Chapter C - Problems

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Problem C.1

Two points have cartesian coordinates (3 m, −8 m) and (−5 m, 5 m).

(a) Convert these points to polar coordinates.

(b) What is the distance between these points?

Solution to C.1

(a) To convert from cartesian coordinates to polar use:

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \begin{cases} \tan^{-1}\left(\frac{y}{x}\right) & \text{when } x > 0 \\ 180\,° + \tan^{-1}\left(\frac{y}{x}\right) & \text{when } x < 0 \end{cases}$$

$(x_1, y_1) = (3, -8) \implies r_1 = \sqrt{3^2 + 8^2} = 8.54\text{ m}$ and $\theta_1 = \tan^{-1}\left(\frac{-8}{3}\right) = -69.4\,°$

$(x_2, y_2) = (-5, 5) \implies r_2 = \sqrt{5^2 + 5^2} = 7.07\text{ m}$ and $\theta_2 = 180\,° + \tan^{-1}\left(\frac{3}{-3}\right) = 135\,°$

(b) The distance between two points is:

$$\sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{233} = 15.3\text{ m}.$$

Problem C.2

Two points have polar coordinates $(r, \theta) = (5\text{ m}, 25\,°)$ and $(r, \theta) = (8\text{ m}, 120\,°)$.

(a) Convert these points to cartesian (rectangular) coordinates.
(b) What is the distance between these points?

Solution to C.2

(a) To convert from polar to cartesian coordinates use:

$$x = r\cos\theta \text{ and } y = r\sin\theta$$

$$(r_1, \theta_1) = (5\text{ m}, 25\,°) \implies (x_1, y_1) = (4.53\text{ m}, 2.11\text{ m})$$

$$(r_2, \theta_2) = (8\text{ m}, 120\,°) \implies (x_2, y_2) = (-4\text{ m}, 6.93\text{ m})$$

(b) The distance between points is

$$\sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = 9.77$$

Problem C.3

A car drives around a traffic circle with a radius of 20 m.
(a) When the car drives half way around the circle, what is the magnitude of the displacement vector and the total distance driven?
(b) When the car drives once around a full circle, what is the magnitude of the displacement vector and the total distance driven?

Solution to C.3

(a) The displacement vector is the vector from the starting position to the stopping point. It's magnitude is then the distance between these two point. In this case the distance is

$$2\,r = 2 \times 20 = 40\text{ m}.$$

The total distance driven is half the circumference.

$$\frac{1}{2}\,2\,\pi\,r = \pi\,20\,m = 62.8\,m$$

(b) For a full circle the magnitude of the displacement is $\vec{0}$, so its magnitude is 0. The total distance is the circumference.

$$2\,\pi\,r = 2\,\pi\,20\,m = 126\,m$$

Problem C.4

(a) Obtain a component expression for a position vector when its magnitude and direction are given by $r = 40$ m and $\theta = 155\,°$.
(b) A car moves at 70 mi/hr in the direction 12° south of west. Obtain a component expression for the velocity of the car.
(c) A rope pulls with a force of 40 N in the direction given by $\theta = -75\,°$. Obtain a component expression for the force vector.

Solution to C.4

The magnitude and direction of a position vector in two dimensions are just its polar coordinates and the component expression for it is

$$\vec{r} = x\,\hat{x} + y\,\hat{y} = \langle x,\, y\rangle = \langle r\cos\theta,\, r\sin\theta\rangle.$$

For any other 2D vector $\vec{A}$ we can write it in terms of its magnitude A and direction θ

$$\vec{A} = A_x\,\hat{x} + A_y\,\hat{y} = \langle A_x,\, A_y\rangle = \langle A\cos\theta,\, A\sin\theta\rangle.$$

(a) $r = 40\text{ m},\ \theta = 155\,° \implies \vec{r} = \langle r\cos\theta,\, r\sin\theta\rangle = \langle -36.3,\, 16.9\rangle\text{ m}$

(b) $v = 70$ mi/hr. If the direction is 12° south of west then the angle, measured counterclockwise from the positive x axis (taken as East) is $\theta = 192\,°$. This gives

$$\vec{v} = \langle v_x,\, v_y\rangle = \langle v\cos\theta,\, v\sin\theta\rangle = \langle -68.5,\, -14.6\rangle\text{ mi/hr}$$

(c) $F = 40\text{ N},\ \theta = -75\,° \implies \vec{F} = \langle F_x,\, F_y\rangle = \langle F\cos\theta,\, F\sin\theta\rangle = \langle 10.4,\, -38.6\rangle\text{ N}$

Problem C.5

A person walks in successive displacements of 20 m west and then 30 m in the direction 25 ° East of North. What is the person's net displacement?

