Chapter G - Problems

Blinn College - Physics 2425 - Terry Honan

Problem G.1

A 0.12 kg ball dropped from a height of 2.0 m rebounds to 1.8 m.

(a) What is the change in the ball's momentum during its collision with the floor?

(b) Suppose the ball is in contact with the floor for 0.08 s. What is the average force of the floor on the ball?

Solution to G.1

(a) We can relate the speed at the ground to the height of the ball by equating the energy at the top and the bottom. In both cs This gives the speeds before and after hitting the ground.

$$v_{\text{bottom}}^2 = v_{\text{top}}^2 + 2\,g\,h \implies v = \pm\sqrt{2\,g\,h}$$

Choose upward as the positive direction.

$$v_i = -\sqrt{2\,g\,h_i} = -\sqrt{2\times 9.8\times 2} = -6.2610$$

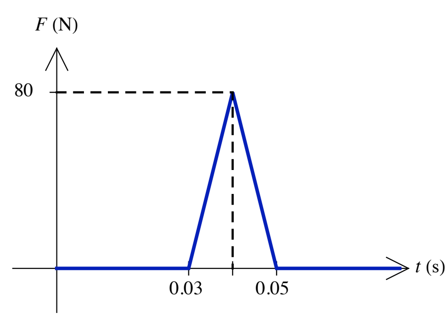
$$v_f = +\sqrt{2\,g\,h_f} = \sqrt{2\times 9.8\times 1.8} = 5.9397$$

$$\Delta \text{p} = \text{m}\left(v_f - v_i\right) = 0.12\times(5.9397 -- 6.2610) = 1.464\ \text{kg}\ \frac{\text{m}}{\text{s}}$$

(b)The impulse-momentum gives the average force

$$\overline{F}\,\Delta t = \Delta p \implies \overline{F} = \frac{1.464}{0.08} = 18.3\,N$$

Problem G.2



The graph above is the force vs. time for some collision on a 0.25 kg ball that is initially at rest.
(a) What is the impulse given to the ball?
(b) What is the average force acting on the ball during the collision?
(c) What is the velocity of the ball after the collision?

Solution to G.2

(a) The impulse $I = \int_{t_i}^{t_f} F(t)\, dt$ has the graphical interpretation of the area under the F vs. t graph. In this case

$$I = \frac{1}{2}\text{ base}\times\text{height} = \frac{1}{2}\,0.02\times 80 = 0.8\ N\cdot s\ .$$

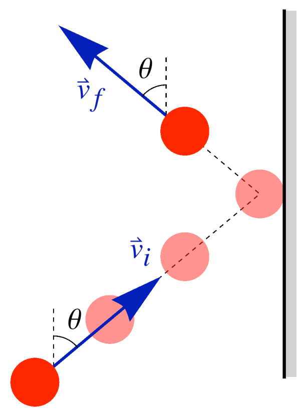
(b) $I = \overline{F}\,\Delta t$ gives the average force.

$$\overline{F} = \frac{0.8}{0.02} = 40\ N$$

(c) The impulse momentum theorem gives the final velocity.

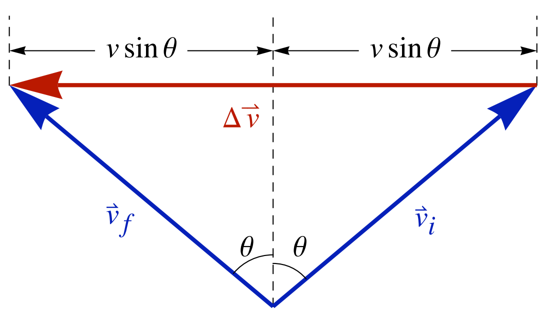
$$I = \Delta p = m\left(v_f - v_i\right) \implies 0.8 = 0.25\left(v_f - 0\right) \implies v_f = 3.2\ \frac{\text{m}}{\text{s}}$$

Problem G.3



A ball of mass m bounces elastically with a wall. The ball's speed is v before and after the collision and θ, the angle the ball makes with the wall, is the same before and after the collision. It the ball is in contact with the wall for a time T, then what is the average fore of the wall on the ball?

