# Chapter M

## *Nuclear Physics*

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## M.1 - The Structure of the Nucleus

### Constituents of the Nucleus

We saw in the last chapter that Rutherford discovered the nucleus in 1909 with his experiment scattering alpha particles from thin gold foil. The nucleus was then known to be some very dense positively-charged core that had essentially all of the mass of the atom, except for the much lighter electrons outside the nucleus. Rutherford speculated that hydrogen nuclei were fundamental particles in their own right and he later coined the term proton. But what was this nucleus made of? The mass of the nucleus was larger than the masses of the protons needed to get the appropriate charge, and by a factor about two, typically. The other question was: how can positively charged protons, which repel each other, be bound together? It was considered that somehow electrons could be bound inside the nucleus with the proton but it turns out that combining the size of the nucleus and the uncertainty principle, there would be much too much energy in the electrons and that would contribute too large a mass to the nucleus. Rutherford speculated that there was another neutral particle, the neutron, that accounted for the missing mass. Rutherford hoped that the the existence of neutrons would also assist in binding the nucleus, but a neutral particle could not itself overcome the electrostatic repulsion of the protons. The neutron was discovered in 1932; it is electrically neutral and has a mass that is very close to that of a proton but is slightly more massive. To bind the nucleus there must be some force stronger than the electromagnetic force that can hold the protons and neutrons together. This force is known, cleverly enough, as the strong nuclear force.

The strong nuclear force is well understood at different levels. For our purposes, it is what holds together protons and neutrons, but at a more elementary level we now know it as the force that holds together quarks, which make up protons, neutrons and a plethora of other subnuclear particles known as hadrons. From the perspective of the nuclear force, the protons and neutrons are identical; they differ only in their electromagnetic properties. We refer to both protons and neutrons as nucleons. With the nuclear force there is something called isospin; just as an electron can be spin up and spin down, a nucleon can be isospin up, the proton, and isospin down, the neutron. There is a tendency for the lowest energy states have isospin symmetry, where the number of protons and neutrons are equal. It is observed that the stable light nuclei reflect this but, because of the added energy of the electrostatic repulsion of protons, the heavy nuclei tend to be more neutron rich.

### Notation and Counting

The masses of the proton and neutron are approximately equal and both are *much* larger than the mass of an electron.

$$m_{\text{proton}} \simeq m_{\text{neutron}} \gg m_{\text{electron}}$$

To give the precise values,

$$\begin{aligned} m_{\text{proton}} &= 1.6726 \times 10^{-27}\ \text{kg} \\ m_{\text{neutron}} &= 1.6749 \times 10^{-27}\ \text{kg} \\ m_{\text{electron}} &= 9.109 \times 10^{-31}\ \text{kg} \end{aligned}$$

The atomic number $Z$ is the number of protons in the nucleus, which is the number of electrons in a neutral atom. The number of nucleons is the atomic mass number $A$.

$$\begin{aligned} Z &= \textit{atomic number} = \#\text{ of protons} = \#\text{ of electrons} \\ A &= \textit{atomic mass number} = \#\text{ of nucleons (protons and neutrons)} \\ N &= A - Z = \#\text{ of neutrons} \end{aligned}$$

One would expect, naively, that the mass of an atom should equal the sum of its constituent parts, but since the relativity chapter we know better. It takes energy to break up an atom into its constituent parts. Using the mass-energy equivalence, $E = m\,c^2$, it follows that since the constituent parts have more energy, and thus they have more mass.

$$m_{\text{atom}} = Z\,m_{\text{proton}} + N\,m_{\text{neutron}} + Z\,m_{\text{electron}} - \begin{pmatrix}\text{Binding}\\ \text{Energy}\end{pmatrix} \Big/ c^2$$

In this expression, the Binding Energy is the amount of energy required to break up the atom into is constituent parts; for example, $E_0 = 13.6$ eV is the binding energy for the hydrogen atom.

Because we cannot simply add the masses of the constituent parts, we introduce the atomic mass unit, u, which is defined to be the approximate contribution to the mass of an atom due to each proton and neutron.

$$\text{u} \simeq m_{\text{proton}} \simeq m_{\text{neutron}}$$

Thus the approximate mass of an atom is $A\,\text{u}$.

$$m_{\text{atom}} \simeq A\,\text{u}$$

We precisely define u in terms of the carbon-12 isotope, ${}^{12}_{6}\text{C}$. Carbon has $Z = 6$. The 12 refers to the mass number, $A = 12$. u is defined as $1/12^{\text{th}}$ the mass of ${}^{12}_{6}\text{C}$.

$$\text{u} = \frac{1}{12}\,\text{mass}\left({}^{12}_{6}\text{C}\right) = 1.6605\times10^{-27}\ \text{kg} = \frac{1}{6.0221\times10^{23}}\ \text{g}$$

The expression above shows the connection between u and Avogadro's number, $N_A\,\text{u} = 1\ \text{g}/\text{mol}$, but this is not critical for our discussion here.