Solution to C.5

$$\vec{s}_1 = \langle -20, 0 \rangle$$

$s_2 = 30$ m and $\theta_2 = 65°$ gives

$$\vec{s}_2 = \langle s_{2x}, s_{2y} \rangle = \langle s_2 \cos\theta_2, s_2 \sin\theta_2 \rangle = \langle 12.679, 27.189 \rangle.$$

The net displacement is the sum of the two:

$$\vec{s} = \vec{s}_1 + \vec{s}_2 = \langle -7.32, 27.2 \rangle \text{ m}$$

Problem C.6

Define the vectors $\vec{A}$ and $\vec{B}$ by

$$\vec{A} = \langle 5, -6, 2 \rangle = 5\,\hat{x} - 6\,\hat{y} + 2\,\hat{z} \text{ and}$$
$$\vec{B} = \langle -3, 5, -7 \rangle = -3\,\hat{x} + 5\,\hat{y} - 7\,\hat{z}$$

(a) What is the vector $\vec{C} = \vec{A} + \vec{B}$? Also give the magnitude of $\vec{C}$ and specify its direction by finding $\hat{C}$, a unit vector in the direction of $\vec{C}$.

(b) What is the vector $\vec{D} = \vec{A} - 2\,\vec{B}$? Also give the magnitude of $\vec{D}$ and specify its direction by finding $\hat{D}$, a unit vector in the direction of $\vec{D}$.

Solution to C.6

(a) $\vec{C} = \vec{A} + \vec{B} = \langle 5, -6, 2 \rangle + \langle -3, 5, -7 \rangle = \langle 2, -1, -5 \rangle = \langle C_x, C_y, C_z \rangle$

$$C = \|\vec{C}\| = \sqrt{C_x^2 + C_y^2 + C_z^2} = 5.47$$

$$\hat{C} = \frac{\vec{C}}{\|\vec{C}\|} = \langle 0.365, -0.183, -0.913 \rangle$$

(b) $\vec{D} = \vec{A} - 2\,\vec{B} = \langle 5, -6, 2 \rangle - 2\,\langle -3, 5, -7 \rangle = \langle 11, -16, 16 \rangle = \langle D_x, D_y, D_z \rangle$

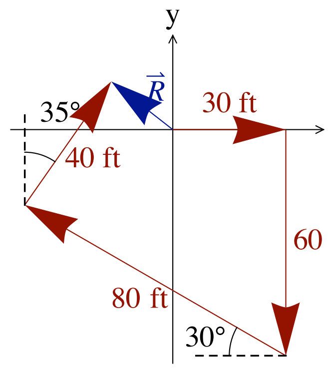
$$D = \|\vec{D}\| = \sqrt{D_x^2 + D_y^2 + D_z^2} = 25.2$$

$$\hat{D} = \frac{\vec{D}}{\|\vec{D}\|} = \langle 0.437, -0.636, 0.636 \rangle$$

Problem C.7

A soccer ball is kicked in four consecutive displacements as shown. What is the resultant displacement of the ball? Also give the magnitude and direction of the resultant.

Solution to C.7



Call the four displacements $\vec{A}$, $\vec{B}$, $\vec{C}$ and $\vec{D}$. The resultant $\vec{R}$ is the sum

$$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$$

$\vec{A}$ is in the positive x-direction and $\vec{B}$ is in the negative y-direction.

$$\vec{A} = \langle 30, 0 \rangle \text{ and } \vec{B} = \langle 0, -60 \rangle$$

Properly defined angles are measured counterclockwise from the positive x axis. The angles for the vectors $\vec{C}$ and $\vec{D}$ are defined differently. Relative to the positive x-axis we get $\theta_C = 150\,°$ and $\theta_D = 55\,°$. The vectors, in component form, can be found using the usual formulas.

