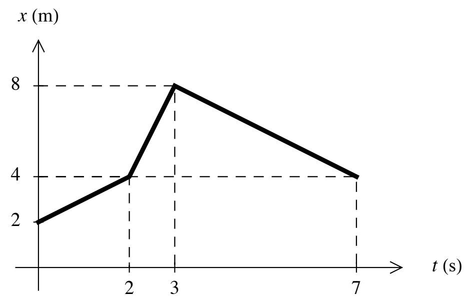
Chapter B - Problems

Blinn College - Physics 2425 - Terry Honan

Problem B.1



(a) What is the average velocity between 0 s and 2 s, between 3 s and 7 s, between 2 s and 7 s?

(b) What is the instantaneous velocity at 1 s and and at 4 s?

(c) What is the average acceleration between 1 s and 4 s?

(b) What is the instantaneous acceleration at 1 s and at 4 s?

Solution to B.1

(a) The average velocity is defined as $\overline{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{x_f - x_i}{t_f - t_i}$.

Between 0 s and 2 s: $\overline{v} = \dfrac{x(2) - x(0)}{2 - 0} = \dfrac{4 - 2}{2} = 1\ \dfrac{\text{m}}{\text{s}}$

Between 3 s and 7 s: $\overline{v} = \dfrac{x(7) - x(3)}{7 - 3} = \dfrac{4 - 8}{4} = -1\ \dfrac{\text{m}}{\text{s}}$

Between 2 s and 7 s: $\overline{v} = \dfrac{x(7) - x(2)}{7 - 2} = \dfrac{4 - 4}{5} = 0$

(b) The instantaneous velocity is the slope of the x vs. t graph. The instantaneous velocity of a straight-line segment is the average velocity for that segment.
The instantaneous velocity at $t = 1$ s is the same as the average velocity between 0 and 2 s, which is $1\ \dfrac{\text{m}}{\text{s}}$.
The instantaneous velocity at $t = 4$ s is the same as the average velocity between 3 s and 7 s, which is $-1\ \dfrac{\text{m}}{\text{s}}$.

(c) The average acceleration is defined as $\overline{a} = \dfrac{\Delta v}{\Delta t} = \dfrac{v_f - v_i}{t_f - t_i}$.

Between 1 s and 4 s: $\overline{a} = \dfrac{v(4) - v(1)}{4 - 1} = \dfrac{-1 - 1}{3} = -\dfrac{2}{3}\ \dfrac{\text{m}}{\text{s}^2}$

(d) During periods of constant velocity the acceleration is zero, so at both 1 s and 4 s, $a = 0$.

Problem B.2

A particle moves in one dimension. It's position as a function of time is given, in SI units, by: $x(t) = 2\,t^4 - 5\,t^2 + 18$.

(a) What is the average velocity between 2s and 4s?
(b) What is the instantaneous velocity at 3s?
(c) What is the average acceleration between 2s and 4s?
(d) What is the instantaneous acceleration at 3s?

Solution to B.2

(a) $x(2) = 2\cdot 2^4 - 5\cdot 2^2 + 18 = 30$ and $x(4) = 2\cdot 4^4 - 5\cdot 4^2 + 18 = 450$

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x(4)-x(2)}{4-2} = \frac{450-30}{4-2} = 210\ \mathrm{m/s}$$

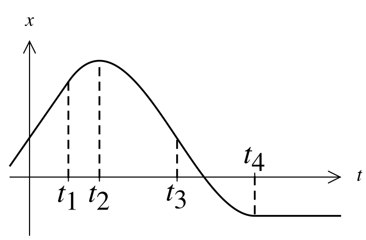
(b) $v = \frac{d}{dt}\left(2t^4 - 5t^2 + 18\right) = 2\cdot 4t^3 - 5\cdot 2t + 0 = 8t^3 - 10t \implies \mathrm{v}(3) = 8\cdot 3^3 - 10\cdot 3 = 186\ m/s$

(c) $v(2) = 8\cdot 2^3 - 10\cdot 2 = 44$ and $\mathrm{v}(4) = 8\cdot 4^3 - 10\cdot 4 = 472$

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v(4) - v(2)}{4-2} = \frac{472-44}{4-2} = 214\ \mathrm{m/s^2}$$

(d) $a = \frac{d}{dt}\left(8t^3 - 10t\right) = 8\cdot 3t^2 - 10\cdot 1 = 24t^2 - 10 \implies \mathrm{a}(3) = 24\cdot 3^2 - 10 = 206\ \mathrm{m/s^2}$

Problem B.3



The above diagram is a graph of position versus time. For $t < t_1$ the graph is a straight line. For $t > t_4$ the graph is a horizontal line. (t_3 is a point of inflection.)

(a) Where is the velocity zero? Where is it positive? Where is it negative?

(b) Where is the acceleration zero? Where is it positive? Where is it negative?

