Chapter E - Problems

Blinn College - Physics 2425 - Terry Honan

Problem E.1

(a) What is the centripetal (radial) acceleration of a point on the earth's equator?

(b) Give an expression for the centripetal acceleration as a function of the latitude angle, θ_L. What is this at the latitude of Bryan, Texas, at $\theta_L = 30.7\,°$?

(c) What are the speeds of a point at the equator and at Bryan, Texas due to the eaarth's rotation?

(d) The earth's rotation is slowing at a rate of 2.2 s every 100,000 years..

$$\frac{dT}{dt} = \frac{\Delta T}{\Delta t} = \frac{2.3\text{ s}}{100{,}000\text{ years}}$$

What is the (very small) tangential component of the acceleration of a point on the earth's equator? Hint: use the chain rule.

$$\frac{d}{dt}\frac{2\pi r}{T} = -\frac{2\pi r}{T^2}\frac{dT}{dt}$$

Solution to E.1

$$a_c = \frac{v^2}{r} \text{ and } v = \frac{2\pi r}{T} \implies a_c = \left(\frac{2\pi}{T}\right)^2 r$$

For the earth: $T = \;\; 1\text{ day} = 24 \cdot 3600\, s$

(a) At the equator:

$$r = R_E \implies a_c = \left(\frac{2\pi}{24 \cdot 3600}\right)^2 6.37 \times 10^6 = \;\; 0.0337\,\frac{\text{m}}{s^2}$$

Comment: Compared to the gravitational acceleration of g, this is small but not negligible.

(b) r is the radius of the circle but something on the rotating earth moves along a path that traces a latitude line. At some point other that the equator this is:

$$r = R_E \cos\theta_L \implies a_c = \left(0.0337\,\frac{\text{m}}{s^2}\right) \times \cos\theta_L$$

$$\theta_L = 30.7\,° \implies a_c = 0.0290\,\frac{\text{m}}{s^2}$$

(c) The speed is found form the period: $v = \frac{2\pi r}{T}$.

$$\text{At equator: } v = \frac{2\pi}{24 \cdot 3600}\, 6.37 \times 10^6 = 463\,\frac{\text{m}}{\text{s}}$$

$$\text{At Bryan: } v = \frac{2\pi}{24 \cdot 3600}\, 6.37 \times 10^6 \cos 30.7\,° = 398\,\frac{\text{m}}{\text{s}}$$

(d) To find the tangential acceleration use $a_t = dv/dt$ and the hints.

$$a_t = \frac{dv}{dt} = \frac{d}{dt}\frac{2\pi r}{T} = -\frac{2\pi r}{T^2}\frac{dT}{dt} = -\frac{2\pi r}{T^2}\frac{\Delta T}{\Delta t} = -\frac{2\pi R_E}{(1\text{ day})^2}\frac{2.3\text{ s}}{100{,}000\text{ years}}$$

$$= -\frac{2\pi R_E}{(1\text{ day})^2}\frac{2.3\, s}{100{,}000\text{ year}} = -\frac{2\pi\, 6.37\times 10^6}{(24\times 3600)^2}\frac{2.3}{100{,}000\times(365.24\times 24\times 3600)} = 3.91\times 10^{-15}\,\frac{\text{m}}{s^2}$$

Problem E.2

A car drives around a 200 m radius circle with a speed that decreases uniformly from 30 m/s to 20 m/s in 8 s. At the instant the speed is 25 m/s then:
(a) what is the centripetal acceleration,
(b) what is the tangential acceleration and
(c) what is the magnitude of the acceleration?

Solution to E.2

(a) $a_c = \frac{v^2}{r} = \frac{25^2}{200} = 3.125\,\frac{\text{m}}{\text{s}^2}$

(b) $a_t = \frac{dv}{dt} = \frac{\Delta v}{\Delta t} = \frac{20-30}{8} = -1.25\,\frac{\text{m}}{\text{s}^2}$

(c) $a = \sqrt{a_c^2 + a_t^2} = 3.37\,\frac{\text{m}}{\text{s}^2}$

Problem E.3

While moving in a circle with a 12 m radius the speed of a particle varies with time by $v(t) = 2 + 10\,t - 4\,t^2$ in SI units. At $t = 2$ s what are the centripetal and tangential components of th acceleration? Also give the magnitude of the total acceleration and the angle of this acceleration measured relative to the centripetal direction.

