

# Physics 2325 - Formula List

## 1D Kinematics

General 1D Motion:  $x$  as a function of  $t$

$$\bar{v} = \frac{\Delta x}{\Delta t}, \quad v = \frac{dx}{dt}, \quad \bar{a} = \frac{\Delta v}{\Delta t}, \quad a = \frac{dv}{dt}$$

Constant Vel.:  $x(t) = x_0 + vt \Rightarrow \Delta x = vt$

Constant Acc.:  $v(t) = v_0 + at$  and  $x(t) = x_0 + v_0 t + \frac{1}{2} at^2$

$$v = v_0 + at \quad \Delta x = \frac{1}{2}(v_0 + v)t$$

$$\Delta x = v_0 t + \frac{1}{2} at^2 \quad v^2 = v_0^2 + 2a \Delta x$$

Free Fall:  $x \rightarrow y$  ( $y$  is up) and  $a = -g$

**General Vectors**  $\vec{A} = A_x \hat{x} + A_y \hat{y} = \langle A_x, A_y \rangle$

**Mag. & Dir. angle**  $\Rightarrow$  Components  $A_x = A \cos \theta$  and  $A_y = A \sin \theta$ .

**Components**  $\Rightarrow$  **Mag. & Dir. angle**

$$A = \sqrt{A_x^2 + A_y^2} \quad \text{and } \theta = \begin{cases} \tan^{-1}\left(\frac{A_y}{A_x}\right) & \text{for } A_x > 0 \\ 180^\circ + \tan^{-1}\left(\frac{A_y}{A_x}\right) & \text{for } A_x < 0 \end{cases}$$

**General 2D Kinematics**  $\vec{r}$  as a function of  $t$

$$\bar{v} = \frac{\Delta \vec{r}}{\Delta t}, \quad \vec{v} = \frac{d\vec{r}}{dt}, \quad \bar{a} = \frac{\Delta \vec{v}}{\Delta t}, \quad \vec{a} = \frac{d\vec{v}}{dt}, \quad \text{Ave. Speed} = \frac{\text{Total Distance}}{\text{Total Time}}$$

Constant Acc.:  $\vec{v}(t) = \vec{v}_0 + \vec{a}t$  and  $\vec{r}(t) = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a}t^2$

## Projectiles

Horizontal:  $a_x = 0 \Rightarrow v_x$  is const. Vertical:  $a_y = -g$

$v_{0x} = v_0 \cos \theta$  and  $v_{0y} = v_0 \sin \theta$ .

$$R = \frac{v_0^2}{g} \sin(2\theta), \quad (R = \Delta x \text{ when } \Delta y = 0)$$

**Relative Motion**  $\vec{v} = \vec{v}' + \vec{v}_0$

## Newton's Laws

First Law:  $\vec{v}$  is const., unless net force.

Second Law:  $\vec{F}_{\text{net}} = m\vec{a}$

Third Law:  $\vec{F}_{12} = -\vec{F}_{21}$

Weight  $\propto$  Mass:  $W = mg$

**Friction between surfaces**  $f_s \leq \mu_s N$  (static),  $f_k = \mu_k N$  (kinetic)

## Circular Motion

Uniform Circular Motion:  $a_c = \frac{v^2}{r}$ , Also  $v = \frac{2\pi r}{T} \Rightarrow a_c = \left(\frac{2\pi}{T}\right)^2 r$

General Circular Motion:  $a_c = \frac{v^2}{r}$ ,  $a_t = \frac{dv}{dt}$

**Accelerated Frames**  $\vec{g}_{\text{art}} = -\vec{a}$  (artificial gravity)

accelerated frame  $\Rightarrow$  false force opposite acc.