The notation used for carbon-12 is more generally used. We write an isotope with atomic number $Z$, and mass number $A$

$${}^{A}_{Z}X$$

where $X$ is the symbol for that element. For instance

$${}^{235}_{92}\text{U}$$

is uranium-235, where uranium has $Z = 92$ and this is the $A = 235$ isotope.

### The Size of the Nucleus

Inside a nucleus the protons and neutrons are quite densely packed and there is not mush empty space between the nucleons. It is analogous to water molecules in liquid water where there the total volume occupied by the water molecules is the total volume occupied by the water. Water is, to a good approximation, an incompressible fluid. The nucleus also behaves like an incompressible fluid and the total volume of the nucleus is proportional to the total number of nucleons. An approximate expression relating the radius to the number of nucleons is

$$r = 1.2\ \text{fm}\times A^{1/3}\ \ \text{where}\ \ \text{fm} = 10^{-15}\ \text{m is a femto meter or fermi.}$$

The volume of a sphere is $V = \frac{4}{3}\pi r^3$, so it follows that the volume is proportional to $A$, the number of nucleons.

**Example M.1 - The Neutron Star**

A neutron star is a state of collapse after a star explodes in a supernova. The density of a neutron star is that of nuclear matter. A star the size of the sun is too small to supernova but heavier stars do. After a supernova, there are two final states, there is the neutron star but very large stars turn out to be too massive to stop at the neutron star state; they continue collapsing to a black hole.

What is the radius of a neutron star with a mass of three solar masses, $M = 3\,M_{\text{sun}}$, where $M_{\text{sun}} = 1.99\times10^{30}$ kg?

**Solution**

$$M_{\text{sun}} = 1.99\times10^{30}\ \text{kg} \implies M = 3\,M_{\text{sun}} = 5.97\times10^{30}\ \text{kg}$$

Because the neutron star is the density of nuclear matter the size formula applies. We need to find the mass number $A$.

$$\text{u} = 1.6605\times10^{-27}\ \text{kg} \implies M = A\,\text{u} \implies A = M/\text{u} = 3.60\times10^{57}$$

Using the size formula we can find its radius.

$$r = 1.2\ \text{fm}\times A^{1/3} = 1.2\times10^{-15}\ \text{m}\times A^{1/3} = 18\,400\ \text{m} = 18.4\ \text{km}$$

## M.2 - Radioactive Decay

### Conservation Laws in Nuclear Reactions

There are three types of radioactive decay in a nucleus: alpha decay, beta decay and gamma decay. Before discussing each we should put constraints on what is possible in nuclear reactions. The fundamental conservation laws of physics apply: energy, linear momentum, angular momentum and electric charge are conserved. In addition the number of nucleons $A$ will always stay the same. As a practical matter, we can concentrate on the nucleon number $A$ and the total electric charge being conserved.

## Alpha Decay

An alpha particle $\alpha$ is just the nucleus of helium-4, the nucleus of the standard isotope of helium.

$$\alpha = {}^{4}_{2}\text{He}$$

It follows than in any alpha decay decreases $A$ by 4 and $Z$ by 2.

$${}^{A}_{Z}X \longrightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}\text{ He}$$

Where $X$ is called the parent and $Y$ is the daughter

$\alpha$-rays are easily blocked. A small amount of paper is often sufficient. Does that mean people should not be concerned with $\alpha$-emitters? It is not your proximity to the $\alpha$-emitting atoms it is the possibility for inhalation or ingestion. If an $\alpha$-emitter is airborne or in the food supply then it is very dangerous. The radiation inside the body is very dangerous and can easily cause cancer.

### Example M.2 - $\alpha$ Decay

What is the resulting daughter nucleus when a ${}^{238}_{92}\text{Ur}$ (Uranium-238) undergoes $\alpha$ decay.

