Chapter D - Problems

Blinn College - Physics 2425 - Terry Honan

Problem D.1

The same force acts at different times on two masses. m_1 is given an acceleration of $4\,\text{m}/\text{s}^2$ and m_2 is given an acceleration of $1\,\text{m}/\text{s}^2$.

(a) What is the ratio of the masses, m_2/m_1?

(b) Suppose the two masses are combined and the same force is acted on the combination. What is its acceleration?

Solution to D.1

(a) $F = m_1\,a_1 = m_2\,a_2,\ a_1 = 4\,\frac{\text{m}}{s^2}$ and $a_2 = 1\,\frac{\text{m}}{s^2}$

$$\frac{m_2}{m_1} = \frac{a_1}{a_2} = \frac{4}{1} = 4$$

(a) $F = m_1\,a_1 = (m_1 + m_2)\,a_3,\ a_1 = 4\,\frac{\text{m}}{s^2}$ and $a_2 = 1\,\frac{\text{m}}{s^2}$

$$m_1\,a_1 = (m_1 + m_2)\,a_3 \ \text{ and } \ m_2 = 4\,m_1 \implies m_1\,4 = (m_1 + 4\,m_1)\,a_3 \implies a_3 = \frac{4}{5}\,\frac{\text{m}}{\text{s}^2}$$

Problem D.2

A 2 ton truck provides an acceleration of $3\,\text{ft}/s^2$ to a 4 ton trailer. If the truck exerts the same force on the road while pulling a 16 ton trailer, what acceleration results? Assume there are no resistive forces.

Solution to D.2

Like pounds, tons are units of weight. lb-mass is often used to refer to the mass equivalent of 1 lb, so we will use the term ton-mass to refer to its mass equivalent.

$$F = m_1\,a_1 = m_2\,a_2 \implies \frac{a_2}{a_1} = \frac{m_1}{m_2} \implies \frac{a_2}{3} = \frac{2+4}{2+16} \implies a_2 = 1\,\frac{\text{ft}}{\text{s}^2}$$

Note that the ratio of two masses in ton-mass is the same as in kg and the ratio of two accelerations in ft/s^2 is the same as in m/s^2 .

Problem D.3

A 5 g bullet is accelerated to $300\,m/s$ down a 72 cm gun barrel. Assuming a uniform acceleration, what is the net force on the bullet in the gun barrel?

Solution to D.3

Use one dimensional kinematics to find the acceleration.

$$v_0 = 0,\ \ v = 300\ \frac{\text{m}}{\text{s}}\ \ \text{and}\ \ \Delta x = 0.72\ \text{m}$$

$$v^2 - v_0^2 = 2\,a\,\Delta x \implies a = \frac{300^2}{2\times 0.72} = 62\,500\ \text{N}$$

Newton's second law gives the net force.

$$F_{\text{net}} = m\,a = 0.005\times 62\,500 = 312.5\ \text{N}$$

Problem D.4

The displacement of 20 kg mass as a function of time in SI units is given by:

$$\vec{r}(t) = \left\langle 2\,t^3 - 9\,t - 15,\ -6\,t^2 + 5\,t + 18 \right\rangle.$$

What is the net force acting on the mass as a function of time? What is the net force at $t = 2$ s?

Solution to D.4

The acceleration is the second derivative of the position vector. Newton's second law then gives the net force.

$$\vec{v}(t) = \frac{d}{dt}\vec{r} = \frac{d}{dt}\left\langle 2\,t^3 - 9\,t - 15,\ -6\,t^2 + 5\,t + 18 \right\rangle = \left\langle 6\,t^2 - 9,\ -12\,t + 5 \right\rangle$$

$$\vec{a}(t) = \frac{d}{dt}\vec{v} = \frac{d}{dt}\left\langle 6\,t^2 - 9,\ -12\,t + 5 \right\rangle = \langle 12\,t,\ -12 \rangle$$

At $t = 2$ s we get

$$\vec{a}(2) = \langle 24,\ -12 \rangle\ \frac{\text{m}}{\text{s}^2} \implies \vec{F}_{\text{net}} = m\,\vec{a} = 20\,\vec{a} = \langle 480,\ -240 \rangle\ \text{N}.$$

Problem D.5

Two forces $\vec{F}_1$ and $\vec{F}_2$ act on a 12 kg mass to produce an acceleration of magnitude $3\ \text{m}/\text{s}^2$ in the negative y-direction. If the first force is given by $\vec{F}_1 = \langle -30,\ 20 \rangle$ N then what is the value of the second force $\vec{F}_2$?