Solution to E.3

$$v(t) = 2 + 10\,t - 4\,t^2 \implies v(2\text{ s}) = 6 \implies a_c = \frac{v^2}{r} = \frac{6^2}{12} = 3\,\frac{\text{m}}{\text{s}^2}$$

$$\frac{dv}{dt}(t) = 10 - 8\,t \implies \frac{dv}{dt}(2\text{ s}) = -6 \implies a_t = \frac{dv}{dt} = -6\,\frac{\text{m}}{\text{s}^2}$$

$$a = \sqrt{a_c^2 + a_t^2} = 6.71\,\frac{\text{m}}{\text{s}^2}$$

$$\theta = \tan^{-1}\frac{a_t}{a_c} = -63.4\,°$$

Problem E.4

In the Bohr model of the hydrogen atom an electron moves in a circle of radius 5.29×10^{-11} m with a speed of $2.20\times 10^6\,m/s$. What net force is needed to produce this motion.

Solution to E.4

$$a = \frac{v^2}{r} = \frac{\left(2.20\times 10^6\right)^2}{5.29\times 10^{-11}} = 9.14\times 10^{22}\ \frac{\text{m}}{s^2}$$
$$\Longrightarrow\ F_{\text{net}} = m\,a = 9.11\times 10^{-31}\ a = 8.34\times 10^{-8}\ \text{N}$$

Problem E.5

A small block sits on a turntable that rotates with a period of 3 s. If the coefficient of static friction between the turntable and the block is 0.4 then what is the largest distance the block can be from the center without slipping?

Solution to E.5

Newton's second law in the centripetal direction gives:

$$F_{\text{net},c} = m\,a_c \ \Longrightarrow\ f_s = m\left(\frac{2\pi}{T}\right)^2 r$$

and in the vertical direction

$$F_{\text{net,ver}} = m\,a_{\text{ver}} = 0 \ \Longrightarrow\ N = m\,g.$$

The constraint force of static friction satisfies the inequality $f_s \le \mu_s N$. Inserting our information from above into this inequality gives

$$m\left(\frac{2\pi}{T}\right)^2 r \le \mu_s\, m\, g.$$

The largest radius r_{max} saturates this inequality.

$$m\left(\frac{2\pi}{T}\right)^2 r_{\text{max}} = \mu_s\, m\, g \ \Longrightarrow\ r_{\text{max}} = \mu_s\ g\left(\frac{T}{2\pi}\right)^2 = 0.4\times 9.80\left(\frac{3}{2\pi}\right)^2 = 0.894\ \text{m}$$

Problem E.6

A Ferris Wheel has a radius of 30 m and rotates once every 40 s. What are the minimum and maximum normal force of the seat on a 160 lb man. A Ferris wheel rotates in a vertical circle and a rider always sits upright.

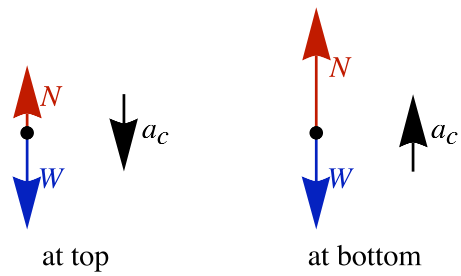
Solution to E.6

The extreme cases should be at the bottom and at the top so that is all we need to consider. The magnitude of the acceleration is the same everywhere in the circle.

$$a_c = \left(\frac{2\pi}{T}\right)^2 r = \left(\frac{2\pi}{40}\right)^2 30 = 0.74022$$

Since we are given the weight the mass is W/g and we can write

$$m\,a_c = W\,\frac{a_c}{g} = 160 \times \frac{0.74022}{9.80} = 12.085\text{ lb}$$



At the top the acceleration is downward.

$$F_{\text{net}} = m\,a \implies W - N = m\,a_c \implies N = W - m\,a_c = 160 - 12.085 = 148\text{ lb}$$

At the bottom the acceleration is upward.