$$C = 80 \text{ and } \theta_C = 150\,° \implies \vec{C} = \left\langle C \cos\theta_C,\, C \sin\theta_C \right\rangle = \langle -69.282, 40 \rangle$$

$$D = 40 \text{ and } \theta_D = 55\,° \implies \vec{D} = \langle D \cos\theta_D,\, D \sin\theta_D \rangle = \langle 22.943, 32.766 \rangle$$

To find $\vec{R}$ we then perform the sum:

$$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D} = \langle -16.339, 12.766 \rangle = \left\langle R_x, R_y \right\rangle.$$

The resultant displacement is

$$\vec{R} = \langle -16.3, 12.8 \rangle \text{ ft}.$$

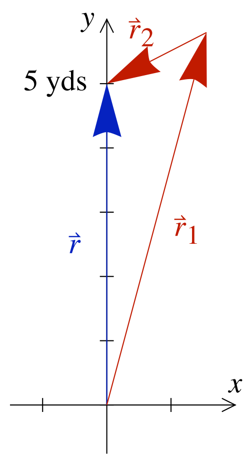
To find the magnitude and direction of $\vec{R}$ use the usual formulas.

$$R = \sqrt{R_x^2 + R_y^2} = 20.7 \text{ ft} \;\text{ and } \theta = 180\,° + \tan^{-1}\frac{R_y}{R_x} = 142.0\,°$$

Problem C.8

A golfer putts a ball toward a hole 5 yards to the north. If his first putt is 6 yards in the direction $15\,°$ east of north, then what displacement vector is needed for the second putt to hit the hole. Also give the magnitude and direction of the second putt's displacement.

Solution to C.8



$$\vec{r} = \vec{r}_1 + \vec{r}_2$$

Here we are given $\vec{r}$ and $\vec{r}_1$ and we are looking for $\vec{r}_2$.

$$\vec{r} = \langle 0, 5\rangle \text{ and } \vec{r}_1 = \langle 6\cos 75°, 6\sin 75°\rangle = \langle 1.5528, 5.7956\rangle$$

$$\vec{r}_2 = \vec{r} - \vec{r}_1 = \langle x_2, y_2\rangle = \langle -1.5528, -.7956\rangle = \langle -1.55, -.796\rangle \text{ yards}$$

For the magnitude and direction:

$$r_2 = \sqrt{x_2^2 + y_2^2} = 1.74 \text{ yards} \quad \text{and} \quad \theta = 180° + \tan^{-1}\frac{y_2}{x_2} = 207.1°$$

Problem C.9

(a) Show that if the angle between vectors $\vec{A}$ and $\vec{B}$ is θ, then the magnitude of their sum is:

$$\|\vec{A} + \vec{B}\| = \sqrt{A^2 + B^2 + 2\,A\,B\cos\theta}$$

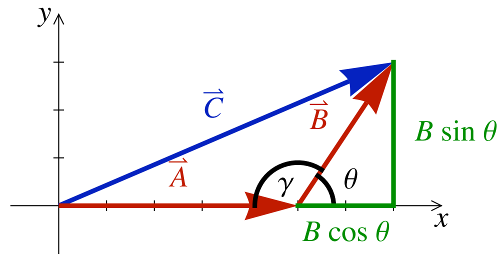
(Hint: Since this is independent of the choice of coordinates, one can, without loss of generality, choose coordinates so that A is in the x direction.)

(b) The *law of cosines* says that if a triangle has sides of lengths A, B and C, and the angle opposite the side of length C is γ then:

$$C^2 = A^2 + B^2 - 2\,A\,B\cos\gamma$$

Use the result of part (a) to prove this.

Solution to C.9



(a) Define $\vec{C} = \vec{A} + \vec{B}$ and refer to the diagram.

$$C^2 = [A + (B\cos\theta)]^2 + (B\sin\theta)^2 = A^2 + 2\,A\,B\cos\theta + B^2(\cos^2\theta + \sin^2\theta) = A^2 + B^2 + 2\,A\,B\cos\theta$$

(b) Writing this in terms of the angle γ gives the law of cosines.

$$\gamma = 180° - \theta \implies \cos\theta = -\cos\gamma$$

This proves the result.

Problem C.10

A car drives West at 20 m/s for 5 min, then drives 30 m/s to the North for 2 min and, finally, drive in the direction 30° East of South at 40 m/s for 3 min.

