Chapter A - Problems

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Problem A.1

Suppose that x is a length, t is a time and a is an acceleration. To get a general expression for x as a function of both t and a, we choose the general form:

$$x = \kappa\, a^m\, t^n$$

where κ is a dimensionless constant. For this expression to be dimensionally correct what must m and n be?

Solution to A.1

Since x is a length, t is a time and a is an acceleration we get: $[x] = \mathrm{L}$, $[t] = \mathrm{T}$ and $[a] = \frac{\mathrm{L}}{\mathrm{T}^2}$. A dimensionless constant has dimension 1, so $[\kappa] = 1$. Equating the dimensions of both sides of our general expression gives:

$$[x] = [\kappa]\,[a]^m\,[t]^n \implies \mathrm{L}^1\,\mathrm{T}^0 = 1 \cdot \left(\frac{\mathrm{L}}{\mathrm{T}^2}\right)^m \times \mathrm{T}^n = \mathrm{L}^m \times \mathrm{T}^{n-2\,m}$$

When we equate the powers of L and T we get two equations:

$$1 = m \;\text{ and }\; 0 = n - 2\,m$$

which has the unique solution $m = 1$ and $n = 2$.

Problem A.2

Three fundamental constants G, c and $\hbar$ have dimensions:

$$[G] = \frac{\mathrm{L}^3}{\mathrm{M}\cdot\mathrm{T}^2},\; [c] = \frac{\mathrm{L}}{\mathrm{T}} \text{ and } [\hbar] = \frac{\mathrm{M}\cdot\mathrm{L}^2}{\mathrm{T}}.$$

(a) What must m, n and p be to make L_0a length, when

$$L_0 \;=\; G^m \cdot c^n \cdot \hbar^p.$$

(b) $\hbar$, called "h bar", is a rescaled version of Plank's constant h; it is usually also called Plank's constant.

$$\hbar = \frac{h}{2\,\pi} = 1.054 \times 10^{-34}\ \mathrm{J}\cdot\mathrm{s}$$

Using this and the values of G and c given in the book evaluate L_0.

Comment: G is *Newton's Gravitational constant*, c is the *speed of light* which plays a central role in Relativity and $\hbar$ is called *Plank's constant*, which appears in Quantum theory. L_0 is known as the *Plank length*; it is the very small distance that will set the scale of some ultimate theory of *Quantum Gravity*. Quantum Gravity is some yet undiscovered theory that consistently combines Gravity, Relativity and Quantum theory.

Solution to A.2

(a) The dimensions of both sides of the equation must be equal. This gives three linear equations for the three unknowns.

$$[L_0] = [G]^m \cdot [c]^n \cdot [\hbar]^p \implies \mathrm{L} = \left(\frac{\mathrm{L}^3}{\mathrm{M} \cdot \mathrm{T}^2}\right)^m \cdot \left(\frac{\mathrm{L}}{\mathrm{T}}\right)^n \cdot \left(\frac{\mathrm{M} \cdot \mathrm{L}^2}{\mathrm{T}}\right)^p \implies \mathrm{L}^1 \cdot \mathrm{M}^0 \cdot \mathrm{T}^0 = \mathrm{L}^{3m+n+2p} \cdot \mathrm{M}^{p-m} \cdot \mathrm{T}^{-(2m+n+p)}$$

Equating powers of the three fundamental dimensions L, M and T gives the linear equations for the unknowns m, n and p.

$$1 = 3m + n + 2p\ ,\ 0 = p - m \ \text{ and } \ 0 = 2m + n + p$$

The middle equation allows us to eliminate p. The other 2 equations become $1 = 5m + n$ and $0 = 3m + n$. Subtracting gives $1 = 2m$.

The solution is $m = \frac{1}{2}$, $n = -\frac{3}{2}$ and $p = \frac{1}{2}$.

(b) From part (a) $L_0 = \sqrt{\frac{\hbar \cdot G}{c^3}} = \sqrt{\frac{1.0546\times10^{-34} \cdot 6.673\times10^{-11}}{(2.998\times10^8)^3}} = 1.616\times10^{-35}$ m

Comment: Compare this with the size of atoms $\sim 10^{-10}$m and with the size of atomic nuclei $\sim 10^{-15}$m.

Problem A.3

(a) The period (time) for small oscillations of a simple pendulum T is given by:

$$T = 2\pi\sqrt{\frac{L}{g}}\ ,$$

where L is the length of the pendulum and g is the acceleration due to gravity. Verify that this expression is dimensionally correct.

(b) The velocity as a function of time for small oscillations of a pendulum is given by:

$$v = A \sin(\omega t + \phi),$$

where t is time. A, ω and ϕ are constants, the amplitude, angular frequency and phase. What are the dimensions and SI units of these constants?

Solution to A.3

(a) For the expression to be dimensionally correct it must be true that:

$$[\mathrm{T}\} = \ [2\pi]\sqrt{[\mathrm{L}]/[\mathrm{g}]} \quad \implies \ \mathrm{T} = 1 \times \sqrt{\frac{\mathrm{L}}{\mathrm{L}/\mathrm{T}^2}}$$

The above equation is satisfied, showing that the expression is dimensionally correct. (Note that since 2π is dimensionless, $[2\pi] = 1$.)

