Chapter F - Problems

Blinn College - Physics 2425 - Terry Honan

Problem F.1

A 600 N force pushes a refrigerator 8 m along a floor. What is the work done by the force?

Solution to F.1

For a constant force in one dimension:

$$W = F\,\Delta x = 600 \times 8 = 4800\ \text{J}$$

Problem F.2

Junior lifts a 20 N weight slowly (assume zero acceleration at all times) a distace of 1.5 m. What is the work done by Junior and what is the work done by gravity?

Solution to F.2

The lifting force is the same as the weight in magnitude. $W = F\,\Delta x$ is the work done by a constant force in one dimension. F and Δx should be viewed as one dimensional vectors; the direction of a one dimensional vector is its sign.

$$W_{\text{Junior}} = F\,\Delta x = 20 \times 1.5 = 30\ \text{J} \quad \text{and}$$

$$W_{\text{grav}} = F\,\Delta x = (-20) \times 1.5 = -30\ \text{J}$$

Problem F.3

Consider the vectors $\vec{A} = \langle -2, 5, -3 \rangle$ and $\vec{B} = \langle -1, 0, 2 \rangle$.
(a) What is the angle between the two vectors?
(b) What is the angle between $\vec{A}$ and the positive z axis?

Solution to F.3

The dot product has $\vec{A} \cdot \vec{B} = A\,B \cos\theta$ and $\vec{A} \cdot \vec{B} = A_x\,B_x + A_y\,B_y + A_z\,B_z$ as alternative definitions.

(a) To find the angle between two vectors we equate the two definitons. To evaluate this dot product,. since we are given the components of the two vectors, the second is the definition we need

$$\vec{A} \cdot \vec{B} = A_x\,B_x + A_y\,B_y + A_z\,B_z = (-2)\,(-1) + 5 \cdot 0 + (-3) \cdot 2 = -4$$

The the magnitudes of the two vectors are

$$A = \sqrt{2^2 + 5^2 + 3^2} = \sqrt{38} \text{ and } B = \sqrt{1 + 0 + 2^2} = \sqrt{5}\,.$$

Now we can find the angle.

$$\vec{A}\cdot\vec{B} = A\,B\cos\theta \implies \theta = \cos^{-1}\frac{\vec{A}\cdot\vec{B}}{A\,B} = \cos^{-1}\frac{-4}{\sqrt{38}\,\sqrt{5}} = 106.9\,°$$

(b) The angle between a vector and the z axis can be by repeating the above procedure replacing $\vec{B}$ with $\hat{e}_z$. Generally, the dot product of any vector and a unit vector gives the component of the vector in the direction of the unit vector.

$$\vec{A}\cdot\hat{e}_z = A_x\,0 + A_y\,0 + A_z\,1 = A_z = -3$$

$$\theta = \cos^{-1}\frac{\vec{A}\cdot\hat{e}_z}{A\,1} = \cos^{-1}\frac{-3}{\sqrt{38}\;1} = 119.1\,°$$

Problem F.4

A 20 N block, initially at rest, is dragged 5 m along a horizontal floor by a rope. The rope has a tension of 12 N and makes an angle of 25 ° above horizontal. A friction force of 9 N acts backward.
(a) There are four forces acting: the tension, friction, the normal force and gravity. What is the work done by each force?

(b) What is the final speed of the block?

Solution to F.4

(a) There are three forces acting on the block: the tension, the normal force and gravity. The work done by a constant force is:

$$W = \vec{F}\cdot\Delta\vec{r} = F\,\Delta r\cos\theta.$$

Gravity and the normal force are perpendicular to the direction of motion (horizontal) and thus do zero work.

