Chapter M

Temperature and Heat

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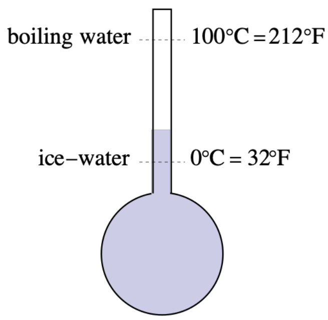
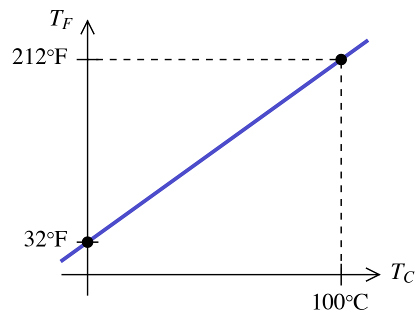
M.1 - Temperature Scales

Fahrenheit and Celsius Scales





Two commonly used temperature scales are Fahrenheit and Celsius. In the Fahrenheit scale the temperature of the freezing point of water (at one atmosphere) is 32 °F and the boiling point is 212 °F. For Celsius these two temperatures are 0 °C and 100 °C. It is straightforward to convert between the two. Consider a graph of T_F vs. T_C. We insist that these two scales are linearly related; this implies that the graph is a line. The slope of the line is

$$\text{slope} = \frac{\Delta T_F}{\Delta T_C} = \frac{212 - 32}{100 - 0} = \frac{9}{5}.$$

The intercept is 32. It follows that the conversion is

$$T_F = \frac{9}{5} T_C + 32.$$

Note that for temperature difference we get

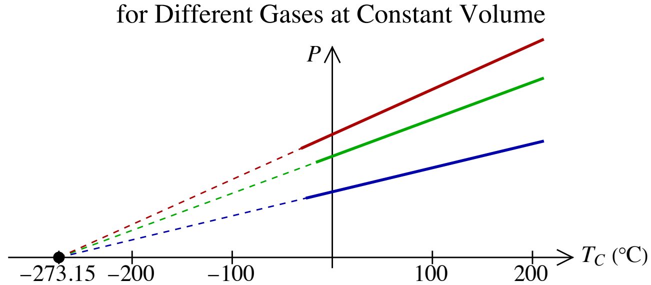
$$\Delta T_F = \frac{9}{5} \Delta T_C.$$

Constant Volume Gas Thermometers

A thermometer based on the expansion of a gas at constant volume was introduced. It was seen that a plot of pressure vs. temperature for different gases gave lines that had a common T-intercept, the temperature at zero pressure, at a very cold negative temperature.

Pressure vs. Temperature

for Different Gases at Constant Volume



This was the first hint of a coldest temperature that we now call absolute zero. It should be mentioned that absolute zero is much more fundamental than just a property of cold gases. It is a very fundamental value and the coldest temperature in *all* thermal experiments. In the

Celsius scale the value of absolute zero is now *defined exactly* as

$$T_{\text{absolute zero}} = -273.15\,^{\circ}\text{C}.$$

Absolute Temperature and Kelvin

An absolute temperature scale is one that is shifted to make absolute zero, zero in that scale. The absolute scale associated with Celsius is called the Kelvin scale. The conversion between Celsius and Kelvin is

$$T_K = T_C + 273.15\ .$$

Temperatures in Kelvin are given as K and not °K. We usually will take the above number to be just 273. Note that temperature differences in Kelvin are the same as in Celsius. We will give these Celsius temperature differences in K.

Example M.1 - Simple Conversions

(a) The temperature of liquid nitrogen at one atmosphere of pressure is 77 K. What is this in Celsius and in Fahrenheit?

Solution

$$T_C = T_K - 273\text{ K} = 77\text{ K} - 273\text{ K} = -196\,^{\circ}\text{C}$$

$$T_F = \frac{9}{5}\,T_C + 32 = -321\,^{\circ}\text{F}$$

(b) The nominal value for healthy human body temperature is 98.6 °F . What is this in Celsius and in Kelvin?

Solution

$$T_F = 98.6\,^{\circ}\text{F} \;\text{ and }\; T_F = \frac{9}{5}\,T_C + 32 \;\Longrightarrow\; T_C = \frac{5}{9}\,(T_F - 32) = 37\,^{\circ}\text{C}$$

$$T_K = T_C + 273\text{ K} = 37\,^{\circ}\text{C} + 273\text{ K} = 310\text{ K}$$

M.2 - Heat

Heat is thermal energy that flows from hot to cold, or more precisely, from higher temperature to lower temperature. It is an essential point that heat is thermal energy that moves. The static notion of thermal energy is a quite different thing which we will define later as *internal energy*. We will use Q to denote heat. We will choose the convention that Q is the heat *added* to a thermodynamic system. When heat is removed from something we take Q to be negative.

