solution

Using the frequency of $f = 1.50\times10^{15}$ Hz and W_0 for each gives the result.

$$W_0 = 4.64\times10^{-19}\text{ J} \implies K_{max} = h\,f - W_0 = 5.31\times10^{-19}\text{ J}$$

$$W_0 = 7.52\times10^{-19}\text{ J} \implies K_{max} = h\,f - W_0 = 2.43\times10^{-19}\text{ J}$$

Example K.4 - Electron-Positron Annihilation

When an electron and positron collide they annihilate into two high-energy photons of equal energies, assuming the initial kinetic energies are negligible. Give the energy of the photons in joules and electron-volts, eV. What are the wavelength and frequency of the resulting photons? Where is this on the electromagnetic spectrum?

Solution

The positron has the same mass as the electron

$$c = 3.00\times10^{8}\text{ m/s}\,,\quad m_{electron} = 9.11\times10^{-31}\text{ kg} \text{ and } h = 6.63\times10^{-34}\text{ J}\cdot\text{s}$$

The total energy is then the total rest energy of both.

$$E_{tot} = 2\,m_{electron}\,c^2 = 1.64\times10^{-13}\text{ J}$$

The energy in each photon is then half this.

$$E = m_{electron}\,c^2 = 8.20\times10^{-12}\text{ J}$$

To convert to electron-volts, an eV is e multiplied by a volt V.

$$1\text{ eV} = e\times1\text{ V} = 1.60\times10^{-19}\text{ C}\cdot\text{V} = 1.60\times10^{-19}\text{ J} \implies E = 512\,000\text{ eV}$$

To find the frequency we use the expression for the energy of a photon

$$E = h\,f \implies f = E/h = 4.10\times10^{33}\text{ Hz}$$

and we can then find the wavelength.

$$\lambda = c/f = 7.32\times10^{-26}\text{ m}$$

Comparing these values with the electromagnetic spectrum we see that these are very energetic gamma rays.

Example K.5 - Photons per Second from a Laser

How many photons per second are emitted by 50-mW He-Ne laser ($\lambda = 632.8$ nm). The power is 50 mW; that is the energy per time.

Solution

The energy of each photon is $E_{photon} = h\,f$. The total energy divided by the energy of each photon (E / E_{photon}) is the number of photons , so power is the energy per time then the power divided by the energy of each photon is the number of photons per time.

$$\mathcal{P} = 0.050\ \text{W}\ ,\ \ \lambda = 632.8 \times 10^{-9}\ \text{m}\ ,\ \ c = 3.00 \times 10^{8}\ \text{m/s}\ \ \text{and}\ \ h = 6.63 \times 10^{-34}\ \text{J} \cdot \text{s}$$

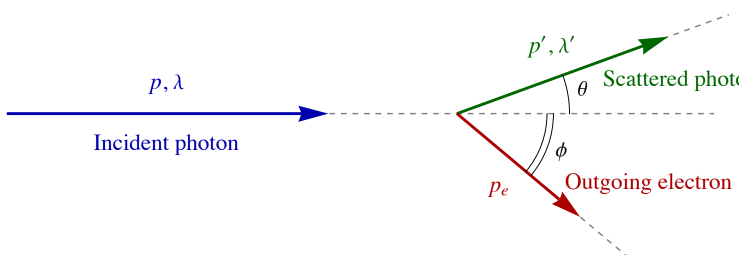
The total energy is then the total rest energy of both.

$$f = c / \lambda\ \text{and}\ E_{photon} = h\,f \implies E_{photon} = h\,c / \lambda = 3.143 \times 10^{-19}\ \text{J}$$

We can then find the number of photons per time.

$$\#\ \text{of photons} = E / E_{photon} \implies \#\ \text{of photons/time} = \mathcal{P} / E_{photon} = 1.59 \times 10^{17}\ \text{photons/s}$$

Compton Scattering



An important application and verification of the photon hypothesis was Compton scattering. A photon of wavelength λ bounces off an electron, initially at rest. The photon scatters (bounces off, using Physics I language) and leaves with a new wavelength λ' at an angle θ from its initial direction. The electron moves off in a different direction with its own momentum. By conserving energy and momentum and eliminating the final momentum of the electron, Compton was able to derive the following result that relates the change in the wavelength of the photon to its scattering angle. The proof is too tedious to include here.

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_e\,c}\,(1 - \cos\theta)\ \text{where}\ \frac{h}{m_e\,c} = \lambda_C = 2.43 \times 10^{-12}\ \text{m}$$

λ_C defined above is called the Compton wavelength . Don't confuse it with the other wavelengths in the formula, but it does set the scale for Compton scattering. As a wavelength this is somewhere between x-rays and γ-rays.