(b) The argument of sine (ωt in this case) must be dimensionless. Thus we must have $[\omega] = 1/[t] = T^{-1}$ and units$(\omega) = \mathrm{s}^{-1}$. It also follows that ϕ is dimensionless, $[\phi] = 1$ and units$[\phi] = 1$. Also the result of the sine function is dimensionless, so the dimension of v must be the same as what multiplies the sine, $[A] = \mathrm{L}/\mathrm{T}$ and units$(A) = \mathrm{m}/\mathrm{s}$.

Problem A.4

Which of the following equations are dimensionally correct?

(a) $v^2 - v_0^2 = 2\,a\,x$ (b) $x = v_0\,t + \frac{1}{3}\,a\,t^3$ (c) $x = \frac{1}{2}\left(v + v_0\right)t^2$

v and v_0 are velocities, a is acceleration, t is time and x is a distance.

Solution to A.4

(a) For this to be dimensionally correct then we must have
$[v]^2 = [v_0]^2 = [a]\cdot[x]$. This becomes: $\frac{L^2}{T^2} = \frac{L^2}{T^2} = \frac{L}{T^2}\cdot L$. This is satisfied so the expression is dimensionally correct.

(b) For this to be dimensionally correct then we must have
$[x] = [v_0]\,[t] = [a]\,[t]^3$. This becomes: $L = \frac{L}{T}\,T \neq \frac{L}{T^2}\,T^3$, which is not satisfied so the expression is not dimensionally correct.

(c) The two terms added must be dimensionally the same. They are, $[v] = [v_0] = L/T$. It also must be true that: $[x] = \left[\frac{1}{2}\right][v]\,[t]^2$. This becomes: $L \neq 1\cdot\frac{L}{T}\,T^2$, which is not satisfied so the expression is not dimensionally correct.

Problem A.5

Newton's Law of Universal Gravitation $F = G\,\frac{m_1\,m_2}{r^2}$ will be discussed in Chapter I. It relates the gravitational force F between point masses m_1 and m_2 separated by a distance r. G is Newton's Universal constant. What are the SI units of G?

Solution to A.5

$$\text{units}(G) = \frac{\text{units}(F)\times\text{units}(r)^2}{\text{units}(M)\times\text{units}(m)} = \frac{\text{N m}^2}{\text{kg}^2}$$

Problem A.6

Suppose volume V varies with time t by the relation: $V = \alpha\,t^2 + \beta\,t + \gamma$. What are the SI units of the constants α, β and γ?

Solution to A.6

$$\text{units}(\gamma) = \text{units}(V) = \text{m}^3$$

$$\text{units}(\beta) = \frac{\text{units}(V)}{\text{units}(t)} = \frac{m^3}{s}$$

$$\text{units}(a) = \frac{\text{units}(V)}{\text{units}(t)^2} = \frac{m^3}{s^2}$$

Problem A.7

One acre is 43 560 ft^2. How many acres are in a square mile? How many square meters in an acre?

Solution to A.7

$$1\ \text{mile}^2 = (5280\ \text{ft})^2\ \frac{1\ \text{acre}}{43\,560\ \text{ft}^2} = 640\ \text{acre}$$

$$1\ \text{acre} = 43\,560\ \text{ft}^2 \times \left(\frac{.3048\ \text{m}}{1\ \text{ft}}\right)^2\ =\ 4047.\ \text{m}^2$$

Problem A.8

Some units are obscure. In horse racing the length unit of furlongs is used. There are 8 furlongs to a mile. A fortnight is an obscure unit for time, where 1 fortnight = 14 days.

Suppose these units are used to study velocity and acceleration. Convert $1\ \frac{\text{furlong}}{\text{fortnight}}$ and $1\ \frac{\text{furlong}}{\text{fortnight}^2}$ to SI units.

Solution to A.8

$$\begin{aligned} & 1\ \frac{\text{furlong}}{\text{fortnight}} \times \frac{1\ \text{mile}}{8\ \text{furlong}} \times \frac{5280\ \text{ft}}{1\ \text{mile}} \times \frac{.3048\ \text{m}}{1\ \text{ft}} \\ & \quad \times \frac{1\ \text{fortnight}}{14\ \text{days}} \times \frac{1\ \text{day}}{24\ \text{hours}} \times \frac{1\ \text{hour}}{3600\ s} \\ = &\ \ 1.66 \times 10^{-4}\ \frac{\text{m}}{\text{s}} \end{aligned}$$

$$\begin{aligned} & 1\ \frac{\text{furlong}}{\text{fortnight}^2} \times \frac{1\ \text{mile}}{8\ \text{furlong}} \times \frac{5280\ \text{ft}}{1\ \text{mile}} \times \frac{.3048\ \text{m}}{1\ \text{ft}} \\ & \quad \times \left(\frac{1\ \text{fortnight}}{14\ \text{days}}\right)^2 \times \left(\frac{1\ \text{day}}{24\ \text{hours}}\right)^2 \times \left(\frac{1\ \text{hour}}{3600\ s}\right)^2 \\ = &\ \ 1.37 \times 10^{-10}\ \frac{m}{\text{s}^2} \end{aligned}$$