Solution to B.3

(a) Velocity is the slope of the x vs. t graph.
$v = 0$: $t = t_2$ and $t \geq t_4$
$v > 0$: $t < t_2$
$v < 0$: $t_2 < t < t_4$

(b) Acceleration is the slope of the v vs. t graph. One can get information about the acceleration from the original x vs. t graph; it is related to its concavity. For $t < t_1$ and $t > t_4$ the velocity is constant, so the acceleration is zero. After t_1 the slope is decreasing so the acceleration is negative. Even after t_2 the acceleration is negative because it is negative and becomes increasingly more negative. It reaches its most negative value at t_3, the point of inflection; this is where the graph changes from downward convexity ($a < 0$) to concave upward ($a > 0$). Thus at t_3 the acceleration is zero. The slope after $t_3$is negative but becomes less negative and gives a postive acceleration, up to t_4.

$a = 0$: $t < t_1$, $t = t_3$ and $t > t_4$

$a > 0$: $t_3 < t < t_4$

$a < 0$: $t_1 < t < t_3$

Problem B.4

Suppose the maximum stopping acceleration for a car on a given surface is fixed. Suppose the minimum stopping distance for this car moving at 30 mi/hr is 35 ft.

(a) What is its minimum stopping distance for this car travelling at 60 mi/hr?

(b) What is its stopping acceleration in ft$/s^2$?

Solution to B.4

(a) First we need to derive a formula relating the stopping distance S to the initial speed V and the acceleration a. To do this use $v_0 = V$, $v = 0$ and $\Delta x = S$.

$$v^2 - v_0^2 = 2\,a\,\Delta x \implies V^2 = -2\,a\,\text{s}$$

One can solve this problem by using the initial speed of 30 mi/hr and the distance of 35 ft to solve for the acceleration and then use this acceleration and the second initial speed of 60 mi/hr to solve for the second stopping distance. The difficulty of this method is that one must convert units twice to get the answer. A much easier method is to use ratios. From the equation relating speed and distance, it is clear that doubling the speed will quadruple the distance if the acceleration is kept fixed. This implies that the stopping distance is 140 ft.

We can formalize this discussion: Dividing $V_1^2 = -2\,a\,S_1$ into $V_2^2 = -2\,a\,S_2$ gives:

$$\left(\frac{V_2}{V_1}\right)^2 = \frac{S_2}{S_1} \implies \left(\frac{60}{30}\right)^2 = \frac{S_2}{35} \implies S_2 = 140\text{ ft.}$$

Note that the ratio of two speeds is the same in mi/hr as in m/s so there is no need for unit conversion here.

(b) We first need to convert mi/hr to ft/s:

$$30\,\frac{\text{mi}}{\text{hr}} \times \frac{5280\text{ ft}}{\text{mi}} \times \frac{1\text{ hr}}{3600\text{ s}} = 44\,\frac{\text{ft}}{\text{s}}$$

$$v^2 - v_0^2 = 2\,a\,\Delta x \implies 0 - 44^2 = 2\,a\,35 \implies a = 27.7\,\frac{\text{ft}}{\text{s}^2}$$

To put this in context, the acceleration due to gravity is $g = 32.0\,\frac{\text{ft}}{\text{s}^2}$

Problem B.5

A particle moves along a line with a constant acceleration. At $t = 0$ it is at $x = 0.3$ m moving in the positive direction at $1.2\,m/s$. If at $t = 2$ s it is at $x = -0.5$ m then what is its acceleration.

Solution to B.5

$v_0 = 1.2\, m/s,\ \Delta x = -0.5\ - 0.3 = -0.8$ and $t = 2$ s.

$$\Delta x = v_0\, t + \frac{1}{2}\, a\, t^2 \implies -0.8 = 1.2\times 2 + \frac{1}{2}\, a\, 2^2 \implies a = -1.6\ \frac{\text{m}}{\text{s}^2}$$

Problem B.6

A drag racer accelerates uniformly from rest at $8\ \frac{\text{m}}{s^2}$ over a distance of 400 m.

(a) What is the velocity of the car after the acceleration?
(b) How long does it take for the car to accelerate?

Solution to B.6

$v_0 = 0,\ a = \ 8\,\text{m}/\text{s}^2$ and $\Delta x = 400$ m.

(a) $v = ?\quad v^2 - v_0^2 = \ 2\, a\, \Delta x \implies v^2 - 0 = 2\cdot 8\cdot 400 \implies v = \ 80\, m/s.$

(b) $t = \ ?\quad \Delta x = v_0\,\text{t} + \frac{1}{2}\, a\, t^2 \implies 400 \ = \ 0 \ + \ \frac{1}{2}\, 8\ t^2 \implies t \ = 10\,\text{s}\ .$

Problem B.7

A ball is thrown straight upward from the ground at the base of a building. A person hanging out a 4 m high window catches the ball 1.5 s later.
(a) What was the velocity of the ball just after it was thrown?
(b) What was the velocity of the ball just before it was caught?

Solution to B.7

$\Delta y \ = \ 4\,\text{m}$ and $t = \ 1.5\, s$.