(a) What is the net displacement of the car?

(b) What is the average velocity of the car?

(c) What is the average speed of the car?

Solution to C.10

(a) The distance for each segment is the speed multiplied by the time in seconds.

$$\Delta\vec{r} = \langle -20\times 300,\, 0\rangle + \langle 0,\, 30\times 120\rangle + \langle 40\times 180\cos(-60°),\, 40\times 180\sin(-60°)\rangle$$

$$\implies \Delta\vec{r} = \langle -2400,\, -2635.4\rangle = \langle -2400. - 2640\rangle \text{ m.}$$

(b) The average velocity is $\overline{\vec{v}} = \frac{\Delta\vec{r}}{\Delta t}$.

$$\overline{\vec{v}} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\langle -2400, -2635.4\rangle}{300+120+180} = \langle -4.00,\, -4.39\rangle\,\frac{\text{m}}{\text{s}}$$

(c) The average speed is defined by:

$$\text{Average Speed} = \frac{\text{Total Distance}}{\text{Total Time}}.$$

This becomes:

$$\text{Average Speed} = \frac{20\times 300 + 30\times 120 + 40\times 180}{300+120+180} = 28\,\frac{\text{m}}{\text{s}}.$$

Problem C.11

The displacement of a body as a function of time in SI units is given by:

$$\vec{r}(t) = \left\langle t^3 + 9\,t - 5,\ 2\,t^4 - 5\,t^2 + 18 \right\rangle.$$

(a) What is the average velocity of the body between 1s and 3s?

(b) What is the speed of the body at 2s?

(c) In what direction is the body moving at 2s?

(d) What is the acceleration at 2s?

Solution to C.11

(a) To find the average velocity, we need the position vecotors at 1 s and 3 s: $\vec{r}(1) = \langle 5, 15 \rangle$ and $\vec{r}(3) = \langle 49, 135 \rangle$

$$\bar{\vec{v}} = \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r}(3) - \vec{r}(1)}{3-1} = \langle 22, 60 \rangle\ \frac{\text{m}}{\text{s}}$$

(b) The speed is the magnitude of the velocity. Differentiate the position vector to get the velocity, plug in the time of 2 s and then find the magnitude.

$$\vec{v} = \frac{d\vec{r}}{d\,t} = \left\langle 3\,t^2 + 9,\ 8\,t^3 - 10\,t \right\rangle$$

$$\vec{v}(2) = \langle 21, 44 \rangle \implies v = \sqrt{v_x^2 + v_y^2} = \sqrt{21^2 + 44^2} = 48.8\ \frac{\text{m}}{\text{s}}$$

(c) The direction of motion is the direction of the velocity vector.

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{44}{21}\right) = 64.5\,°$$

(d) The derivative of the velocity vector is the acceleration.

$$\vec{a} = \frac{d\vec{v}}{d\,t} = \left\langle 6\,t,\ 24\,t^2 - 10 \right\rangle \implies \vec{a}(2) = \langle 12, 86 \rangle\ \frac{\text{m}}{s^2}$$

Problem C.12

At time zero a particle starts at the origin with a velocity of $\langle 30, -20 \rangle\, m/s$. It moves with a constant acceleration so that after 5 s the velocity is $\langle -20, 10 \rangle\, m/s$.

(a) Obtain expressions for the velocity as a function of time $\vec{v}(t)$ and the position vector as a function of time $\vec{r}(t)$.

(b) What is the speed after 12 s?

Solution to C.12

(a) The velocity as a function of time for constant acceleration has the general form.

$$\vec{v}(t) = \vec{v}_0 + \vec{a}\, t$$

$$\vec{v}(5) = \vec{v}_0 + \vec{a}\, t \implies \langle -20, 10\rangle = \langle 30, -20\rangle + \vec{a}\, 5 \implies \vec{a} = \langle -10, 6\rangle \frac{\text{m}}{\text{s}^2}$$

The velocity as a function of time becomes:

$$\vec{v}(t) = \vec{v}_0 + \vec{a}\, t = \langle 30, -20\rangle + \langle -10, 6\rangle\, t$$
$$= \langle 30 - 10\, t, -20 + 6\, t\rangle. \text{ (in SI units)}$$