What is the effect of heat on a system? Suppose you add heat to a pot of water. The heat will increase the temperature of the water, usually. But if the water is at the boiling point the heat doesn't change the temperature; it changes the phase. Thus, when heat is added to a system it can either change the temperature of the system or change it's phase.

Temperature Change - Specific Heat

To raise the temperature of a fixed quantity (mass) of a material by some amount ΔT requires heat that is roughly proportional to the temperature change $Q \propto \Delta T$. This is precise as the ΔT becomes small. Moreover, to raise a substance by a fixed temperature requires an amount of heat proportional to m, the mass of the substance, $Q \propto m$. We can combine these proportionalities and get

$$Q = m\,c\,\Delta T.$$

c, the constant of proportionality is called the specific heat; it is a property of a material. Generally, it varies somewhat with the temperature and pressure of the substance, but we will typically neglect this change as small.

The specific heat c is a property of a material. The heat capacity C is a property of an object.

$$Q = C\,\Delta T$$

For example, a thermos has a heat capacity; the glass in a thermos has a specific heat. If an object is made of one material then $C = m\,c$. If it is made of several then $C = m_1\,c_1 + m_2\,c_2 + \ldots$.

Units: The SI unit for heat is: J = joule

Alternative units for heat are cal (calories), kcal or BTU.

- 1 cal raises 1 g of water by 1 K.
- 1 kcal raises 1 kg of water by 1 K.
- 1 BTU raises 1 lb_m (pound mass) by 1 F° (Fahrenheit degree).

Water has a high specific heat. Its value is:

$$c_{\text{water}} = 4186 \, \frac{\text{J}}{\text{kg}\cdot\text{K}} = 1 \, \frac{\text{kcal}}{\text{kg}\cdot\text{K}} = 1 \, \frac{\text{cal}}{\text{g}\cdot\text{K}} = 1 \, \frac{\text{BTU}}{\text{lb}_\text{m}\,\text{F}^\circ}$$

Note that the specific heat of ice is different than liquid water $c_{\text{ice}} = 2100 \, \frac{\text{J}}{\text{kg}\cdot\text{K}}$

Substance	Specific Heat $\left(\frac{\text{J}}{\text{kg K}}\right)$
Water	4186
Ice ($\lesssim$ 0 °C)	2100
Lead	130
Copper	390
Silver	234
Aluminum	910
Ethanol	2428
Iron	470

Specific Heats at 20 °C and 1 atm

Example M.2 - Hot Coffee, Cold Mug

0.25-kg of coffee at 82 °C, the official Starbucks temperature, is added to a 0.15-kg glass coffee mug at 20 °C. Assuming no heat exchange with the environment, what is the equilibrium temperature of the coffee/mug system? Take the specific heat of coffee to be that of water. For glass: $c_G = 0.20 \, \text{kcal}/(\text{kg}\cdot\text{K})$.

Solution

$$c_W = 1 \, \frac{\text{kcal}}{\text{kg}\cdot\text{K}} \, , \;\; c_G = 0.20 \, \frac{\text{kcal}}{\text{kg}\cdot\text{K}} \, , \;\; m_W = 0.25 \, \text{kg} \, , \;\; m_G = 0.15 \, \text{kg} \, , \;\; T_W = 82 \, ^\circ\text{C} \;\; \text{and} \;\; T_G = 20 \, ^\circ\text{C}$$

The total heat exchange between the coffee and mug is zero. Since Q is the heat added, it will be negative for the coffee.

$$0 = Q_{\text{tot}} = m_W \, c_W (T - T_W) + m_G \, c_G (T - T_G)$$

We can solve for the final temperature T.

$$T = \frac{m_W \, c_W \, T_W + m_G \, c_G \, T_G}{m_W \, c_W + m_G \, c_G} = 75.4 \, ^\circ\text{C}$$

Note that since the specific formulas involve temperature differences we can do the calculations without converting to Kelvins.

Example M.3 - Time to Boil Water

(a) How long does it take to boil a pot of water? To make this a good question we must know more information. Take there to be 2 kg of water (2 liters) initially at room temperature, 20 °C. We also need to know the rate at which heat is delivered; take the burner to be 2000 W, meaning that it delivers 2000 J of heat each second. Furthermore, we will neglect the heat capacity of the pot and ignore heat exchange with the environment.