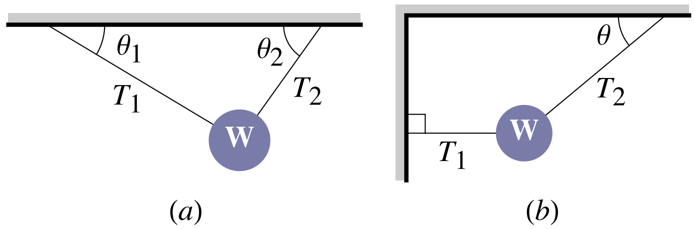
Solution to D.5

The acceleration is $\vec{a} = \langle 0,\ -3 \rangle\ \text{m}/\text{s}^2$. The second law involves the net force, which in this case is the sum of the two forces.

$$\vec{F}_{\text{net}} = m\,\vec{a} \implies \vec{F}_1 + \vec{F}_2 = m\,\vec{a} \implies \langle -30,\ 20 \rangle + \vec{F}_2 = 12\,\langle 0,\ -3 \rangle \implies \vec{F}_2 = \langle 30,\ -56 \rangle\ \text{N}$$

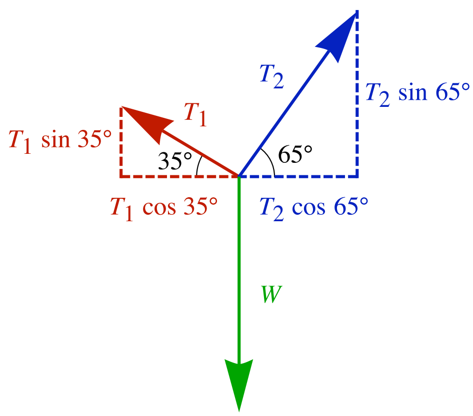
Problem D.6

A weight W hangs from two strings as shown. What are both tensions T_1 and T_2? Leave your answers as multiples of W.
(a) $\theta_1 = 35°$ and $\theta_2 = 65°$
(b) Solve using θ as arbitrary.



Solution to D.6

(a) For any two dimensional problem we resolve the second law into a pair a perpendicular directions, in this case we take then as horizontal and vertical.



In this problem the acceleration is zero. Whenever the acceleration is zero, or even zero in one of the directions, we can rewrite the second law by balancing the forces. In this case we can replace the second law resolved in the horizontal direction with the condition that the horizontal forces balance; the total force to the left equals the total force to the right.

$$F_{\text{net,hor}} = 0 \implies T_1 \cos 35° = T_2 \cos 65° \implies T_2 = T_1 \frac{\cos 35°}{\cos 65°}$$

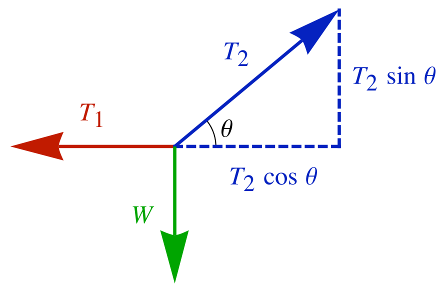
In the vertical direction we similarly get the total force up equals the total force down. We can then solve for the tensions.

$$F_{\text{net,ver}} = 0 \implies T_1 \sin 35° + T_2 \sin 65° = W$$

$$\implies T_1 \sin 35° + T_1 \frac{\cos 35°}{\cos 65°} \sin 65° = W \implies T_1 = 0.429\, W$$

$$T_2 = T_1 \frac{\cos 35°}{\cos 65°} \implies T_2 = 0.832\, W$$

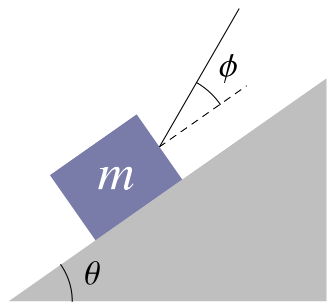
(b) Here we similarly balance the horizontal and vertical components.



$$F_{\text{net,ver}} = 0 \implies T_2 \sin\theta = W \implies T_2 = \frac{W}{\sin\theta}$$

$$F_{\text{net,hor}} = 0 \implies T_1 = T_2 \cos\theta \implies T_1 = \frac{W}{\sin\theta}\cos\theta = \frac{W}{\tan\theta}$$