The position as a function of time is:

$$\vec{r}(t) = \vec{r}_0 + \vec{v}_0\, t + \frac{1}{2}\vec{a}\, t^2 = \langle 0, 0\rangle + \langle 30, -20\rangle\, t + \frac{1}{2}\langle -10, 6\rangle\, t^2 = \left\langle 30\, t - 5\, t^2, -20\, t - 3\, t^2\right\rangle \text{ (in SI units)}$$

(b) The speed at 12 s is the magnitude of the velocity at that time.

$$\vec{v} = \vec{v}(12) = \langle 30 - 10\times 12, -20 + 6\times 12\rangle = \langle -90, 52\rangle$$
$$\implies v = \sqrt{v_x^2 + v_y^2} = \sqrt{90^2 + 52^2} = 104\ \frac{\text{m}}{\text{s}}$$

Problem C.13

A bartender slides a beer mug to Joe at the end of a bar. Joe, not being at his best, misses the mug which then tragically slides off the bar onto the floor below. Suppose the bar is height h and the mug lands a distance d from the base of the bar.

(a) With what speed did the mug leave the bar?
(b) What was the speed of the mug when it hit the floor?
(c) In what direction was the mug moving when it hit the floor?
(d) What are the answers to parts (a), (b) and (c) when $h = 80$ cm and $d = 1.2$ m?

Solution to C.13

If a projectile is shot with a horizontal initial velocity, $v_{0y} = 0$, then the height, initial velocity and horizontal range, h, v_{0x} and Δx are related by:

$$h = \frac{1}{2} g\, t^2 \quad (\text{since } \Delta y = -h)$$
$$\Delta x = v_{0x}\, t$$

This represents a common class of problems. Of the three variables h, v_{0x} and Δx two will be given and the third will be asked for.

This problem has an horizontal initial velocity and is in this class.

(a) Since velocity is purely horizontal when it leaves, the initial speed is just v_{0x}.

$$h = \frac{1}{2} g\, t^2 \implies t = \sqrt{\frac{2\, h}{g}}$$

$$\Delta x = v_{0x}\, t \implies v_{0x} = \frac{\Delta x}{t} = d\sqrt{\frac{g}{2\, h}}$$

(b) The speed is the magnitude of the velocity, se we first need the velocity when the mug hit the floor?

$$v_x = v_{0x} = d\sqrt{\frac{g}{2\,h}} \;\;\text{and}\;\; v_y = v_{0y} - g\,t = 0 - g\sqrt{\frac{2\,h}{g}} = -\sqrt{2\,g\,h}$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{d^2\,\frac{g}{2\,h} + 2\,g\,h}$$

(c) The direction of motion is just the direction of the velocity vector.

$$\theta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\frac{-\sqrt{2\,g\,h}}{d\sqrt{\frac{g}{2\,h}}} = -\tan^{-1}\frac{2\,h}{d}$$

(d) Turning to the numerical part of this problem, we have: $d = 1.2\,m$, $h = 0.80$ m and $g = 9.8\,\frac{\text{m}}{\text{s}^2}$

$$v_{0x} = d\sqrt{\frac{g}{2\,h}} = 2.97\,\frac{\text{m}}{\text{s}}$$

$$v = \sqrt{d^2\,\frac{g}{2\,h} + 2\,g\,h} = 4.95\,\frac{\text{m}}{\text{s}}$$

$$\theta = -\tan^{-1}\frac{2\,h}{d} = 53.1\,°$$

Problem C.14

An astronaut walking on some new planet finds that with an itital speed of $3\,\frac{\text{m}}{\text{s}}$ he can jump a maximum distance of 6 m. What is the acceleration due to gravity on this planet?

Solution to C.14

This is a simple application of the Range formula for projectiles:

$$R = \frac{v_0^2}{g}\sin(2\,\theta)$$

The maximum range is at $45\,°$.

$$R_{\text{max}} = \frac{v_0^2}{g} \Longrightarrow g = \frac{v_0^2}{R_{\text{max}}} = \frac{3^2}{6} = 1.5\,\frac{\text{m}}{s^2}$$

Problem C.15

A projectile is shot so that its horizontal range is five times its maximum height. What initial angle is needed achieve this? You have enough information to give a numerical answer.

