Physics 2325 - Formula List

- **1D Kinematics**

General 1D Motion: x as a function of t

$v_{\text{ave}} = \frac{\Delta x}{\Delta t}$. $v = \frac{dx}{dt}$, $a_{\text{ave}} = \frac{\Delta v}{\Delta t}$. $a = \frac{dv}{dt}$

Constant Vel.: $x(t) = x_0 + v\,t \implies \Delta x = v\,t$

Constant Acc.: $v(t) = v_0 + a\,t$ and $x(t) = x_0 + v_0\,t + \frac{1}{2}\,a\,t^2$

$v = v_0 + a\,t$ $\quad \Delta x = \frac{1}{2}\left(v_0 + v\right)\,t$

$\Delta x = v_0\,t + \frac{1}{2}\,a\,t^2$ $\quad v^2 = v_0^2 + 2\,a\,\Delta x$

Free Fall : $x \to y$ (y is up) and $a = -g$

- **General Vectors** $\vec{A} = A_x\,\hat{x} + A_y\,\hat{y} = \langle A_x, A_y \rangle$

Mag. & Dir. angle $\Longrightarrow$ Components $A_x = A\cos\theta$ and $A_y = A\sin\theta$.

Components $\Longrightarrow$ Mag. & Dir. angle

$$A = \sqrt{A_x^2 + A_y^2} \text{ and } \theta = \begin{cases} \tan^{-1}\left(\frac{A_y}{A_x}\right) & \text{for } A_x > 0 \\ 180° + \tan^{-1}\left(\frac{A_y}{A_x}\right) & \text{for } A_x < 0 \end{cases}$$

- **General 2D Kinematics** $\vec{r}$ as a function of t

$\vec{v}_{\text{ave}} = \frac{\Delta\vec{r}}{\Delta t}$, $\vec{v} = \frac{d\vec{r}}{dt}$, $\vec{a}_{\text{ave}} = \frac{\Delta\vec{v}}{\Delta t}$, $\vec{a} = \frac{d\vec{v}}{dt}$, Ave. Speed $= \frac{\text{Total Distance}}{\text{Total Time}}$

Constant Acc.: $\vec{v}(t) = \vec{v}_0 + \vec{a}\,t$ and $\vec{r}(t) = \vec{r}_0 + \vec{v}_0\,t + \frac{1}{2}\,\vec{a}\,t^2$

- **Projectiles**

Horizontal: $a_x = 0 \implies v_x$ is const. $\quad$ Vertical: $a_y = -g$

$v_{0x} = v_0\cos\theta$ and $v_{0y} = v_0\sin\theta$.

$R = \frac{{v_0}^2}{g}\sin(2\,\theta)$, ($R = \Delta x$ when $\Delta y = 0$)

- **Relative Motion** $\vec{v} = \vec{v}' + \vec{v}_0$
- **Newton's Laws**

First Law: $\vec{v}$ is const., unless net force.

Second Law: $\vec{F}_{\text{net}} = m\,\vec{a}$

Third Law: $\vec{F}_{12} = -\vec{F}_{21}$

Weight $\propto$ Mass: $W = m\,g$

- **Friction between surfaces** $f_s \le \mu_s\,N$ (static), $f_k = \mu_k\,N$ (kinetic)
- **Circular Motion**

Uniform Circular Motion: $a_c = \frac{v^2}{r}$, Also $v = \frac{2\,\pi\,r}{T} \implies a_c = \left(\frac{2\,\pi}{T}\right)^2 r$

General Circular Motion: $a_c = \frac{v^2}{r}$, $a_t = \frac{dv}{dt}$

- **Accelerated Frames** $\vec{g}_{\text{art}} = -\vec{a}$ (artificial gravity)

accelerated frame $\Longrightarrow$ false force opposite acc.