#### Solution

We lower $A$ by 4 and $Z$ by 2.

$${}^{238}_{92}\text{Ur} \longrightarrow {}^{234}_{90}Y + {}^{4}_{2}\text{ He}$$

Looking up what $Z = 90$ is on a periodic table (You would not be expected to memorize this.) you get Thorium. So the answer is

$${}^{234}_{90}\text{Th}$$

## Beta Decay and the Weak Nuclear Force

The neutron is more massive than the proton and can decay into a proton. Of course, there has to be more to that decay because charge is not conserved. It was found that a neutron decays into a proton and electron. This is called a beta decay. The half-life of this decay for a free neutron, where free means not in a nucleus, is about 10.3 min. A proton is less massive than a neutron so a free proton decaying into a neutron (and other things) could not conserve energy. Inside a nucleus the situation is different. The nucleus itself can provide the added energy to allow the proton to beta decay; the proton becomes a neutron and a positron, the antiparticle of the electron.

Beta decay was discovered well before the neutron, however. It was observed that a nucleus would emit an electron or positron and the nucleus would then change to a different nucleus with the appropriate $A$ and $Z$. When studying this decay, they discovered an anomalous result. If nucleus decayed into just those two particles, the daughter nucleus and electron (or positron), then to conserve both energy and momentum, both the energy and momentum of the ejected electron would be fixed; every decay of a given nucleus would give the same electron energies and momenta. That was not observed. The ejected electron had a continuous distribution of different energies and momenta. In 1930 after this anomaly was discovered, Wolfgang Pauli proposed a solution, that this implied there was a light neutral particle being emitted. This was later called the neutrino and we use the Greek letter $\nu$ (nu) for it. Niels Bohr actually suggested that energy was not conserved in beta decay; calmer heads prevailed. Beta decay could not be described with the three known fundamental forces: gravity, electromagnetism and the strong nuclear force. It required the addition of a fourth force, the weak nuclear force.

The reaction in a nucleus becomes this

$${}^{A}_{Z}X \longrightarrow {}_{Z+1}^{\;\;A}Y + e^{-} + \overline{\nu}$$

The bar over the neutrino indicates it is actually an antineutrino. This is sometimes written as $\beta^{-}$ decay. When a proton in a nucleus decays to a neutron and positron then the reaction takes the form:

$${}^{A}_{Z}X \longrightarrow {}_{Z-1}^{\;\;A}Y + e^{+} + \nu$$

Here the $e^{+}$is a positron and the neutrino lacks its antiparticle bar. This is $\beta^{+}$decay. Neutrinos only interact by the weak nuclear force. Because of this they are very hard to detect or absorb. Every nanosecond around a million neutrinos pass through each of us without interacting at all.

As an example of beta decay, consider ${}^{14}_{6}\text{C}$, carbon-14. This has a $\beta^{-}$ decay to nitrogen-14 with a half-life of 5730 yr and is useful for radioactive dating, which we will discuss in a bit. The reaction is

$${}^{14}_{6}\text{C} \longrightarrow {}^{14}_{7}\text{N} + e^{-} + \overline{\nu}\,.$$

If it decays with that half-life, why do we observe it. The earth is old and essentially all of it would be gone by now. Well, it is created in the upper atmosphere by the reverse (almost) of this process, beta absorption by nitrogen-14. Nitrogen-14 is the most common element in the atmosphere and the solar wind contains high energy electrons. Here is the reaction.

$$^{14}_{7}\mathrm{N} + e^- \longrightarrow\ ^{14}_{6}\mathrm{C} + \nu\,.$$

So carbon-14 is continually replenished in the upper atmosphere.

Beta rays travel further through matter than alpha rays.

**Example M.3 - Beta Decay Examples**

Write the nuclear reaction expression for the $\beta^+$ decay of $^{10}_{6}C$, carbon-10 and for the $\beta^-$ decay of $^{234}_{90}\mathrm{Th}$, thorium-234.

**Solution**

For the $\beta^+$ decay carbon will go to something with a lower $Z$ and that is $Z = 5$ which is boron.

$$^{10}_{6}\mathrm{C} \longrightarrow\ ^{10}_{5}\mathrm{B} + e^+ + \nu$$

The $\beta^-$ decay will give a higher $Z$ which, as everyone knows, is Protactinium Pa. (You would not be expected to know the element names for different $Z$.)

$$^{234}_{90}\mathrm{Th} \longrightarrow\ ^{234}_{91}\mathrm{Pa} + e^- + \overline{\nu}$$

## Gamma Decay

Gamma decay is just the result of an excited nucleus emitting a photon, just as atoms do. The difference is that there is a lot more possible energy available in the nucleus so these are very energetic photons, gamma rays. Often the result of some other nuclear decay leaves a nucleus in an excited state and that will eventually decay down. As an example, the nitrogen-14 nucleus after the beta decay discussed above could be in an excited state, which we denote with the asterisk *

$$^{14}_{6}\mathrm{C} \longrightarrow\ ^{14}_{7}\mathrm{N}^* + e^- + \overline{\nu}$$

and then that can undergo a $\gamma$-decay.

$$^{14}_{7}\mathrm{N}^* \longrightarrow\ ^{14}_{7}\mathrm{N} + \gamma$$

Note that generally for gamma decay $A$ and $Z$ are unchanged.

$$^{A}_{Z}X^* \longrightarrow\ ^{A}_{Z}X + \gamma$$

$\gamma$-rays are much more penetrating that both $\alpha$ and $\beta$. It often takes lead shielding to stop them.