Solution

$$c = 4186 \, \frac{\text{J}}{\text{kg}\cdot\text{K}} \, , \;\; m = 2 \, \text{kg} \, , \;\; \Delta T = T_f - T_i = 100 \, ^\circ\text{C} - 20 \, ^\circ\text{C} = 80 \, \text{K} \;\; \text{and} \;\; \mathcal{P} = 2000 \, \text{W}$$

The heat is the power times the time: $Q = \mathcal{P} \, t$.

$$\mathcal{P}\, t = m\, c\, \Delta T \implies t = m\, c\, \Delta T / \mathcal{P} = 335\ \text{s} = 5.58\ \text{min}$$

Phase Change - Latent Heat

The phase change between solids and liquids is called fusion. The liquid-gas transition is called vaporization. At the temperature of a phase transition the latent heat is the amount of heat per mass needed to change the phase.

$$Q = \pm m\, L$$

The sign must be added by hand. When going from solid to liquid heat is added, so Q is positive and from liquid to solid it is negative. Similarly, from liquid to gas $Q > 0$ and from gas to liquid $Q < 0$.

Substance	Melting Point (Fusion) Temperature	L_f – Latent Heat of Fusion (J/K)	Boiling Point (Vaporization) Temperature	L_v – Latent Heat of Vaporization (J/K)
Helium	–	–	-269 °C = 4.2 K	20.7×10^3
Hydrogen	-259 °C = 14 K	59.5×10^3	-252.9 °C = 20.3 K	445×10^3
Nitrogen	-210.0 °C = 63.2 K	25.3×10^3	-195.8 °C = 77.3 K	201×10^3
Oxygen	-219 °C = 54 K	13.7×10^3	-183 °C = 90 K	213×10^3
Methane	-182 °C = 91 K	58.4×10^3	-164 °C = 109 K	112×10^3
Ethanol	-114 °C = 159 K	104×10^3	78 °C = 351 K	854×10^3
Water	0 °C = 273 K	334×10^3	100 °C = 373 K	2260×10^3
Lead	327 °C = 600 K	24.5×10^3	1750 °C = 2023 K	179.5×10^3
Silver	962 °C = 1235 K	105×10^3	2163 °C = 2436 K	2390×10^3
Copper	1084 °C = 1357 K	209×10^3	2562 °C = 2835 K	4730×10^3
Gold	1064 °C = 1337 K	63.7×10^3	2856 °C = 3229 K	1645×10^3
Iron	1538 °C = 1811 K	247×10^3	2861 °C = 3134 K	6090×10^3
Silicon	1414 °C = 1687 K	1790×10^3	3265 °C = 3538 K	$12\,800 \times 10^3$

Latent heats and phase transition temperatures for different substances at 1 atm. (Reference)

From the table above, we can see the following.

- For water: fusion at 0 °C $L_f = 334 \times 10^3$ J/kg
  vaporization at 100 °C $L_v = 2260 \times 10^3$ J/kg
- For lead: fusion at 327 °C $L_f = 24.5 \times 10^3$ J/kg
  vaporization at 1750 °C $L_v = 179.5 \times 10^3$ J/kg

Note that L_f for lead is much less than for that for water. One could easily conclude, absurdly, that it is easier to melt lead than water, however it takes considerable heat to raise its temperature to 327 °C.

Not all materials have all three phases at one atmosphere. At that pressure, helium does not have a solid phase. Also at one atmosphere, carbon dioxide lacks a liquid phase; the direct transition from solid to gas is known as sublimation and there is a latent heat of sublimation.

- For CO_2: sublimation at -78.5 °C $L_s = 571 \times 10^3$ J/kg

Example M.4 - Time to Boil Water (Continued)

(b) To continue the previous example: How much longer does it take to boil away all the water?

Solution

$$L_v = 2260 \times 10^3\ \frac{\text{J}}{\text{kg}}, \quad m = 2\ \text{kg} \quad \text{and} \quad \mathcal{P} = 2000\ \text{W}$$

The heat is the power times the time: $Q = \mathcal{P}\, t$.

$$\mathcal{P}\, t = m\, L_v \implies t = m\, L_v / \mathcal{P} = 2260\ \text{s} = 37.7\ \text{min}$$

(c) What is the ratio of the time it takes to boil away water to the time it takes to bring it from 20 °C to a boil? This is the ratio of the time for part (b) to the time for part (a). This will be independent of both the mass of water and the power of the burner.