$$W_N = 0 = W_{\text{grav}}$$

The tension force does positive work:

$$W_T = T\,\Delta r\cos\theta = 12\times 5\cos 25\,° = 54.379 = 54.4\text{ J}.$$

The friction force f_k opposes the direction of motion and does negative work:

$$W_f = f_k\,\Delta r\cos 180\,° = -f_k\,\Delta r = -9\times 5 = -45\text{ J}.$$

(b) The work energy theorem relates the net work to the change in the kinetic energy. The net work is the sum of the works done by all force acting on the body (all forces int he free-body diagram). In this case we have $W_{\text{net}} = W_T + W_f + W_{\text{grav}} + W_N$

$$W_{\text{net}} = \Delta K \implies W_T + W_f + W_{\text{grav}} + W_N = \frac{1}{2}\,m\left(v_f^2 - v_i^2\right)$$
$$\implies T\,\Delta r\cos\theta - f_k\,\Delta r + 0 + 0 = \frac{1}{2}\,m\left(v_f^2 - 0\right)$$
$$\implies 54.379 - 45 = \frac{1}{2}\,\frac{20}{9.8}\left(v_f^2 - 0\right) \implies v_f = 3.03\,\frac{\text{m}}{\text{s}}$$

Problem F.5

Consider a force of $F(x) = 6\,x^2 - 20$ (in SI units). If this acts on a body that moves from $x = 2$ m to $x = 4$ m then what is the work done by the force?

Solution to F.5

For a varying force in one dimension the work is

$$W = \int_{x_i}^{x_f} F(x)\, dx.$$

Heere we get:

$$W = \int_2^4 \left(6\,x^2 - 20\right) dx = \left(2\,x^3 - 20\,x\right)\Big|_2^4 = 48 - (-24) = 72\ \text{N}$$

Problem F.6

It takes a 60 N force to compress a spring 4 cm.
(a) What is the work done by the spring when it is compressed from 0 to $x = -4$ cm?
(b) What is the work done in compressing the spring in part (a)?
(c) What is the work done by the spring when it is compressed from the stretched position of $x = 2$ cm to $x = -4$ cm?

Solution to F.6

We first need to find the spring constant.

$$F = k\,x \implies k = \frac{F}{x} = \frac{60}{0.04} = 1500\ N/m$$

The work done by a spring between x_i and x_f is

$$W_{\text{spring}} = -\frac{1}{2}\,k\left(x_f^2 - x_i^2\right)$$

(a) $x_i = 0$ and $x_f = -0.04$ m $\implies$ $W_{\text{spring}} = -1.2$ J

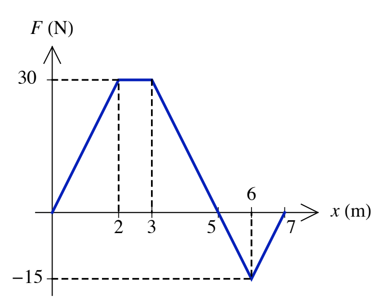
(b) When some agent compresses the spring it does the negative of this.

$$W_{\text{agent}} = -W_{\text{spring}} = +1.2\ \text{J}$$

(c) $x_i = 0.02$ m and $x_f = -0.04$ m $\implies$ $W_{\text{spring}} = -0.9$ J

Problem F.7

A 3 kg mass intially at rest at $x = 0$ is acted upon by a single force given by the graph below. What is the speed of the mass at $x = 2$ m, $x = 5$ m and $x = 7$ m.



Solution to F.7

We need to use the work energy theorem where the net work is the work done by the only force

$$W_{\text{net}} = \Delta K \implies W = \frac{1}{2} m \left(v_f^2 - v_i^2\right) = \frac{1}{2} m v_f^2 \implies v_f = \sqrt{\frac{2W}{m}}$$

For a varying force in one dimension the work is

$$W = \int_{x_i}^{x_f} F(x)\, dx.$$

This has the graphical interpretation as the area under the graph of F vs. x. Here we can find the areas using simple geometry. Remember that when a function is below the axis the dontribution to the area is negative. Denote the work from 0 to x by W_{0x}. Using $m = 3$ kg we can find v_f.