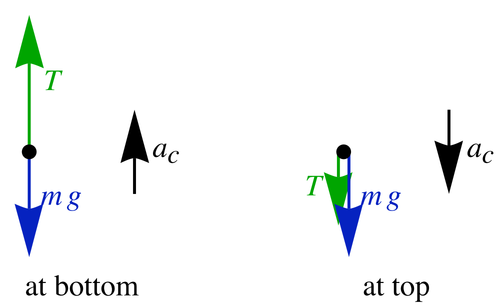
$$F_{\text{net}} = m\,a \implies N - W = m\,a_c \implies N = W + m\,a_c = 160 + 12.085 = 172\text{ lb}$$

It follows that the minimum and maximum normal forces are 148 lb and 162 lb.

Problem E.7

A mass m is moves in a vertical circle at the end of a string of length L.
(a) If at the bottom the mass has a speed v then what is the tension T in the string?
(b) If at the top the mass has a speed v then what is the tension T in the string? What is the minimum speed the mass can have without the string losing its tension?

Solution to E.7



(a) The acceleration is directed toward the center of the circle, which is upward in this case.

$$F_{\text{net}} = m\,a \implies T - m\,g = m\,a_c \implies T = m\,a_c + m\,g = m\left(\frac{v^2}{L} + g\right)$$

(b) At the top the acceleration is downward.

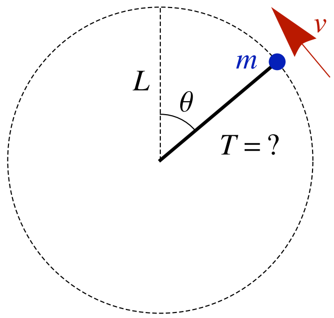
$$F_{\text{net}} = m\,a \implies T + m\,g = m\,a_c \implies T = m\,a_c - m\,g = m\left(\frac{v^2}{L} - g\right)$$

The minimum speed at the top is when the tension is zero.

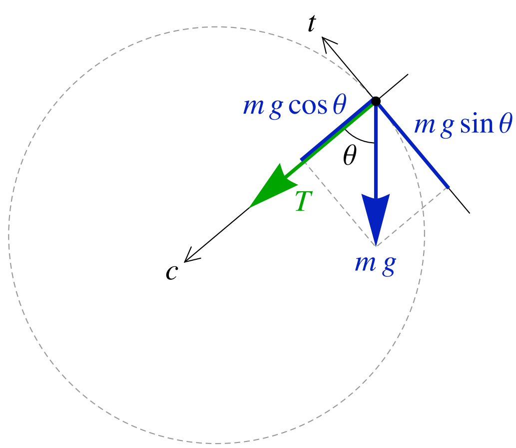
$$T = 0 \implies v_{\min} = \sqrt{g\,L}$$

Problem E.8

A mass m is moves in a vertical circle at the end of a string of length L. If the mass has a speed v, at angle θ measured from vertical as shown, then what is the tension T in the string at that position. Also, find the tangential componet of the acceleration at that position.



Solution to E.8



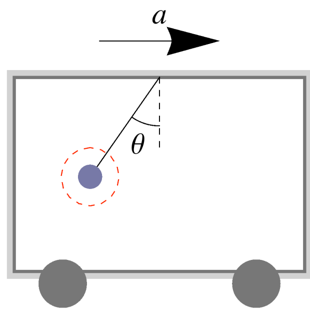
Here there are both tangential and centripetal components of the acceleration. The tension force is in the centripetal direction, so that is what we must consider.

$$F_{\text{net},c} = m\,a_c \implies T + m\,g\cos\theta = m\,\frac{v^2}{r} \implies T = m\,\frac{v^2}{r} - m\,g\cos\theta$$

The tangential component of the acceleration comes from the other component of the weight. The tangential direction is the direction of the velocity; this gives a negative component.

$$F_{\text{net},t} = m\,a_t \implies -m\,g\sin\theta = m\,a_t \implies a_t = -g\sin\theta$$

Problem E.9

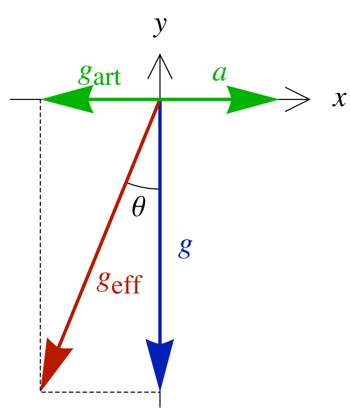


We saw in the Chapter D notes that a pendulum in an accelerating cart will hang at an angle given by

$$\tan\theta = \frac{a_c}{g}.$$

Treating the accelerating cart as an accelerating frame reanalyze the problem to derive the same result. What is the tension in the string?