- **Dot or Scalar Product**

$\vec{A}\cdot\vec{B} = A\,B\cos\theta$ where $A = \sqrt{A_x^2 + A_y^2 + A_z^2}$

$\vec{A}\cdot\vec{B} = A_x\,B_x + A_y\,B_y + A_z\,B_z$

- **Work** $W = \int\vec{F}\cdot d\vec{r}$

const. force: $W = F\,\Delta x$ (1D) $\quad W = \vec{F}\cdot\Delta\vec{r}$ (2D or 3D)

$W_{\text{grav}} = -m\,g\,\Delta y$ (work done by gravity)

$W = \int_{x_i}^{x_f} F(x)\,dx$ (variable force in 1D)

- **Springs: Hooke's Law and Work**

$F = -k\,x$ (Hooke's Law), $W = -\frac{1}{2}\,k\left(x_f^2 - x_i^2\right)$

- **Work-Energy Theorem** $W_{\text{net}} = \Delta K$

W_{net} (net work). $K = \frac{1}{2}\,m\,v^2$ (kinetic energy)

$\vec{F}$ is conservative $\Longleftrightarrow 0 = \oint\vec{F}\cdot d\vec{r}$

conservative forces $\implies \Delta U = -W$ (U is potential energy)

Gravity: $U = m\,g\,y$ $\quad$ Spring: $U = \frac{1}{2}\,k\,x^2$

W_{nc} is work of all nonconservative forces.

$E = E^{\text{mech}} = K_{\text{tot}} + U_{\text{tot}} \implies E_i + W_{\text{nc}} = E_f$, $W_{\text{nc}} = 0 \implies E_i = E_f$

$W_{\text{nc}} = 0$, one mass, gravity is only cons. force

$\implies E_{\text{bottom}} = E_{\text{top}} \implies v_{\text{bottom}}^2 = v_{\text{top}}^2 + 2\,g\,h$

Power: $\mathcal{P} = \frac{dW}{dt} = \vec{F}\cdot\vec{v}$

- **Potential Energy $\implies$ Force**

1D: $F = -\frac{d}{dx}\,U$, $\quad$ 3D: $F_x = -\frac{\partial}{\partial x}\,U$, $F_y = -\frac{\partial}{\partial y}\,U$, $F_z = -\frac{\partial}{\partial z}\,U$

- **Momentum and Impulse-Momentum Theorem**

$\vec{p} = m\,\vec{v}$ (mom.) $\quad \vec{I} = \int_{t_i}^{t_f}\vec{F}\,dt$ (impulse)

$\vec{F}_{\text{net,ave}}\,\Delta t = \vec{I}_{\text{net}} = \Delta\vec{p} = m\left(\vec{v}_f - \vec{v}_i\right)$

- **Center of Mass for a System of Particles**

Discrete : $\quad M = \sum_i m_i \quad \vec{r}_{\text{cm}} = \frac{1}{M}\sum_i m_i\,\vec{r}_i$

Continuous : $M = \int dm \quad \vec{r}_{\text{cm}} = \frac{1}{M}\int\vec{r}\,dm$

- **Second Law for a Particle and System**

particle : $\vec{F}_{\text{net}} = m\,\vec{a} \quad \vec{F}_{\text{net}} = \frac{d}{dt}\vec{p}$

system : $\vec{F}_{\text{net}}^{\text{ext}} = M\,\vec{a}_{\text{cm}} \quad \vec{F}_{\text{net}}^{\text{ext}} = \frac{d}{dt}\vec{p}_{\text{tot}}$

$\vec{p}_{\text{tot}} = m_1\,\vec{v}_1 + m_2\,\vec{v}_2 + \ldots = M\,\vec{v}_{\text{cm}}$

- **Conservation of Momentum**

$\vec{F}_{\text{net}}^{\text{ext}} = 0 \implies \Delta\vec{p}_{\text{tot}} = 0 \implies \vec{p}_{\text{tot},i} = \vec{p}_{\text{tot},f}$

$F_{\text{net},x}^{\text{ext}} = 0 \implies \Delta p_{\text{tot},x} = 0 \implies p_{\text{tot},i,x} = p_{\text{tot},f,x}$

- **Collisions**

$\vec{p}_{\text{tot},i} = \vec{p}_{\text{tot},f} \implies m_1\,\vec{v}_{1i} + m_2\,\vec{v}_{2i} = m_1\,\vec{v}_{1f} + m_2\,\vec{v}_{2f}$

Elastic $\Longleftrightarrow K_{\text{tot},i} = K_{\text{tot},f}$

1D Elastic trick: $K_{\text{tot},i} = K_{\text{tot},f} \implies v_{1i} + v_{1f} = v_{2i} + v_{2f}$

Totally Inelastic: $\vec{v}_{1f} = \vec{v}_{2f} = \vec{v}_f \implies m_1\,\vec{v}_{1i} + m_2\,\vec{v}_{2i} = \left(m_1 + m_2\right)\vec{v}_f$