Example K.6 - Compton Scattering

A photon with an incident wavelength of 0.032 nm scatters at a 55° angle off a stationary electron. What is the wavelength of the scattered photon? Also, what are the frequency, energy and momentum of the scattered photon?

Solution

$$\theta = 55°,\ \ \lambda = 0.032 \times 10^{-9}\ \text{m and}\ \lambda_C = 2.46 \times 10^{-12}\ \text{m}$$

Given the information above we can find the wavelength of the scattered photon.

$$\lambda' - \lambda = \frac{h}{m_e\,c}\,(1 - \cos\theta) \implies \lambda' = \lambda + \frac{h}{m_e\,c}\,(1 - \cos\theta) = \lambda + \lambda_C\,(1 - \cos\theta) = 3.58 \times 10^{-11}\ \text{m}$$

To find the frequency f', energy E' and the momentum p' of the scattered photon, we need the constants c and h..

$$c = 3.00 \times 10^{8}\ \text{m/s}\ \ \text{and}\ \ h = 6.63 \times 10^{-34}\ \text{J} \cdot \text{s}$$

$$f' = c / \lambda' = 8.36 \times 10^{18}\ \text{Hz}$$

$$E' = h\,f' = 5.44 \times 10^{-15}\ \text{J}$$

$$p' = E'/c = 1.85 \times 10^{-23}\ \mathrm{kg \cdot m/s}$$

Note that combining the formulas above, and dropping the primes, we get an expression for the momentum of a photon. We will see this expression in the next section but used more generally.

$$p = E/c = h\,f/c = h/\lambda$$

K.3 - Wave Mechanics

de Broglie Wavelength

In 1924 Louis de Broglie (pronounced "de broy") was a graduate student working on his PhD thesis. In his thesis he introduced the notion of matter waves. We saw in the last section that the momentum of a photon and its wavelength were related by $p = h/\lambda$. de Broglie reasoned that if waves have a particle nature, as Einstein's photon hypothesis showed, then all particles should have a wave nature. We will discuss the deBroglie wavelength again when we cover the Bohr atom. de Broglie was able to interpret the Bohr atom in terms of standing matter waves.

$$\lambda = h/p$$

The timeline here is of interest. Einstein's photon hypothesis was in 1905. Bohr introduced his model of the atom in 1911. After de Broglie's 1924 paper things rapidly fell into place. If there are matter waves then what wave equation do they satisfy. A year later Schrodinger built on de Broglie's wave hypothesis and found the wave equation that matter waves satisfied. That was the birth of modern quantum mechanics. Between de Broglie and Schrodinger, Heisenberg published an alternative version of quantum mechanics that proved to be equivalent to the Schrodinger approach.

We now have seen that waves have a particle nature and particles have a wave nature. This is often described as wave-particle duality. Look at interference of light. We saw a continuous intensity pattern when light shines through two-slits or a single slit. But what does that continuous pattern have to do with photons. Light is emitted and absorbed as a particle but travels between to positions more like a wave. The intensity pattern is proportional to the probability of a photon landing in a particular position. We cannot predict the outcome of an experiment but the probabilities of the outcomes. What about matter waves? Electrons will also show the same interference or diffraction pattern when shot through two slits or a single slit. We cannot predict where it will land just the probabilities of it landing in different positions.

An electron microscope is based on this wave property of electrons. With visual wavelengths of light one cannot make out details that are smaller than the wavelength of the light, and for visible light that is, as we have seen, on the order of hundreds of nanometers or less than a micron = μm. For example, the corona virus has a diameter of, on average $125\ \mathrm{nm} = 0.125\ \mu\mathrm{m}$, so it is not something you can observe with visible light. Electrons can, with sufficient momentum, have much smaller wavelengths and thus can used to view objects much smaller. Often one sees colors added to electron microscope images but these are added after the fact. There is no such thing as color on those scales.

Example K.7 - Accelerating Voltage for an Electron Microscope

To design an electron microscope with a wavelength of around 0.05 nm, what voltage is needed to accelerate electrons to the necessary speed? Assume this is nonrelativistic. We will see this is the case after the fact.

Solution

First the given information

$$\lambda = 0.05 \times 10^{-9}\ \mathrm{m}$$

and necessary constants.

$$e = 1.60 \times 10^{-19}\ \mathrm{C}\ ,\quad m_e = 9.11 \times 10^{31}\ \mathrm{kg}\ ,\quad c = 3.00 \times 10^{8}\ \mathrm{m/s}\ \text{ and }\ h = 6.63 \times 10^{-34}\ \mathrm{J \cdot s}$$

Find the momentum, then velocity and then the kinetic energy.

$$p = h/\lambda = 1.33 \times 10^{-23}\ \mathrm{kg \cdot m/s} \implies v = p/m_e = 1.46 \times 10^{7}\ \mathrm{m/s} \implies K = \frac{1}{2}\, m_e\, v^2 = 9.65 \times 10^{-17}\ \mathrm{J}$$

To find the voltage conservation of energy gives $K = e\,V$

$$V = K/e = 603\ \mathrm{V}$$

The speed is about $1/20^{\text{th}}$ the speed of light which justifies us to use nonrelativistic formulas.