Solution to E.9



A forward acceleration corresponds to a backward artificial gravity. A suspended object will hang in the direction of the effective gravity $\vec{g}_{\text{eff}} = \vec{g} + \vec{g}_{\text{art}}$. The angle is given by:

$$\theta = \tan^{-1}\left(\frac{a}{g}\right)$$

The tension is just the effective weight.

$$T = \;W_{\text{eff}} = \;m\,g_{\text{eff}} = \;m\sqrt{a^2 + g^2}$$

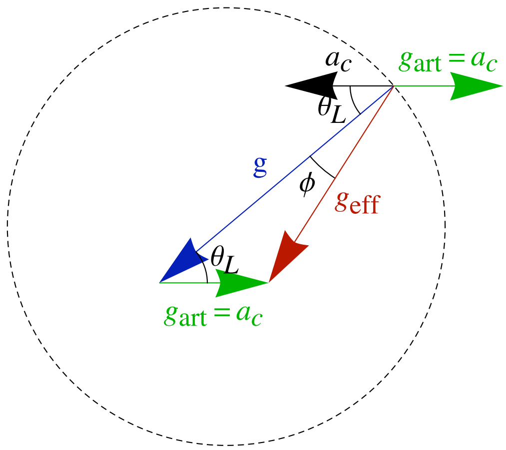
Problem E.10

This is an extension of problem E.1. Define *true vertical* to be the direction pointing toward the center of the earth and assume the earth is a perfect sphere. A plumb bob will not hang in the direction of true vertical but will hang in the dirction of the effective gravity $\vec{g}_{\text{eff}}$. The latitude of Bryan Texas is $\theta_L = 30.7\,°$.

(a) What is the strength of the effective gravity g_{eff} at Bryan Texas, taking $g = 9.80\ \text{m}/\text{s}^2$ as exact?

(b) What is the angle between true vertical and the direction of $\vec{g}_{\text{eff}}$ at Bryan Texas?

Solution to E.10



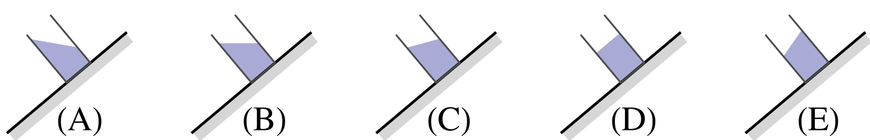
(a) Use the law of cosines to find g_{eff}.

$$g_{\text{eff}} = \sqrt{g^2 + g_{\text{art}}^2 - 2\, g\, g_{\text{art}} \cos\theta_L} = \sqrt{9.80^2 + 0.0290^2 - 2\times 9.80\times 0.0289 \cos 30.7°} = 9.78\ \frac{\text{m}}{\text{s}^2}$$

(b) To get the angle, ϕ in the diagram, use the law of sines.

$$\frac{\sin\phi}{g_{\text{art}}} = \frac{\sin\theta_L}{g_{\text{eff}}} \implies \frac{\sin\phi}{0.0290} = \frac{\sin 30.7°}{9.78} \implies \phi = 0.0867°$$

Problem E.11



The above graphics represent a glass of water sliding down an incline. View the glass as an accelerated frame and answer the following:

(a) Which describes a glass sliding down a frictionless incline?

(b) Which describes a glass sliding down an incline in the case where the friction is such that the speed of the glass is constant?

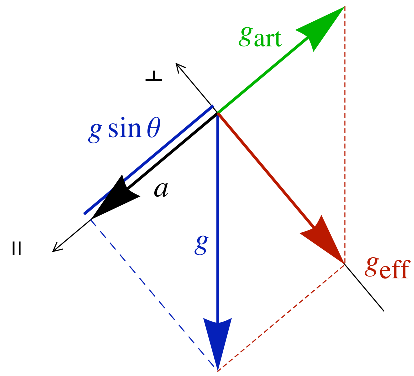
(c) Which describes a glass sliding down an incline in the case where there is slight friction, so that the speed of the glass is increasing?

(d) Which describes a glass sliding down an incline in the case where there is high friction, so that the speed of the glass is decreasing?