Problem D.7

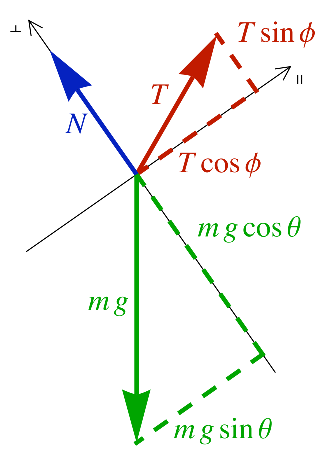


(a) A block of mass m is lowered down an incline at a constant speed v by a rope as shown above. What is the tension in the rope T?

(b) A block of mass m is lowered down an incline at a constant acceleration a (down the incline) by a rope as shown above. What is the tension in the rope T?

(c) A block of mass m is pulled up an incline at a constant acceleration a (up the incline) by a rope as shown above. What is the tension in the rope T?

Solution to D.7



(a) If the speed is constant then the acceleration is zero; the value of v is unimportant. We can then balance the forces in the directions parallel and perpendicular to the incline. Here we only need the parallel component to get the tension.

$$F_{\text{net},\parallel} = m\,a_{\parallel} = 0 \implies m\,g\sin\theta = T\cos\phi \implies T = m\,g\,\frac{\sin\theta}{\cos\phi}$$

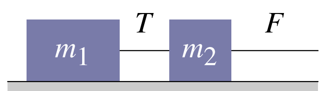
(b) If the acceleration is down the incline then choose that as the positive direction. Again we only need the parallel component to get the tension.

$$F_{\text{net},\parallel} = m\,a_{\parallel} \implies m\,g\sin\theta - T\cos\phi = m\,a \implies T = \frac{m\,g\sin\theta - m\,a}{\cos\phi}$$

(c) When the acceleration is up the incline then choose that as the positive direction.

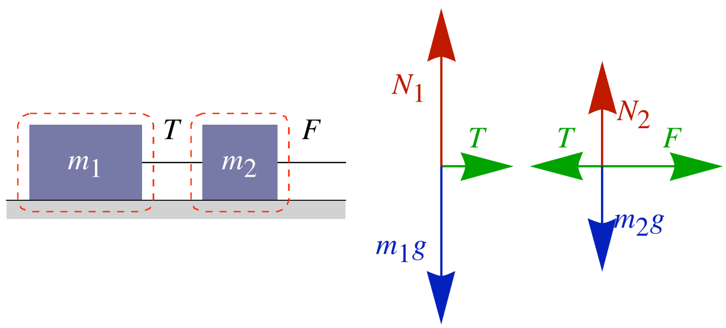
$$F_{\text{net},\parallel} = m\,a_{\parallel} \implies T\cos\phi - m\,g\sin\theta = m\,a \implies T = \frac{m\,g\sin\theta + m\,a}{\cos\phi}$$

Problem D.8



Two blocks with masses m_1 and m_2 are dragged along a frictionless surface as shown. They are pulled by a force F and are connected by a string. What is the tension T between the blocks?

Solution to D.8



In this problem the vertical forces cancel and are uninteresting. We will only consider the horizontal equations. Both masses will have the same acceleration. The tensions are equal and opposite due to the third law. Applying the second law to both masses gives:

$$F_{\text{net},1} = m_1\,a \implies T = m_1\,a$$

$$F_{\text{net},a} = m_2\,a \implies F - T = m_2\,a$$

Adding these two equations together gives a in terms of F.

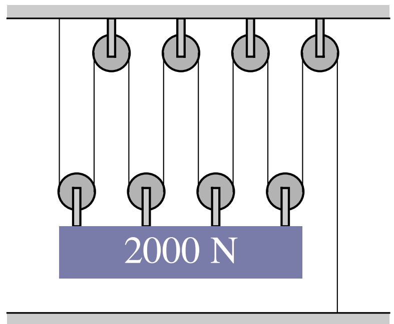
$$F = (m_1 + m_2)\,a \implies a = \frac{F}{m_1 + m_2}$$

We can then find T.

$$T = m_1\,a \implies T = \frac{m_1}{m_1 + m_2}\,F$$

Problem D.9

Assume all pulleys are ideal. This means they are frictionless and massless.