## Dot or Scalar Product

$$\vec{A} \cdot \vec{B} = AB \cos \theta \quad \text{where } A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

**Work**  $W = \int \vec{F} \cdot d\vec{r}$

const. force:  $W = F \Delta x$  (1D)  $W = \vec{F} \cdot \Delta \vec{r}$  (2D or 3D)

$W_{\text{grav}} = -mg \Delta y$  (work done by gravity)

$W = \int_{x_i}^{x_f} F(x) dx$  (variable force in 1D)

## Springs: Hooke's Law and Work

$F = -kx$  (Hooke's Law),  $W = -\frac{1}{2}k(x_f^2 - x_i^2)$

## Work-Energy Theorem

$$W_{\text{net}} = \Delta K$$

$W_{\text{net}}$  (net work).  $K = \frac{1}{2}mv^2$  (kinetic energy)

$\vec{F}$  is conservative  $\Leftrightarrow 0 = \oint \vec{F} \cdot d\vec{r}$

conservative forces  $\Rightarrow \Delta U = -W$  ( $U$  is potential energy)

Gravity:  $U = mgy$  Spring:  $U = \frac{1}{2}kx^2$

$W_{\text{nc}}$  is work of all nonconservative forces.

$$E = E^{\text{mech}} = K_{\text{tot}} + U_{\text{tot}} \Rightarrow E_i + W_{\text{nc}} = E_f, \quad W_{\text{nc}} = 0 \Rightarrow E_i = E_f$$

$W_{\text{nc}} = 0$ , one mass, gravity is only cons. force

$$\Rightarrow E_{\text{bottom}} = E_{\text{top}} \Rightarrow v_{\text{bottom}}^2 = v_{\text{top}}^2 + 2gh$$

Power:  $\mathcal{P} = \frac{dW}{dt} = \vec{F} \cdot \vec{v}$

## Potential Energy $\Rightarrow$ Force

$$1D: F = -\frac{d}{dx}U, \quad 3D: F_x = -\frac{\partial}{\partial x}U, F_y = -\frac{\partial}{\partial y}U, F_z = -\frac{\partial}{\partial z}U$$

## Momentum and Impulse-Momentum Theorem

$$\vec{p} = m\vec{v} \text{ (mom.)} \quad \vec{I} = \int_{t_i}^{t_f} \vec{F} dt \text{ (impulse)}$$

$$\vec{F}_{\text{net}} \Delta t = \vec{I}_{\text{net}} = \Delta \vec{p} = m(\vec{v}_f - \vec{v}_i)$$

## Center of Mass for a System of Particles

$$\text{Discrete: } M = \sum_i m_i \quad \vec{r}_{\text{cm}} = \frac{1}{M} \sum_i m_i \vec{r}_i$$

$$\text{Continuous: } M = \int dm \quad \vec{r}_{\text{cm}} = \frac{1}{M} \int \vec{r} dm$$

## Second Law for a Particle and System

$$\text{particle: } \vec{F}_{\text{net}} = m\vec{a} \quad \vec{F}_{\text{net}} = \frac{d}{dt}\vec{p}$$

$$\text{system: } \vec{F}_{\text{net}}^{\text{ext}} = M\vec{a}_{\text{cm}} \quad \vec{F}_{\text{net}}^{\text{ext}} = \frac{d}{dt}\vec{p}_{\text{tot}}$$

$$\vec{p}_{\text{tot}} = m_1 \vec{v}_1 + m_2 \vec{v}_2 + \dots = M \vec{v}_{\text{cm}}$$

## Conservation of Momentum

$$\vec{F}_{\text{net}}^{\text{ext}} = 0 \Rightarrow \Delta \vec{p}_{\text{tot}} = 0 \Rightarrow \vec{p}_{\text{tot},i} = \vec{p}_{\text{tot},f}$$

$$F_{\text{net},x}^{\text{ext}} = 0 \Rightarrow \Delta p_{\text{tot},x} = 0 \Rightarrow p_{\text{tot},i,x} = p_{\text{tot},f,x}$$

## Collisions

$$\vec{p}_{\text{tot},i} = \vec{p}_{\text{tot},f} \Rightarrow m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f}$$

Elastic  $\Leftrightarrow K_{\text{tot},i} = K_{\text{tot},f}$

1D Elastic trick:  $K_{\text{tot},i} = K_{\text{tot},f} \Rightarrow v_{1i} + v_{1f} = v_{2i} + v_{2f}$

Totally Inelastic:  $\vec{v}_{1f} = \vec{v}_{2f} = \vec{v}_f \Rightarrow m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = (m_1 + m_2) \vec{v}_f$

## General Rotations about fixed axis:

$$\bar{\omega} = \frac{\Delta \theta}{\Delta t}, \quad \omega = \frac{d\theta}{dt}, \quad \bar{\alpha} = \frac{\Delta \omega}{\Delta t}, \quad \alpha = \frac{d\omega}{dt}$$