Solution to E.11

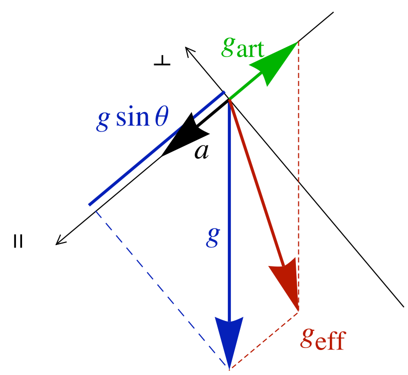
The surface of the water will be perpendicular to the effective gravity $\vec{g}_{\text{eff}}$ at that position.

(a) If the incline is frictionless then the acceleration is $a = g \sin\theta$ and water level is parallel to the surface. This gives case (D).

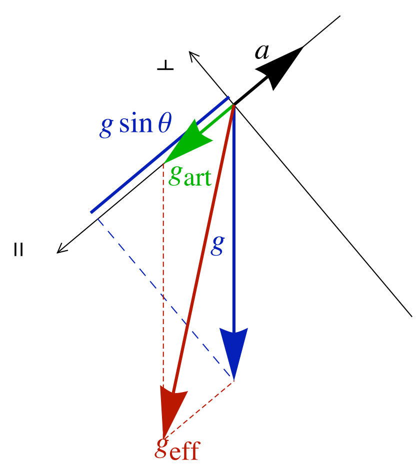


(b) If the friction causes a constant velocity, then the acceleration is zero and $\vec{g}_{\text{eff}} = \vec{g}$. This gives a horizontal water level and case (B).

(c) If the friction causes an increasing velocity, then $\vec{g}_{\text{eff}}$ is between normal to the surface and vertical, and thus gives case (C).

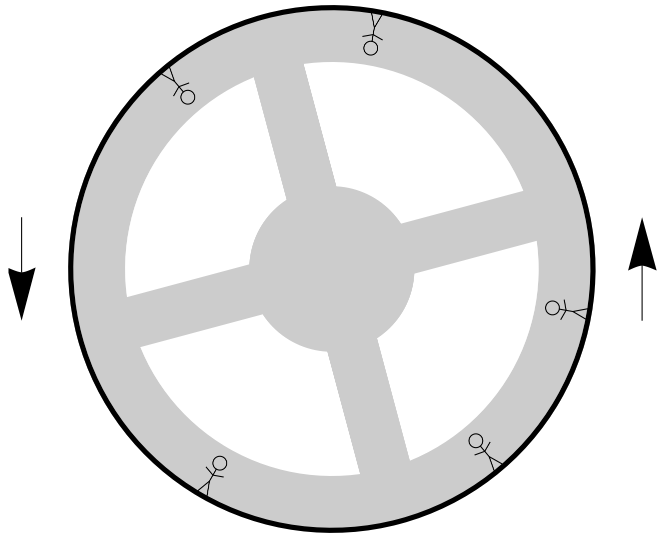


(c) If the friction causes a decreasing velocity, then the acceleration is up the incline and $\vec{g}_{\text{art}}$ is down the incline. This skews the water level to case (A).



Note that case (E) would correspond to some external force, for example a tension, pulling the glass down the incline with an acceleration greater than $g \sin\theta$.

Problem E.12



The outside rim of a rotating space station has a 150 m radius. It is desired to create an artificial gravity where a 150 lb man has an artificial weight of 50lb. What period of rotation is needed to do this?

Solution to E.12

Using $W = m\, g$, we can relate the artificial weight to the earth weight.

$$\frac{W_{\text{art}}}{W} = \frac{g_{\text{art}}}{g} \implies \frac{50}{150} = \frac{g_{\text{art}}}{g} \implies g_{\text{art}} = \frac{1}{3}\, g$$

Since $\vec{g}_{\text{art}} = -\vec{a}$ we have $g_{\text{art}} = a_c$ and $a_c = \left(\frac{2\,\pi}{T}\right)^2 r$.

$$g_{\text{art}} = a_c = \left(\frac{2\,\pi}{T}\right)^2 \text{r} \implies T = 2\,\pi \sqrt{\frac{r}{g_{\text{art}}}} = 2\,\pi \sqrt{\frac{150}{9.80/3}} = 42.6\text{ s}$$