## Radioactive Decay and Half-lives

Radioactive decay is an example of an exponential decay. We previously saw an exponential decay for a discharging capacitor. This is mathematically identical. The difference is that we typically talk about half lives here. Suppose $N_0$ is the initial number of atoms of some radioactive isotope. After one half life we have $N_0/2$ after two half lives we have $N_0/4$ and after $n$ half-lives we have $2^{-n}\,N_0$. After a time $t$ with a half-life of $T_{1/2}$ we have:

$$N(t) = N_0\,2^{-t/T_{1/2}}$$

Writing $2 = e^{\ln 2}$ and using the properties of log functions we get

$$N(t) = N_0\left(e^{\ln(2)}\right)^{-t/T_{1/2}} = N_0\,e^{-\ln(2)\,t/T_{1/2}}$$

We can then write this to look more like a standard exponential decay.

$$N(t) = N_0\,e^{-\lambda\,t} \text{ where } \lambda = \ln(2)/T_{1/2} = 0.691/T_{1/2}$$

This shows how $\lambda$, the decay constant, is related to $T_{1/2}$ the half life. Note that to mathematically solve this for $t$ or $\lambda$ you must take the natural log of both sides.

$$N(t) = N_0\,e^{-\lambda\,t} \implies e^{-\lambda\,t} = \frac{N}{N_0} \implies \lambda\,t = -\ln\left(\frac{N}{N_0}\right)$$

Since $N$ must be less than $N_0$, the natural log of something less than one gives a negative, making the value of $\lambda t$ positive.

#### Example M.4 - Archeological Dig

An archeologist discovers human remains in a dig. An analysis of the carbon shows a fraction of carbon-14 that is 0.15 the fraction of carbon-14 in the environment. The half life of carbon-14 is 5730 yr. How long has it been since the death of the human?

**Solution**

We can find the decay constant $\lambda$ from the half-life.

$$\lambda = \ln(2)/T_{1/2} \implies \lambda = \frac{\ln(2)}{T_{1/2}} = \frac{\ln(2)}{5730\text{ yr}} = 1.210 \times 10^{-4}\text{ yr}^{-1}$$

Because, as we saw earlier, carbon-14 is continually replenished in the environment, the fraction of the carbon-14 isotope stays constant. However, when dating something no new carbon is added after death. When comparing the fraction of carbon-14 in the sample to the fraction in the environment we can get a good estimate of the time since death. The ratio of $N$ to $N_0$ is the same as the fraction of carbon-14 in the sample to the environment.

$$N(t) = N_0\, e^{-\lambda t} \implies \lambda t = -\ln\left(\frac{N}{N_0}\right) = -\ln(0.15) \implies t = \frac{-\ln(0.15)}{\lambda} = 16\,700\text{ yr}$$

## M.3 - Fission and Fusion - the Curve of the Binding Energy

### Nuclear Binding Energy

We have mentioned binding energy before; it is the energy needed to bust up something into its constituents. For nuclear binding energy, it is the energy to break up a nucleus into separate constituents. If you know the mass of the nucleus and the and the mass of the proton and neutron then the missing mass gives, by $E = m c^2$, the binding energy. Usually we are given the atomic masses so we must then consider the small contribution of the electrons. To do so, instead of using the proton's mass use the mass of the hydrogen atom $^1$H, since that has one electron per proton. So to find the binding energy of ${}^{A}_{Z}X$

$$\text{BE}({}^{A}_{Z}X) = \left[Z\,\text{mass}({}^{1}\text{H}) + (A - Z)\,\text{mass}(n) - \text{mass}({}^{A}_{Z}X)\right] \times c^2$$

Because we are subtracting things to give small results it is essential that you keep many digits to get an accurate result.