## Constant Angular Acceleration

$$\omega = \omega_0 + \alpha t \quad \Delta \theta = \frac{1}{2}(\omega + \omega_0)t$$

$$\Delta \theta = \omega_0 t + \frac{1}{2} \alpha t^2 \quad \omega^2 = \omega_0^2 + 2\alpha \Delta \theta$$

■ **Rotational and Linear Quantities**

$$\vec{v} = r\omega \hat{u}_t \text{ or } v_t = r\omega, \quad v_c = 0$$

$$\vec{a} = r\alpha \hat{u}_t + \omega^2 r \hat{u}_c \text{ or } a_t = r\alpha, \quad a_c = \omega^2 r$$

$$\alpha = 0 \iff \omega = \text{const} = \frac{2\pi}{T}$$

■ **Moment of Inertia**

$I$  for a distribution -  $r$  is  $\perp$  dist. from axis

$$\text{Discrete: } I = \sum m_i r_i^2, \quad \text{Continuous: } I = \int r^2 dm$$

**Perpendicular-axis Theorem:** Planar object in  $xy$ -plane:  $I_z = I_x + I_y$

**Parallel-axis Theorem:**  $I = I_{\text{cm}} + M d^2$

**Moments for uniform bodies:**

Thin rod about  $\perp$  axis

$$\text{thru. end: } I = \frac{1}{3} M L^2, \quad \text{thru. center: } I = \frac{1}{12} M L^2$$

$$a \times b \text{ rectangular plate about } \perp \text{ axis thru. center: } I = \frac{1}{12} M (a^2 + b^2)$$

Sphere about axis thru. center:

$$\text{thin shelled hollow: } I = \frac{2}{3} M R^2, \quad \text{solid: } I = \frac{2}{5} M R^2$$

Hoop about  $\perp$  axis thru. center:  $I = M R^2$

(same as thin-shelled hollow cylinder)

Disk about  $\perp$  axis thru. center:  $I = \frac{1}{2} M R^2$  (same as solid cylinder)

■ **Rotational Energy**

$$K = \frac{1}{2} I \omega^2, \quad U = M g y_{\text{cm}}, \quad K_{\text{tot}} = \frac{1}{2} I_{\text{cm}} \omega^2 + \frac{1}{2} M v_{\text{cm}}^2$$

■ **Cross or Vector Product**  $\vec{A} \times \vec{B} = \hat{u} AB \sin \theta$ , right hand rule  $\Rightarrow \hat{u}$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{x} \begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} - \hat{y} \begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} + \hat{z} \begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix}$$

■ **Torque**

About Origin:  $\vec{\tau} = \vec{r} \times \vec{F}$ , About Axis:  $\tau = r F_{\perp} = r_{\perp} F = r F \sin \theta$

Torque due to gravity:  $\vec{\tau}_{\text{gravity}} = \vec{r}_{\text{cm}} \times M \vec{g}$

■ **Angular Momentum of Particle**

About Origin:  $\vec{L} = \vec{r} \times \vec{p}$ , About Axis:  $L = r p_{\perp} = r_{\perp} p = r p \sin \theta$

■ **General Rigid Body Dynamics**

$$\text{2nd Law: } \tau_{\text{net}} = I \alpha \text{ and } \tau_{\text{net}} = \frac{dL}{dt}$$

Angular Momentum:  $L = I \omega$

■ **System of Particles**  $\vec{\tau}_{\text{net}}^{\text{ext}} = \frac{d\vec{L}_{\text{tot}}}{dt}$

$$\vec{\tau}_{\text{net}}^{\text{ext}} = 0 \implies \Delta \vec{L}_{\text{tot}} = 0 \text{ (Conservation)}$$

■ **Equilibrium**  $\vec{F}_{\text{net}} = \vec{0}$  and  $\vec{\tau}_{\text{net}} = \vec{0}$