$$W_{02} = \frac{1}{2} 2 \times 30 = 30 \text{ J} \implies v_f = \sqrt{\frac{2W}{m}} = 4.47 \ \frac{\text{m}}{\text{s}}$$

$$W_{05} = W_{02} + W_{23} + W_{35} = 30 + 1 \times 30 + 30 = 90 \text{ J} \implies v_f = 7.75 \ \frac{\text{m}}{\text{s}}$$

$$W_{07} = W_{05} + W_{57} = 90 - \frac{1}{2} 2 \times 15 = 75 \text{ J} \implies v_f = 7.07 \ \frac{\text{m}}{\text{s}}$$

Problem F.8

A man does work W pushing a car of mass m from rest to a speed v over a distance d on a horizontal surface. A resistive force acts backward.
(a) What is the work done by the resistive force?
(b) What are the pushing force and the magnitude of the resistive force?
(c) Give the answers to parts (a) and (b) using the numbers:

$$m = 1600 \text{ kg}, v = 1.5 \ \frac{\text{m}}{\text{s}}, W = 2400 \text{ J and } d = 6 \text{ m}$$

Solution to F.8

(a) The only two forces that do work on the car are the pushing force and the resistive force; the normal force and gravity are perpendicular to the drection of motion. To do this use the work-energy theorem with $W_{\text{net}} = W + W_R$.

$$W_{\text{net}} = \Delta K \implies W + W_R = \frac{m}{2} v^2 - 0 \implies W_R = \frac{m}{2} v^2 - W$$

(b) From the works it is easy to get the forces.

$$W = F \Delta r \cos\theta = F d \implies F = W/d$$

$$W_R = F \Delta r \cos\theta = -R d \implies R = -W_R/d = W/d - \frac{m}{2d} v^2$$

(c) Using the values $m = 1600$ kg, $v = 1.5 \ \frac{\text{m}}{\text{s}}$, $W = 2400$ J and $d = 6$ m we get:

$$W_R = \frac{m}{2} v^2 - W = 1800 - 2400 = -600 \text{ J},$$

$$F = W/d = 2400/6 = 400\text{ N} \text{ and } R = W_R/d = 600/6 = 100\text{ N}$$

Problem F.9

(a) A block of mass m slides from rest a distance D down an incline at angle θ before hitting a spring that is parallel to the incline. It compresses the spring an additional d before stopping and turning around. If the coefficient of kinetic friction is μ then what is the spring constant?

(b) Solve part (a) numerically using $m = 20$ kg, $D = 40$ cm, $d = 10$ cm, $\theta = 35°$ and $\mu = 0.20$.

Solution to F.9

Here the mechanical energy is $E = \frac{1}{2} m v^2 + m g y + \frac{1}{2} k x^2$. Since there is friction it is not conserved.

$$E_i + W_{\text{nc}} = E_f \implies \frac{1}{2} m v_i^2 + m g y_i + \frac{1}{2} k x_i^2 + W_{\text{nc}} = \frac{1}{2} m v_f^2 + m g y_f + \frac{1}{2} k x_f^2$$

W_{nc} is the work done by friction. Friction acts opposite to the direction of motion so its work is $W_f = -f_k (D + d)$, where $D + d$ is the total distance it slides. The magnitude of the force of friction is $f_k = \mu_k N = \mu N$. Since only the normal force and the perpendicular component of gravity act perpendicular to the surface then $N = m g \cos\theta$. Putting this all together gives

$$W_{\text{nc}} = W_f = -f_k (D + d) = -\mu m g \cos\theta (D + d).$$

The block is at rest in both the initial and final cases.

$$v_i = 0 = v_f$$

The total distance it slides is $D + d$ so the vertical part of that is $(D + d)\sin\theta$. Since the net motion is downward we get:

$$y_i = (D + d)\sin\theta \text{ and } y_f = 0$$

Initially the spring is uncompressed and in the end it is compressed by d.

$$x_i = 0 \text{ and } x_f = d$$

The expression for $E_i + W_{\text{nc}} = E_f$ becomes

$$[0 + m g (D + d)\sin\theta + 0] - \mu m g \cos\theta (D + d) = 0 + 0 + \frac{1}{2} k d^2$$

Solving for k gives the somewhat ugly expression

$$k = \frac{2 m g}{d^2} (D + d)(\sin\theta - \mu\cos\theta)$$

(b) Using $m = 20$ kg, $D = 0.40$ m, $d = 0.10$ m, $\theta = 35°$ and $\mu = 0.20$ gives

$$k = 8030 \frac{\text{N}}{\text{m}}.$$

Problem F.10

A mass swings in a vertical circle at the end of a string of length L. What is the minimum speed the mass must have at the bottom of the circle for the mass to make it over the top without the string collapsing.