#### Example M.5 - Carbon-12 Binding Energy

What is the binding energy of carbon-12 with a mass of exactly 12 u? The mass of $^1$H is 1.007825 u and the mass of $n$, the neutron is 1.008665 u

**Solution**

$$\begin{aligned}\text{BE}({}^{12}_{6}\text{C}) &= \left[6 \times \text{mass}({}^{1}\text{H}) + 6 \times \text{mass}(n) - \text{mass}({}^{12}_{6}\text{C})\right] \times c^2 \\ &= (6 \times 1.007825 + 6 \times 1.008665 - 12.000000)\text{ u } c^2 \\ &= 0.09894\text{ u } c^2\end{aligned}$$

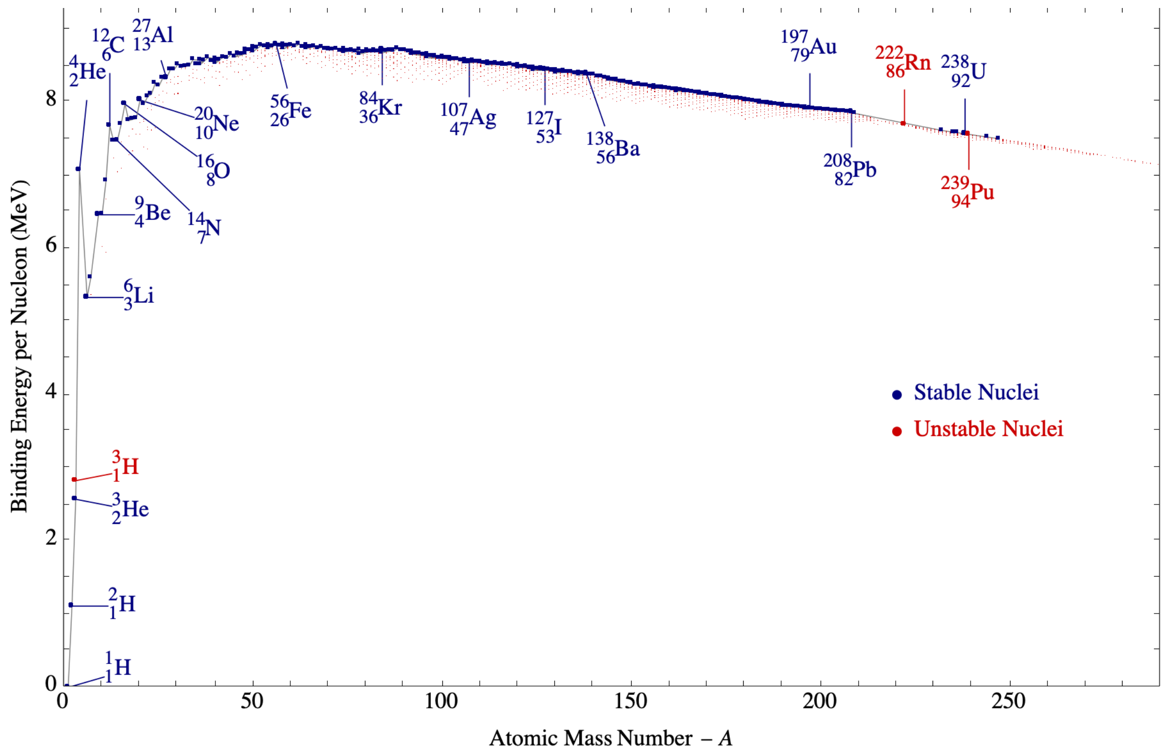
Using the values of constants and conversions we can evaluate our result in joules and then in eV.

$$\text{u} = 1.6605 \times 10^{-27}\text{ kg}, \quad \text{eV} = 1.60 \times 10^{-19}\text{ J and } c = 3.00 \times 10^{8}\text{ m/s}$$

$$\text{BE}({}^{12}_{6}\text{C}) = 1.48 \times 10^{-11}\text{ J} = 92.4\text{ MeV}$$

### The Curve of the Binding Energy

Lower energies are more stable but that means higher binding energies. It is instructive to plot the binding energy per nucleon. This is what is called the curve of the binding energy.




The Curve of the Binding Energy - The Graph shows Binding Energy per Nucleon for all isotopes.
Stable isotopes are shown in blue and unstable in red. Only half-lives longer than 1 s are included.

What this graph shows is that it is energetically favorable for smaller nuclei to combine and for larger nuclei to split. The combination of smaller nuclei to form larger ones is called fusion and the splitting of larger nuclei into smaller ones is called fission.

Where do elements come from? The big bang produced hydrogen and helium. The first generation of stars were fueled by fusion of these two light elements and that produced heavier elements. All elements except hydrogen and helium come from previous generation stars. But the peak of the curve of the binding energy is $^{56}_{26}Fe$; so this means fusion inside stars cannot produce elements heavier than iron. It requires a huge source of energy to make these heavier elements and that source is a supernova. So elements heavier than iron are remnants of previous supernovas.