■ **Newton's Law of Gravity**

$$\text{Magnitude: } F = G \frac{m_1 m_2}{r^2}, \quad \text{Vector: } \vec{F}_{21} = -G \frac{m_1 m_2}{r_{12}^2} \hat{r}_{12}$$

$$\text{Discrete Distribution: } \vec{F} = -G m \sum \frac{m_i}{r_i^2} \hat{r}_i$$

$$\text{Continuous Distribution: } \vec{F} = -G m \int \frac{\hat{r}}{r^2} dm$$

$$\text{Sph. Shell: } F = G \frac{M m}{r^2} (r > R), \quad F = 0 (r < R)$$

$$g = G \frac{M}{R^2} \text{ (at surface of spherical planet)}$$

■ **Gravitational Potential Energy**

$$\text{Two masses: } U = -G \frac{M m}{r}, \quad \text{Several masses: } U = -G \sum_{i < j} \frac{m_i m_j}{r_{ij}}$$

$$\text{Escape speed: } v_{\text{esc}} = \sqrt{\frac{2GM}{R}}$$

$$\text{■ Circular Orbits } v^2 = G \frac{M}{r} \text{ and } T^2 = \frac{4\pi^2}{GM} r^3$$

$$\text{■ Simple Harmonic Motion } \frac{d^2 x}{dt^2} = -\omega^2 x, \quad \omega = 2\pi f = \frac{2\pi}{T}$$

$$x(t) = A \cos(\omega t + \phi) = x_0 \cos \omega t + \frac{v_0}{\omega} \sin \omega t$$

$$\text{■ Energy } E = \frac{1}{2} m v^2 + \frac{1}{2} k x^2 = \begin{cases} \frac{1}{2} k A^2 \\ \frac{1}{2} m v_{\text{max}}^2 \end{cases} \text{ (mass/spring)}$$

$$v = \pm \omega \sqrt{A^2 - x^2} \text{ and } v_{\text{max}} = \omega A \text{ (in general)}$$

■ **Examples of Simple Harmonic Motion**

$$\text{Mass/Spring: } \omega = \sqrt{k/m}$$

$$\text{Physical Pendulum: } \omega = \sqrt{m g d / I}$$

$$\text{Simple Pendulum: } \omega = \sqrt{g/L}$$

$$\text{■ 1D Wave Equation } \frac{\partial^2 u}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 u}{\partial t^2}$$

General Solution:  $u(x, t) = f(x - vt) + g(x + vt)$

■ **Sinusoidal Waves**  $u(x, t) = A \cos(kx \pm \omega t + \phi)$

$$\lambda = \frac{2\pi}{k}, \quad f = \frac{\omega}{2\pi}, \quad v = f \lambda = \frac{\omega}{k}$$

■ **Waves on a String**  $u(x, t) \Rightarrow y(x, t)$

$$\text{Speed: } v = \sqrt{T/\mu} \text{ where } T = \text{Tension, Power: } \mathcal{P} = \frac{1}{2} \mu v \omega^2 A^2$$

■ **Temperature Scales**

$$T_F = \frac{9}{5} T_C + 32, \quad \Delta T_F = \frac{9}{5} \Delta T_C, \quad T_K = T_C + 273$$

■ **Heat**  $Q =$  Heat added to system

$$Q = m c \Delta T \text{ (Temp. change), } Q = \pm m L \text{ (phase change)}$$

■ **Ideal Gas Law**  $P V = N k T$  and  $P V = n R T$

$n =$  # of moles,  $N = N_A n =$  # of molecules

Masses:  $m_{\text{tot}} = n m_{\text{mole}} = N m_{\text{molecule}}$

■ **Work**  $W = \int P dV = \pm \text{Area}$  (done by system)

Constant  $P$ :  $W = P \Delta V$ ,

Ideal gas at constant  $T$ :  $W = n R T \ln(V_f/V_i)$

■ **First Law**

$$\Delta U = Q - W, \quad dU = \delta Q - \delta W, \text{ (}\delta \text{ is inexact differential)}$$

$$\text{■ Entropy } dS = \frac{\delta Q}{T} \implies \Delta S = \int \frac{\delta Q}{T}$$

$$\text{Const. } T: \Delta S = \frac{Q}{T}, \quad \text{Changing } T: \Delta S = m c \ln \frac{T_f}{T_i}$$

■ **Second Law** For a thermally isolated system:  $\Delta S_{\text{tot}} \geq 0$

■ **Heat Engines**  $Q_H = Q_C + W$ , Efficiency:  $e = \frac{W}{Q_H} = 1 - \frac{Q_C}{Q_H}$

Max. Eff.:  $e_{\text{max}} = 1 - \frac{T_C}{T_H}$  (Carnot Engine is H.E. of max. eff.)