The Uncertainty Principle

Suppose you try to pinpoint the position of some particle using light or some other particle with a wavelength. To view it with higher resolution you must use a smaller wavelength. However the smaller the wavelength used, the more momentum is required. This gives a kick to the particle you are trying to observe. The act of measuring something changes what you are trying to measure. This is a very fundamental limitation on measurement in quantum mechanics is called the uncertainty principle.

No measurement is exact, there is always uncertainty. Label the uncertainty in the position, more precisely the x-component of the position as Δx. Similarly label the uncertainty in the x-component of momentum as Δp_x.

$$\Delta x \, \Delta p_x \geq \frac{h}{4\pi}$$

This illustrates that quantum mechanics puts a fundamental limitation on our ability to simultaneously measure position and momentum. Here we introduced the uncertainty principle as a limitation on measurement. But often discussions on the uncertainty principle overstate this and seem to imply that measurement causes the uncertainty principle. The fact is that these quantities are not clearly defined in quantum mechanics. Position and momentum are inherently fuzzy notions and this fuzziness puts a limit on the measurements. Uncertainty, as mentioned here, sounds vaguely defined; it is not. We can precisely define what we mean by uncertainties.

It should be pointed out that the expression for the uncertainty principle stated above is written incorrectly in the Walker text. They write the right hand side as h divided by 2π instead of 4π. Some books do write it as $\Delta x \, \Delta p_x \gtrsim h/(2\pi)$ where the $\gtrsim$ is used to mean that gives an estimate of the uncertainty but with the $\geq$ the book is wrong.

This seems to imply that all of the mechanics you learned in Physics I is wrong; it is not. For macroscopic things that we study in mechanics the uncertainty principle is irrelevant, because the theoretically minimum uncertainty is much smaller than we would ever worry about. Suppose you have a thrown baseball. You can take an accurate video of the baseball and then plot out its position and velocity as functions of time. You know where it is and how fast it is moving. However, to see it light must be scattering off it and with each scattering it very slightly alters its course. But this effect is too small to be of interest.

Example K.8 - A Baseball

A baseball with a mass of 0.145 kg moves with a velocity 45 m/s. If we take Δx to be some length in the middle of the visible spectrum, say 550 nm, then what is the minimum uncertainty in its momentum and velocity? Calculate the fraction $\Delta v/v$ to show how small this effect is.

Solution

List the constants and given information.

$$m = 0.145 \text{ kg}, \quad v = 45 \text{ m/s}, \quad \Delta x = 550 \times 10^{-9} \text{ m} \quad \text{and} \quad h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s}$$

The minimum uncertainty is when the inequality becomes equal.

$$\min \Delta p = \frac{h}{4\pi \, \Delta x} = 9.59 \times 10^{-29} \text{ kg} \cdot \text{m/s}$$

and using $p = m\,v$ we can find the uncertainty in the velocity

$$\min \Delta v = \frac{\min \Delta p}{m} = 6.62 \times 10^{-22} \text{ kg} \cdot \text{m/s}$$

For the fraction we get

$$\frac{\min \Delta v}{v} = 1.47 \times 10^{-29}$$

As we expected, this is tiny.

Example K.9 - An Electron at Atomic Scales

Now we will apply the uncertainty principle to an electron where Δx is the typical size of an atom 0.10 nm.

(a) What is its minimum uncertainty in momentum and velocity?

Solution

We are given

$$\Delta x = 0.10 \times 10^{-9} \text{ m}, \quad m_e = 9.11 \times 10^{31} \text{ kg} \quad \text{and} \quad h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s}$$

$$\min \Delta p = \frac{h}{4\,\pi\,\Delta x} = 5.28 \times 10^{-25}\ \mathrm{kg \cdot m/s}$$

$$\min \Delta v = \frac{\min \Delta p}{m} = 579\,000\ \mathrm{m/s}$$

(b) Since its velocity will average to zero, take its minimum uncertainty in velocity to be its average speed and calculate its kinetic energy, in eV?

Solution

Now we take $v_{\mathrm{ave}} = \min \Delta v$ and find $K = \frac{1}{2}\,m\,v^2$

$$K = \frac{1}{2}\,m\,v^2 = 1.52 \times 10^{-19}\ \mathrm{J} \times \frac{1\ \mathrm{eV}}{1.60 \times 10^{-19}\ \mathrm{J}} = 0.955\ \mathrm{eV}$$