Solution to F.10

We first need to find the minimum speed at the top to make it over without the string collapsing. This part is a chapter E problem. The two forces are the tension T and the weight $m\,g$, both acting downward. The acceleration is centripetal, which is downward. The second law gives:

$$F_{\text{net},c} = m\,a_c \implies T + m\,g = m\,\frac{v^2}{L}.$$

Setting $T = 0$ gives the minimum speed at the top, $v_{\text{min,top}}$

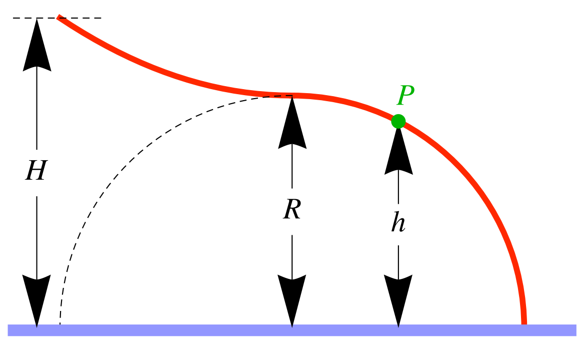
$$0 + m\,g = m\,\frac{v_{\text{min,top}}^2}{L} \implies v_{\text{min,top}} = \sqrt{g\,L}$$

Eneergy is conserved, so we can relate the energy at the bottom to the energy at the top. $v_{\text{bottom}}^2 = v_{\text{top}}^2 + 2\,g\,h$, where the height difference is $h = 2\,L$

$$v_{\text{min,bottom}}^2 = v_{\text{min,top}}^2 + 2\,g\,h = g\,L + 2\,g\,(2\,L) \implies v_{\text{min,bottom}} = \sqrt{5\,g\,L}$$

Problem F.11

Consider a (frictionless) water slide for children. Ths slide is shown in red with the bottom of the slide being a quarter circle of radius R. The water line is shown in blue.



A child starts from rest at the height H above the water (the highest point on the red slide.) At some point P the child will leave the surface. What is the height h of that point P above the water.

Solution to F.11

To relate the speed at the highest point to the speed v at some other position use

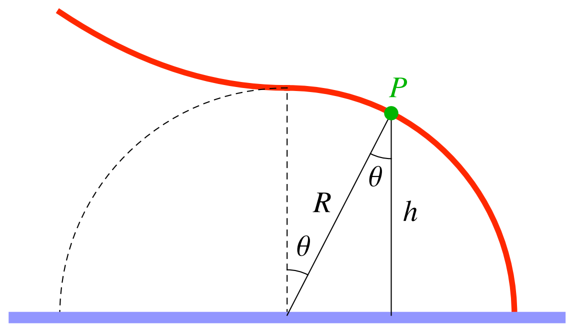
$$v_{\text{bottom}}^2 = v_{\text{top}}^2 + 2\,g\,h'$$

The height difference between the two positions is $h' = H - h$, so the speed at height h is:

$$v^2 = 0 + 2\,g\,(H - h)\,.$$

We can find the normal force N as a function of height by resolve forces into the centripetal direction

$$F_{\text{net},c} = m\,a_c \implies m\,g\cos\theta - N = m\,\frac{v^2}{R}$$



θ is the angle of the child measured from the top of the circle. $\cos\theta = h/R$. Since $N \geq 0$ the child will leave the surface when $N = 0$. This gives: $g\,h = v^2$

Equating the two expressions for v^2 gives:

$$g\,h = v^2 = 0 + 2\,g\,(H - h) \implies h = \frac{2}{3}H$$

Problem F.12

Consider Atwood's machine with a frictionless, light pulley. The smaller mass m_1 is on the floor while the larger mass m_2 is a height h above the floor.
(a) If it is released from rest, then what is the speed of m_2 when it hits the floor.
(b) After m_2 hits the floor m_1 is at height h. It continues upward until it eventually stops. What is the maximum height reached by m_1.