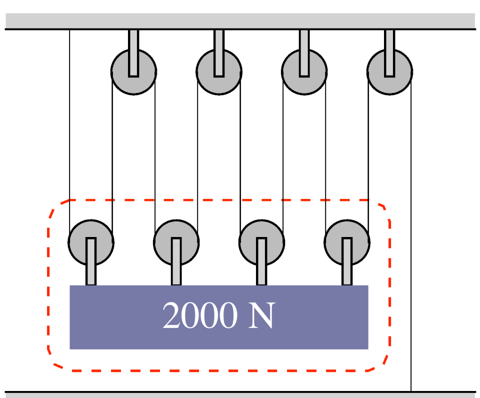
(a) With what tension does the floor pull downward on the rope on the right?

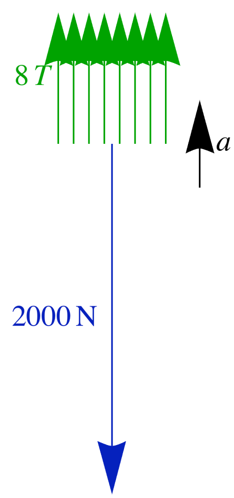
(b) If the rope on the right is disconnected from the floor then what tension is needed to lift the weight at a constant speed of $3\ \frac{\text{m}}{\text{s}}$?

(c) What tension is needed to lift the weight at a constant upward acceleration of $2\ \frac{\text{m}}{s^2}$?

(d) What tension is needed to lower the weight at a constant downward acceleration of $2\ \frac{\text{m}}{s^2}$?

Solution to D.9





With ideal pulleys the tension is constant throughout the arrangement.

(a) $F_{\text{net}} = m\,a = 0 \implies 8\,T = 2000\text{ N} \implies T = 250\text{ N}$

(b) If the speed is constant then $a = 0$ and as above, $T = 250\text{ N}$.

(c) With $a = 2\text{ m}/\text{s}^2$ we get

$$F_{\text{net}} = m\,a \implies 8\,T - 2000 = \frac{2000}{9.8} \times 2 \implies T = 301\text{ N}$$

(d) With $a = -2\ \mathrm{m/s^2}$ we get

$$F_{net} = m\,a \implies 8\,T - 2000 = \frac{2000}{9.8} \times (-2) \implies T = 199\ \mathrm{N}$$

Problem D.10

What is the minimum stopping distance for a car with an initial speed of 30 mi/hr given a coefficient of static friction of 0.55? How would this change on an icy day with a coefficient of 0.15?

Solution to D.10

When a car stops the friction is static friction, unless the car is skidding to a stop.

$$F_{net,or} = m\,a_{hor} \implies f_s = m\,a$$

$$F_{net,ver} = m\,a_{ver} = 0 \implies N = m\,g$$

$$f_s \le \mu_s\,N \implies a \le \mu_s\,g \implies a_{max} = \mu_s\,g$$

In the above expression the direction of the acceleration, backward, is taken as the positive direction. To relate this acceleration to the minimum stopping distance we must use kinematics. If we take the velocity to be positive then we must take the acceleration to be negative. $\implies a_{max} = -\mu_s\,g$

$$2\,a\,\Delta x = v^2 - v_0^2 = 0 - v_0^2 \implies \Delta x_{min} = -\frac{v_0^2}{2\,a_{max}} = \frac{v_0^2}{2\,\mu_s\,g}$$

$$v_0 = 30\ \frac{\mathrm{mi}}{\mathrm{hr}} \times \frac{1609\ \mathrm{m}}{\mathrm{mi}} \times \frac{1\ \mathrm{hr}}{3600\ s} = 13.408\ \frac{\mathrm{m}}{\mathrm{s}}$$

$$\mu_s = 0.55 \implies \Delta x_{min} = 16.7\ \mathrm{m}$$

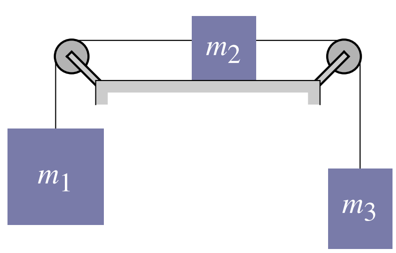
$$\mu_s = 0.15 \implies \Delta x_{min} = 61.2\ \mathrm{m}$$