Solution

$$c = 4186\ \frac{\mathrm{J}}{\mathrm{kg\cdot K}}\ ,\quad \Delta T = 80\ \mathrm{K}\ \text{ and }\ L_v = 2.26\times 10^6\ \frac{\mathrm{J}}{\mathrm{kg}}$$

The heat is the power times the time: $Q = \mathcal{P}\,t$.

$$t_b / t_a = \frac{m\,L_v/\mathcal{P}}{m\,c\,\Delta T/\mathcal{P}} = \frac{L_v}{c\,\Delta T} = 6.75$$

Example M.5 - Add Ice to Water

A quantity of mass m of ice at −25 °C is added to 5-kg of water at 20 °C. There are three possible final states, depending on the value of m. For small m all the ice will melt and you will end up with water at some temperature less than 20°C. If there is a lot of ice then all the water will freeze and this results in ice at a temperature higher than −25 °C. Between the two cases you end up with ice-water, meaning ice and water in equilibrium at 0 °C. Which ranges of values of m will give each of the three outcomes.

Solution

The relevant constants and given information is

$$m_W = 5\ \mathrm{kg}\ ,\ c_I = 2100\ \frac{\mathrm{J}}{\mathrm{kg\cdot K}}\ ,\ c_W = 4186\ \frac{\mathrm{J}}{\mathrm{kg\cdot K}}\ \text{ and }\ L_f = 334\times 10^3\ \frac{\mathrm{J}}{\mathrm{kg}}$$

It suffices to find m_1, the critical initial amount of ice to melt all the ice and end up with all water at 0 °C and m_2, the critical amount to end up with all ice at 0 °C.

The three ranges of m-values are

$$m < m_1 \implies \text{all water}\ ,\ m_1 \le m \le m_2 \implies \text{ice} - \text{water}\ \text{ and }\ m > m_2 \implies \text{all ice}$$

To find m_1 we have three steps to consider

$$m_1 \text{ of ice at } -25°\mathrm{C} \overset{1}{\implies} m_1 \text{ of ice at } 0°\mathrm{C} \overset{2}{\implies} m_1 \text{ of water at } 0°\mathrm{C} \quad \text{and} \quad m_W \text{ of water at } 20°\mathrm{C} \overset{3}{\implies} m_W \text{ of water at } 0°\mathrm{C}$$

The total heat exchange is zero. Summing over the heats added in each of the three steps above gives.

$$0 = Q_{\text{tot}} = Q_1 + Q_2 + Q_3 = m_1\,c_I\,\Delta T_I + m_1\,L_f + m_W\,c_W\,\Delta T_W$$

Solve for m_1 using $\Delta T_I = +25$ K and $\Delta T_W = -20$ K.

$$m_1 = -\frac{m_W\,c_W\,\Delta T_W}{c_I\,\Delta T_I + L_f} = 1.08\ \mathrm{kg}$$

We can find m_2 similarly but the three steps are now

$$m_2 \text{ of ice at } -25°\mathrm{C} \overset{1}{\implies} m_2 \text{ of ice at } 0°\mathrm{C} \ \text{ and }\ m_W \text{ of water at } 20°\mathrm{C} \overset{2}{\implies} m_W \text{ of water at } 0°\mathrm{C} \overset{3}{\implies} m_W \text{ of ice at } 0°\mathrm{C}$$

The total heat exchange is zero. Summing over the heats added in each of the three steps above gives.

$$0 = Q_{\text{tot}} = Q_1 + Q_2 + Q_3 = m_2\,c_I\,\Delta T_I + m_W\,c_W\,\Delta T_W - m_W\,L_f$$

Solve for m_2 using the same ΔT values $\Delta T_I = +25$ K and $\Delta T_W = -20$ K.

$$m_2 = \frac{-m_W\,c_W\,\Delta T_W + m_W\,L_f}{c_I\,\Delta T_I} = 39.8\ \mathrm{kg}$$

Thermal Equilibrium and the Zeroth Law of Thermodynamics

We define two systems to be in *thermal equilibrium* when no heat will flow between them when they are brought into thermal contact.

Thermal contact means that the systems are linked in such a way where heat (and only heat) can flow from one to another. This linking could be some wall that conducts heat or it could be a conducting rod. Two systems are in thermal equilibrium if and only if they are at the same temperature.

After the three laws of thermodynamics were well established, it was realized that there was an additional assumption implicit in the notion of temperature; this became the "zeroth law." If system A is in thermal equilibrium with system B and system B is in thermal equilibrium with system C, then A is in thermal equilibrium with C. Physically, this allows us to make a thermometer. Think of system B as a thermometer. If systems A and C read the same value on B then they must be at the same temperature.

When two systems are brought into thermal contact, heat will flow from one system to the other until they reach thermal equilibrium. The time it takes to reach thermal equilibrium depends on the nature of the thermal contact. Regardless of the details of the thermal contact, the systems will eventually reach thermal equilibrium but the time required will depend on the quality of the thermal contact.