Solution to F.12

(a) There is kinetic energy and gravitational potential energy for both masses. The total energy is:

$$E = \frac{1}{2}m_1\,v_1^2 + \frac{1}{2}m_2\,v_2^2 + m_1\,g\,y_1 + m_2\,g\,y_2.$$

The two speeds are equal $v = v_1 = v_2$. Equating the initial and final energies gives and expression for the speed.

$$E_i = E_f \implies 0 + 0 + 0 + m_2\,g\,h = \frac{1}{2}m_1\,v^2 + \frac{1}{2}m_2\,v^2 + m_1\,g\,h + 0$$

$$\implies v = \sqrt{2\left(\frac{m_2 - m_1}{m_1 + m_2}\,g\right)h}\,.$$

(b) To find the maximum height of m_1 solve for how high it rises after m_2 hits the ground and the string loses its tension. The answer to part (a) is the speed of m_1 when the string loses its tension. Using $v_{\text{bottom}}^2 = v_{\text{top}}^2 + 2\,g\,h$ we get $v^2 = 0 + 2\,g\,h'$, where h' is the distance above h. The maximum height is $h + h'$

$$h + h' = h + \frac{m_2 - m_1}{m_1 + m_2}\,h = \frac{2\,m_2}{m_1 + m_2}\,h$$

Problem F.13

Mass m_1 slides on a horizontal table with a coefficient of kinetic friction of μ. It is connected to a hanging mass m_2 that is initially at a height h. What is the speed of m_2 when it hts the floor?

Solution to F.13

The speeds of the two masses are the same $v_1 = v_2 = v$, so the total kinetic energy is

$$K_{\text{total}} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} (m_1 + m_2) v^2$$

Since m_1 moves horizontally, its potential energy doesn't change; the easiest way to deal with this is to choose $y_1 = 0$ when m_1 is on the table and then choose y_2 differently.

$$U_{\text{total}} = m_1 g y_1 + m_2 g y_2 = m_2 g y_2$$

The total (mechanical) energy becomes

$$E = K_{\text{total}} + U_{\text{total}} = \frac{1}{2} (m_1 + m_2) v^2 + m_2 g y_2.$$

Friction is the only nonconservative force. The sliding mass moves the same distance as the hanging mass. $W_{\text{nc}} = W_f = -f_k \Delta x = -\mu_k m_1 g h$

$$E_i + W_{\text{nc}} = E_f$$

$$\Longrightarrow 0 + m_2 g h - \mu_k m_1 g h = \frac{1}{2} (m_1 + m_2) v_f^2 + 0$$

$$\Longrightarrow v_f = \sqrt{2 g h \frac{m_2 - \mu_k m_1}{m_1 + m_2}}$$

Problem F.14

Given a potential energy as a function of position in two dimensions, is SI units, is :

$$U(x, y) = 4 x^2 y - 7 y^2$$

(a) What is the force as a function of position?
(b) What is the force at $(-2\text{ m}, 3\text{ m})$?

Solution to F.14

(a) We take partial derivatives of the potential energy to get the force as a function of position

$$F_x = -\frac{\partial U}{\partial x} = -(8 x y - 0) = -8 x y$$

$$F_y = -\frac{\partial U}{\partial y} = -\left(4 x^2 - 14 y\right) = 14 y - 4 x^2$$

$$\Longrightarrow \vec{F} = \left\langle -8 x y, 14 y - 4 x^2 \right\rangle$$

(b) At $(x, y) = (-2\text{ m}, 3\text{ m})$:

$$\vec{F} = \langle -48,\, 26 \rangle \text{ N}$$