- **General Rotations about fixed axis:**

$\omega_{\text{ave}} = \frac{\Delta\theta}{\Delta t}$, $\omega = \frac{d\theta}{dt}$, $\alpha_{\text{ave}} = \frac{\Delta\omega}{\Delta t}$, $\alpha = \frac{d\omega}{dt}$

- **Constant Angular Acceleration**

$\omega = \omega_0 + \alpha\,t \quad \Delta\theta = \frac{1}{2}\left(\omega + \omega_0\right)t$

$\Delta\theta = \omega_0\,t + \frac{1}{2}\,\alpha\,t^2 \quad \omega^2 = \omega_0^2 + 2\,\alpha\,\Delta\theta$

- **Rotational and Linear Quantities**

$\vec{v} = r\,\omega\,\hat{u}_t$ or $v_t = r\,\omega$, $v_c = 0$

$\vec{a} = r\,\alpha\,\hat{u}_t + \omega^2\,r\;\hat{u}_c$ or $a_t = r\,\alpha$, $a_c = \omega^2\,r$

$\alpha = 0 \iff \omega = \text{const} = \frac{2\pi}{T}$

- **Moment of Inertia**

I for a distribution - r is $\perp$ dist. from axis
Discrete: $I = \sum m_i\,r_i^2$, Continuous: $I = \int r^2\,dm$

Parallel-axis Theorem: $I = I_{\text{cm}} + M\,d^2$

Moments for uniform bodies:
Thin rod about $\perp$ axis

thru. end: $I = \frac{1}{3}\,M\,L^2$, thru. center: $I = \frac{1}{12}\,M\,L^2$

$a \times b$ rectangular plate about $\perp$ axis thru. center: $I = \frac{1}{12}\,M\left(a^2 + b^2\right)$

Sphere about axis thru. center:

thin shelled hollow: $I = \frac{2}{3}\,M\,R^2$, solid: $I = \frac{2}{5}\,M\,R^2$

Hoop about $\perp$ axis thru. center: $I = M\,R^2$
(same as thin-shelled hollow cylinder)

Disk about $\perp$ axis thru. center: $I = \frac{1}{2}\,M\,R^2$ (same as solid cylinder)

- **Rotational Energy**

$K = \frac{1}{2}\,I\,\omega^2$, $U = M\,g\,y_{\text{cm}}$, $K_{\text{tot}} = \frac{1}{2}\,I_{\text{cm}}\,\omega^2 + \frac{1}{2}\,M\,v_{\text{cm}}^2$

- **Cross or Vector Product** $\vec{A} \times \vec{B} = \hat{u}\,A\,B\sin\theta$, right hand rule $\Rightarrow \hat{u}$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{x}\begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} - \hat{y}\begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} + \hat{z}\begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix}$$

- **Torque**

About Origin: $\vec{\tau} = \vec{r} \times \vec{F}$, About Axis: $\tau = r\,F_\perp = r_\perp\,F = r\,F\sin\theta$

Torque due to gravity: $\vec{\tau}_{\text{gravity}} = \vec{r}_{\text{cm}} \times M\,\vec{g}$

- **Angular Momentum of Particle**

About Origin: $\vec{L} = \vec{r} \times \vec{p}$, About Axis: $L = r\,p_\perp = r_\perp\,p = r\,p\sin\theta$

- **General Rigid Body Dynamics**

2nd Law: $\tau_{\text{net}} = I\,\alpha$ and $\tau_{\text{net}} = \frac{dL}{dt}$

Angular Momentum: $L = I\,\omega$

- **System of Particles** $\vec{\tau}_{\text{net}}^{\text{ext}} = \frac{d\vec{L}_{\text{tot}}}{dt}$

$\vec{\tau}_{\text{net}}^{\text{ext}} = 0 \implies \Delta\,\vec{L}_{\text{tot}} = 0$ (Conservation)

- **Equilibrium** $\vec{F}_{\text{net}} = \vec{0}$ and $\vec{\tau}_{\text{net}} = \vec{0}$
- **Newton's Law of Gravity**

Magnitude: $F = G\,\frac{m_1\,m_2}{r^2}$ Direction is attractive.