Solution to C.15

We must find at which angle $R = 5\,y_{\text{max}}$.

The maximum height is $y_{max} = \Delta y$ when $v_y = 0$.

$$v_y^2 - v_{0\,y}^2 = -2\,g\,\Delta y \implies 0 - v_{0\,y}^2 = -2\,g\,y_{max} \implies y_{max} = \frac{v_{0\,y}^2}{2\,g}$$

We could use the range formula but this would just complicate the trig. We will solve for the range in terms of the components:

$$0 = \Delta\,y = v_{0\,y}\,t - \frac{1}{2}\,g\,t^2 \implies t = \frac{2\,v_{0\,y}}{g}$$

$$\implies R = \Delta\,x = v_{0\,x}\,t = \frac{2\,v_{0\,x}\,v_{0y}}{g}$$

$$R = 5\,y_{max} \implies \frac{2\,v_{0x}\,v_{0y}}{g} = 5\,\frac{v_{0y}^2}{2\,g} \implies \frac{4}{5} = \frac{v_{0y}}{v_{0x}} = \frac{v_0\sin\theta}{v_0\cos\theta} = \tan\theta$$

$$\implies \theta = \tan^{-1}\frac{4}{5} = 36.7\,°$$

Problem C.16

A football is kicked at a speed of 25 m/s at an angle of 50 ° above horizontal.

(a) What is the maximum height reached by the football?

(b) At $t = 2.5$ s, what is the displacement, velocity, speed and acceleration of the football?

(c) What is the total horizontal distance the football travels in the air?

(d) Suppose this kick is toward a crossbar that is 3 m high. For the kick to be a *field goal* if must pass over the crossbar. What is the longest field goal (the horizontal distance to the crossbar) that can be made by this kick?

Solution to C.16

$$v_0 = 25\,\frac{\text{m}}{\text{s}} \text{ and } \theta = 50\,°$$

$$\vec{v}_0 = \langle v_{0x},\,v_{0y}\rangle = \langle v_0\cos\theta,\,v_0\sin\theta\rangle = \langle 16.070,\,19.151\rangle$$

(a) At the highest position $v_y = 0$. $y_{max} = \Delta\,y$ at that position.

$$v_y^2 - v_{0y}^2 = -2\,g\,\Delta\,y \implies 0 - v_{0y}^2 = -2\,g\,y_{max} \implies y_{max} = \frac{v_{0y}^2}{2\,g} = 18.7\text{ m}$$

(b) For the entire time the football is in the air it is a projectile and $\vec{a} = \langle 0,\,-g\rangle$. At $t = 2.5$ s the displacement, velocity and speed are:

$$\Delta\,\vec{r} = \langle \Delta\,x,\,\Delta\,y\rangle = \left\langle v_{0x}\,t,\,v_{0y}\,t - \frac{1}{2}\,g\,t^2\right\rangle = \langle 40.2,\,17.3\rangle\text{ m}$$

$$\vec{v} = \langle v_x,\,v_y\rangle = \langle v_{0x},\,v_{0y} - g\,t\rangle = \langle 16.070,\,-5.3489\rangle = \langle 16.1,\,-5.35\rangle\,\frac{\text{m}}{\text{s}}$$

$$v = \sqrt{v_x^2 + v_y^2} = 16.9\,\frac{\text{m}}{\text{s}}$$

(c) Since we want $\Delta\,x$ when $\Delta\,y = 0$, this is a simple application of the range formula.

$$R = \frac{v_0^2}{g} \sin 2\theta = \frac{25^2}{9.8} \sin 100° = 62.8 \text{ m}$$

(d) Here we must first solve for the time the football is at the height of the crossbar and then solve for the horizontal distance travelled in that time.

$$\Delta y = v_{0y} t - \frac{1}{2} g t^2 \implies 3 = 19.151 t - 4.9 t^2$$

$$\implies t = \frac{19.151 \pm \sqrt{19.151^2 - 4 \times 4.9 \times 3}}{2 \times 4.9} \implies t = 0.16349 \text{ and } t = 3.4745$$

The later time is the one we want. Now we need the horizontal distance travelled in this time.

$$\Delta x = v_{0x} t = 16.070 \times 3.4745 = 60.2 \text{ m}$$

Problem C.17

In making a movie a car drives at a speed of $40\, m/s$ off a 30 m high cliff. How long after leaving the cliff should the sound of the crash be added? Include the time it takes for the sound to return to the top of the cliff. Sound travels at $343\, m/s$.