Problem D.11

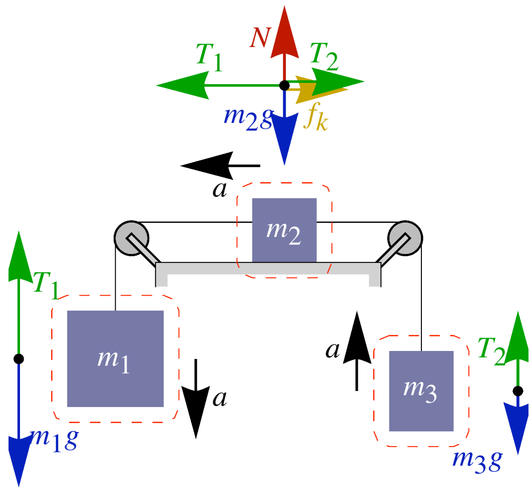
Three masses are connected as shown m_1 is the largest and m_2 slides with a coefficient of friction μ_k.

(a)What is the acceleration of the system?

(b) If $m_1 = 10\ \mathrm{kg}$, $m_2 = 4\ \mathrm{kg}$, $m_3 = 6\ \mathrm{kg}$ and $\mu_k = 0, 35$ then find the value of a. What are the two tensions, between m_1 and m_2 and between m_2 and m_3?



Solution to D.11



(a) We need to apply the second law to all three masses.

For m_1: $F_{\text{net}} = m\,a \implies m_1\,g - T_1 = m_1\,a$

For m_2: $F_{\text{net},\perp} = m\,a_\perp = 0 \implies N = m_2\,g \implies f_k = \mu_k\,N = \mu_k\,m_2\,g$

$$F_{\text{net},\parallel} = m\,a_\parallel \implies T_1 - T_2 - f_k = m_2\,a \implies T_1 - T_2 - \mu_k\,m_2\,g = m_2\,a$$

For m_3: $F_{\text{net}} = m\,a \implies T_2 - m_3\,g = m_3\,a$

The tensions can be eliminated by adding the three equations. This gives:

$$m_1\,g - \mu_k\,m_2\,g - m_3\,g = m_1\,a + m_2\,a + m_3\,a \implies a = \frac{m_1 - \mu_k\,m_2 - m_3}{m_1 + m_2 + m_3}\,g$$

(b) $m_1 = 10\,\text{kg}$, $m_2 = 4\,\text{kg}$, $m_3 = 6\,\text{kg}$ and $\mu_k = 0.35 \implies \text{a} = 1.27\,\frac{\text{m}}{\text{s}^2}$

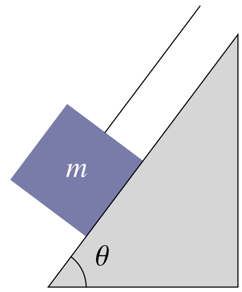
Solve for the tensions by using the acceleration.

$$m_1\,g - T_1 = m_1\,a \implies T_1 = m_1\,g - m_1\,a = 85.3\,N$$

$$T_2 - m_3\,g = m_3\,a \implies T_2 = m_3\,g + m_3\,a = 66.44\,N$$

Problem D.12

A mass m on a steep incline of angle θ with a coefficient of static friction μ_s is held by a rope that is parallel to the incline and pulling up the incline. Without the rope the mass will slide down the incline. For what range of tensions will the mass not slide. Note that if the tension is sufficiently large the mass will slide up the incline.



Solution to D.12

Since the tension is parallel to the surface, $N = m\,g\cos\theta$. This gives the maximum value of the static friction force: $f_s^{max} = \mu_s N = \mu_s\, m\, g\cos\theta$.

$$F_{net,\perp} = m\,a_\perp = 0 \implies N = m\,g\cos\theta \implies f_s^{max} = \mu_s N = \mu_s\, m\, g\cos\theta$$

$$F_{net,\parallel} = m\,a_\parallel = 0 \implies T = m\,g\sin\theta \pm f_s^{max}$$

To prevent the mass from sliding *up* the incline, the friction must act *down* the incline and thus the + sign is used. To prevent the mass from sliding *down* the incline, the friction must act *up* the incline and thus the - sign is used.

$$T_{min} = m\,g\sin\theta - \mu_s\, m\, g\cos\theta \text{ and } T_{max} = m\,g\sin\theta + \mu_s\, m\, g\cos\theta$$

The range of tensions is: $T_{min} \le T \le T_{max}$