Discrete Distribution: $\vec{F} = -G\,m \sum \frac{m_i}{r_i^2}\,\hat{r}_i$

Continuous Distribution: $\vec{F} = -G\,m \int \frac{\hat{r}}{r^2}\,dm$

Sph. Shell: $F = G\,\frac{M\,m}{r^2}$ $(r > R)$, $F = 0$ $(r < R)$

$g = G\,\frac{M}{R^2}$ (at surface of spherical planet)

- **Gravitational Potential Energy**

Two masses: $U = -G\,\frac{M\,m}{r}$, Several masses: $U = -G\sum_{i<j}\frac{m_i\,m_j}{r_{ij}}$

Escape speed: $v_{\text{esc}} = \sqrt{\frac{2\,G\,M}{R}}$

- **Circular Orbits** $v^2 = G\,\frac{M}{r}$ and $T^2 = \frac{4\,\pi^2}{G\,M}\,r^3$

- **Simple Harmonic Motion** $\frac{d^2x}{dt^2} = -\omega^2\,x$, $\omega = 2\,\pi\,f = \frac{2\pi}{T}$

$x(t) = A\cos(\omega\,t + \phi) = x_0\cos\omega\,t + \frac{v_0}{\omega}\sin\omega\,t$

- **Energy** $E = \frac{1}{2}\,m\,v^2 + \frac{1}{2}\,k\,x^2 = \begin{cases} \frac{1}{2}\,k\,A^2 \\ \frac{1}{2}\,m\,v_{\text{max}}^2 \end{cases}$ (mass/spring)

$v = \pm\,\omega\sqrt{A^2 - x^2}$ and $v_{\text{max}} = \omega\,A$ (in general)

- **Examples of Simple Harmonic Motion**

Mass/Spring: $\omega = \sqrt{k/m}$
Physical Pendulum: $\omega = \sqrt{m\,g\,d/I}$
Simple Pendulum: $\omega = \sqrt{g/L}$

- **1D Wave Equation** $\frac{\partial^2}{\partial x^2}\,u = \frac{1}{v^2}\,\frac{\partial^2}{\partial t^2}\,u$

General Solution: $u(x,\,t) = f(x - v\,t) + g(x + v\,t)$

- **Sinusoidal Waves** $u(x,\,t) = A\cos(k\,x \pm \omega\,t + \phi)$

$\lambda = \frac{2\pi}{k}$, $f = \frac{\omega}{2\pi}$, $v = f\,\lambda = \frac{\omega}{k}$

- **Waves on a String** $u(x,\,t) \Rightarrow y(x,\,t)$

Speed: $v = \sqrt{T/\mu}$ where T = Tension, Power: $\mathcal{P} = \frac{1}{2}\,\mu\,\omega^2\,A^2\,v$

- **Temperature Scales**

$T_F = \frac{9}{5}\,T_C + 32$, $\Delta T_F = \frac{9}{5}\,\Delta T_C$, $T_K = T_C + 273$

- **Heat** Q = Heat added to system

$Q = m\,c\,\Delta T$ (Temp. change), $Q = \pm\,m\,L$ (phase change)

- **Ideal Gas Law** $P\,V = N\,k\,T$ and $P\,V = n\,R\,T$

n = # of moles, $N = N_A\,n$ = # of molecules
Masses: $m_{\text{tot}} = n\,m_{\text{mole}} = N\,m_{\text{molecule}}$

- **Work** $W = \int P\,dV = \pm$ Area (done by system)

Constant P: $W = P\,\Delta V$,
Ideal gas at constant T: $W = n\,R\,T\ln\left(V_{\text{f}}/V_{\text{i}}\right)$

- **First Law**

$\Delta U = Q - W$, $dU = \delta Q - \delta W$, (δ is inexact differential)

- **Entropy** $dS = \frac{\delta Q}{T} \implies \Delta S = \int \frac{\delta Q}{T}$

Const. T: $\Delta S = \frac{Q}{T}$, Changing T: $\Delta S = m\,c\ln\frac{T_{\text{f}}}{T_{\text{i}}}$

- **Second Law** For a thermally isolated system: $\Delta S_{\text{tot}} \geq 0$

- **Heat Engines** $Q_H = Q_C + W$, Efficiency: $e = \frac{W}{Q_H} = 1 - \frac{Q_C}{Q_H}$

Max. Eff.: $e_{\text{max}} = 1 - \frac{T_C}{T_H}$ (Carnot Engine is H.E. of max. eff.)