Solution to C.17

This is a two step problem involving a horizontal initial velocity. The height of the cliff is $h = 30$ m. This gives the time of fall of the rock, t_1.

$$h = \frac{1}{2} g t_1^2 \implies t_1 = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 50}{9.80}} = 2.4744 \text{ s}$$

We now need to solve for the horizontal distance the car travels.

$$\Delta x = v_{0x} t = 40 \times t_1 = 98.974$$

The distance the sound travels is $d = \sqrt{h^2 + (\Delta x)^2} = 110.89$. It follows that the return time t_2 and the total time t are:

$$t_2 = \frac{d}{v_{\text{sound}}} = \frac{110.89}{343} = 0.24652 \text{ s}$$

The total time is $t_1 + t_2$.

$$t_1 + t_2 = 2.4744 + 0.24652 = 2.71 \text{ s}$$

Problem C.18

A golfer hits a ball at 40m/s at an angle of 55° above horizontal up an inclined fairway with a 10° slope. How far up the incline does the ball travel before hitting?

(Hint: Solve of the trajectory (y as a function of x) of the ball and equate that to the equation describing the incline.)

Solution to C.18

To simplify the equation for the trajectory (y vs. x) take the initial position to be the origin. This gives: $\Delta x = x$ and $\Delta y = y$.

$$x = \left(v_0 \cos\theta\right) t \implies t = \frac{x}{v_0 \cos\theta}$$

$$y = \left(v_0 \sin\theta\right) t - \frac{1}{2} g\, t^2 \implies y = (\tan\theta)\, x - \frac{g}{2\, v_0^2 \cos^2\theta} x^2$$

The equation for an incline of angle ϕ is: $y = (\tan\phi)\, x$. Eliminating y from the above expressions gives a quadratic expression for x.

$$(\tan\theta - \tan\phi)\, x = \frac{g}{2\, v_0^2 \cos^2\theta} x^2$$

Dividing by x gives the unique nonzero solution:

$$x = \frac{2\, v_0^2 \cos^2\theta}{g} (\tan\theta - \tan\phi)$$

This is the horizontal distance. the distance up the incline is d, where $\frac{x}{d} = \cos\phi$

$$d = \frac{x}{\cos\phi} = \frac{2\, v_0^2 \cos^2\theta}{g \cos\phi} (\tan\theta - \tan\phi)$$

Plugging in the numbers gives:

$$d = \frac{2\cdot 40^2 \cdot \cos^2\ \ 55\,°}{9.80\cdot\cos \times 10\,°} (\tan 55\,° - \tan 10\,°) = 137\ \text{m}.$$

Problem C.19

A car travelling at 100 km/hr is 300 m behind a slower car moving at 80 km/hr. If both cars continue at the same rate how long does it take for the faster car to catch the other?

Solution to C.19

Let us study the motion of the faster car with respect to the slower car and the road. Take $v = 100$ km/hr to be the velocity of the faster car with respect to the road and $v_0 = 80$ km/hr to be the velocity of the slower car with respect to the road. The relative velocity of the faster with respect to the slower is

$$v' = v - v_0 = 20\ \frac{\text{km}}{\text{hr}} \times \frac{1000\ \text{m}}{1\ \text{km}} \times \frac{1\ \text{hour}}{3600\ \text{s}} = \frac{20}{3.6}\ \frac{\text{m}}{\text{s}}$$

The relative distance between the bumpers of the car is $d' = 300$ m.

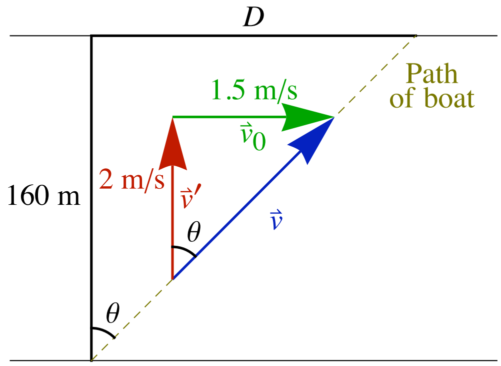
$$d' = v'\, t \implies t = \frac{d'}{v'} = \frac{300}{20/3.6} = 54\ \text{s}$$

Problem C.20

A 160 m wide river has a steady current with a uniform speed of 1.5 m/s. A boat crosses this river by maintaining a bearing, the direction in which the boat points, that is perpendicular to the river's flow and with a speed of 2 m/s relative to the water. (a) What is the speed of the boat relative to the shore? (b) How far downstream from the starting point does the boat land?