(a) $v_0 = ?\quad \Delta y = v_0\,\text{t} \ - \frac{1}{2}\, g\ t^2 \implies +4 = v_0\cdot 1.5 - \frac{1}{2}\, 9.8\cdot(1.5)^2 \implies v_0 = 10.0167 = 10.0\,\text{m/s}$

(b) $v = ?\quad \Delta y = \frac{1}{2}\left(v + v_0\right) t \implies 4 = \frac{1}{2}\,(v + 10.0167)\cdot 1.5 \implies v = \ -4.68\, m/s.$

Note the the negative sign of v indicates that the ball was caught as it was moving downward.

Problem B.8

A river flows at a rate of $5\ \frac{\text{m}}{\text{s}}$. A person standing on a bridge 3 m above the water wants to drop a ball into a boat that moves with the river's current. How far upstream should the boat be from the stream when the person drops the ball for it to land in the boat?

Solution to B.8

First find the time of fall of the ball.

$$h = \frac{1}{2} g t^2 \implies 3 = \frac{1}{2} 9.8 \cdot t^2 \implies t = 0.782461.$$

Next, solve for the distance the boat moves in this time.

$$\Delta x = v t = 5 \times .782461 = 3.91 \text{ m} .$$

Problem B.9

A ball is thrown vertically upward at an initial speed of 20 $\frac{\text{m}}{\text{s}}$ from the ground.

(a) What is the maximum height reached by the ball and how long does it take for the ball to reach the maximum height?
(b) 3 s after the ball is thrown, what is its height, velocity and acceleration?

Solution to B.9

$v_0 = 20 \text{ m}/s$. At the maximum height $v = 0$.

(a) $t = ?$ $v = v_0 - g t \implies 0 = 20 - 9.8 t \implies t = 2.0408 \text{ s}.$

$\Delta y = ?$ $v^2 - v_0^2 = -2 g \Delta y$ $\Delta y = \frac{v_0^2}{2 g} = \frac{20^2}{2 \cdot 9.8} = 20.408 \, m$.

(b) $\Delta y = v_0 t - \frac{1}{2} g t^2 = 20 \cdot 3 - \frac{1}{2} 9.8 \cdot 3^2 = 15.9 \text{ m}$
$v = v_0 - g t = 20 - 9.8 \cdot 3 = -9.4 \frac{\text{m}}{\text{s}}$.

Since it is still in free-fall: $\text{a} = -9.80 \text{ m}/\text{s}^2$.

Problem B.10

A helocopter accelerates straight upward from rest on the ground with a constant acceleration of $4 \frac{\text{m}}{s^2}$. Three seconds after leaving the ground a crate is dropped from the helicopter. How long does it take fo rthe crate to hit the ground after it is dropped.

Solution to B.10

This is a two step problem. The first part is motion with constant acceleration. After the crate is dropped it is in free-fall. The initial height and velocity of the crate are found using the motion of the helicopter.

This first part of the problem is a helicopter at a constant acceleration of $a = 4 \frac{\text{m}}{s^2}$ for t = 3 s. It starts from rest so $v_0 = 0$.

$$\Delta x = v_0 t + \frac{1}{2} a t^2 = 0 + \frac{1}{2} 4 \cdot 3^2 = 18 \text{ m} \text{ and } v = v_0 + a t = 0 + 4 \cdot 3 = 12 \frac{\text{m}}{\text{s}}$$

The second part of the problem is the free-fall of the crate. For the crate the initual velocity of the crate is the velocity of the helicopter when it was dropped and the Δy is the negative of the Δx. $v_0 = 12 \text{ m}/\text{s}$ and $\Delta y = -18 \, m$.

$$\Delta y = v_0 t - \frac{1}{2} g t^2 \implies -18 = 12 t - 4.9 t^2 \implies 4.9 t^2 - 12 t - 18 = 0$$

Use the quadratic formula to find t.

$$t = \frac{-b \pm \sqrt{b^2 - 4\,a\,c}}{2\,a} = \frac{12 \pm \sqrt{12^2 + 4\times4.9\times18}}{9.8} = 3.50 \text{ and } -1.05$$

Throw out the negative solution. (The motion starts at $t = 0$.) Thus $t = 3.50$ s .

Problem B.11

An object is dropped from the roof of a building. A person looking out a 25m high window hears it hit the ground 1.5 s after it passes the window. How high is the roof above the ground? You may neglect the travel time of the sound.

Solution to B.11

This is a two step problem. First consider the motion of the object after passing the window.

$$\Delta y = v_0\,\mathrm{t} - \frac{1}{2}\,g\,t^2 \implies -25 = v_0 \times 1.5 - \frac{1}{2}\times 9.80\times 1.5^2 \implies v_0 = -9.31667$$

This is the velocity of the object as it passes the window. The second step is the motion of the object before passing the window. Here it begins from rest, $v_0 = 0$, and $v = -9.31667$

$$v^2 - v_0^2 = -2\,g\,\Delta y \implies (-9.31667)^2 - 0 = -2\times 9.80\times\Delta \mathrm{y} \implies \Delta y = -4.42859 \implies \text{height} = 4.42859 + 25 = 29.4 \text{ m}$$