Solution to C.20

$\vec{v} = \vec{v}' + \vec{v}_0$, where $\vec{v}_0$ is the velocity of the moving frame with respect to the stationary frame. $\vec{v}$ and $\vec{v}'$ represent some velocity with respect to the fixed and moving frames, respectively.



Here, we are considering the motion of the boat with respect to the shore (the fixed frame) and the water (the moving frame.) The river flows in the direction of $\vec{v}_0$. The direction of the boats bearing is $\vec{v}'$, the boat's velocity relative to the water. $\vec{v}$, the boat's velocity relative to the shore, which is the direction of the bait's trajectory.

(a) Using the Pythagorean theorem we can solve for v, the speed of the boat relative to the shore.

$$v = \sqrt{v'^2 + u^2} = \sqrt{2^2 + 1.5^2} = 2.5 \ \frac{\text{m}}{\text{s}}$$

(b) Using similar triangles we can find D, the downstream distance of the boat.

$$\frac{D}{160} = \frac{1.5}{2} \implies D = \ 120\, m$$

Problem C.21

The pilot of a plane reads his compass and sees his heading is due south. He also sees that the plane's speed with respect to the air is 200 mi/hr. Suppose there is a 50 mi/hr wind blowing to the west.

(a) What is the velocity of the plane with respect to the ground?

(b) What are the speed and direction of the plane relative to the ground?

Solution to C.21

(a) $\vec{u} = -50 \ \frac{\text{mi}}{\text{hr}} \hat{x}$ is the velocity of the wind.

$\vec{v}' = -200 \ \frac{\text{mi}}{\text{hr}} \hat{x}$ is the velocity of the plane relative to the wind.

$\vec{v} = \vec{v}' + \vec{u} = \left(-200\ \hat{y} - 50\ \hat{x}\right) \frac{\text{km}}{\text{hr}} = \langle -50, -200 \rangle \frac{\text{km}}{\text{hr}}$ is the velocity of the plane relative to the ground.

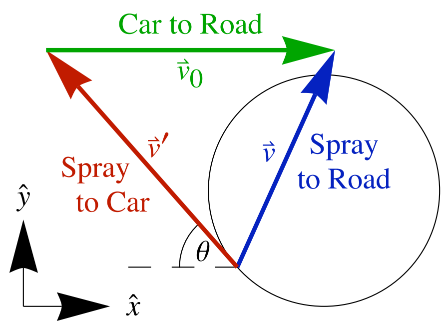
(b) Use the usual rules to get the magnitude and direction of the plane relative to the ground.

$$v = \sqrt{50^2 + 200^2} = 206 \ \frac{\text{km}}{\text{hr}} \ \text{ and } \ \theta = 180\,° + \tan^{-1}\left(\frac{-200}{-50}\right) = 256.0\,°$$

Problem C.22

A car moving at a speed v hits a puddle in the road and sprays water. Relative to the car, the spray is shot backward at speed v and at an angle θ above horizontal. What is the velocity of the spray relative to the road? Take the direction of the car's motion to be the x direction and take y to be vertical.

Solution to C.22



$\vec{v}_0 = v\,\hat{x}$ is the velocity of the car, the moving frame.

$\vec{v}' = -v\cos\theta\,\hat{x} + v\sin\theta\,\hat{y}$ is the velocity of the spray relative to the car.

So, the velocity of the spray relative to the road is:

$\vec{v} = \vec{v}' + \vec{v}_0 = (v - v\cos\theta)\,\hat{x} + v\sin\theta\,\hat{y}$

Comment: This is counterintuitive but true; the spray is actually thrown forward relative to the road. The magnitude of $\vec{v}'$ is the same as the car's speed because that is the speed of the rim of the tire relative to the road.