Solution to G.3



$$\overline{\vec{F}}\,\Delta t = \Delta\vec{p} = m\left(\vec{v}_f - \vec{v}_i\right)$$

Because the speeds and angles before and after the collision are the same the parallel components are unchanged. Consider the perpendicular components.

$$\overline{F}_{\perp}\,\Delta t = \Delta p_{\perp} = m\left(v_{f,\perp} - v_{i,\perp}\right) \implies \overline{F}_{\perp}\,T = m\,(v\sin\theta - -v\sin\theta) \implies \overline{F}_{\perp} = \frac{2\,m\,v}{T}\sin\theta$$

Problem G.4

A 2 kg mass has a velocity of $\langle 4, -3\rangle\, m/s$ and a 3 kg mass has a velocity of $\langle -1, 5\rangle\, m/s$. What is the total momentum of the system and what is the velocity of the center of mass?

Solution to G.4

$$\vec{p}_{\text{tot}} = M\,\vec{v}_{\text{cm}} = \sum m_i\,\vec{v}_i\,, \text{ where } M = \sum m_i$$

$$\vec{p}_{\text{tot}} = m_1\,\vec{v}_1 + m_2\,\vec{v}_2 = 2\,\langle 4, -3\rangle + 3\,\langle -1, 5\rangle = \langle 5, 9\rangle\ \text{kg}\ \frac{\text{m}}{\text{s}}$$

$$M = m_1 + m_2 = 2 + 3 = 5\ \text{kg} \implies \vec{v}_{\text{cm}} = \frac{\vec{p}_{\text{tot}}}{M} = \frac{\langle 5,9\rangle}{5} = \langle 1, 1.8\rangle\ \frac{\text{m}}{\text{s}}$$

Problem G.5

A 5 kg mass is at (0, 3m), a 2 kg mass is at (-2m, 0) and a 3 kg mass is at (1m, -4m). What are the coordinates of the center of mass?

Solution to G.5

$$x_{\text{cm}} = \frac{\sum m_i\,x_i}{\sum m_i} = \frac{5\times 0 + 2\times(-2) + 3\times 1}{5+2+3} = -0.1\ \text{m}$$

$$y_{\text{cm}} = \frac{\sum m_i\,y_i}{\sum m_i} = \frac{5\times 3 + 2\times 0 + 3\times(-4)}{5+2+3} = 0.3\ \text{m}$$

Thus the coordinates are (-0.1 m, 0.3 m)

Problem G.6

The distance between hydrogen and chlorine atoms in an HCl molecule is $1.3\times 10^{-10}\,m$. Where is the center of mass of this molecule, measured from the hydrogen atom? Chlorine is about 35 times more massive than hydrogen.

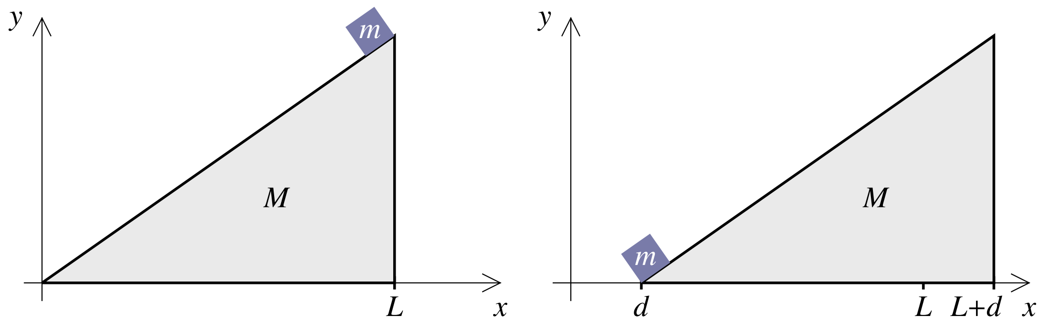
Solution to G.6

Take the x axis to be along the central axis with x=0 at the hydrogen atom.

$$x_{\text{cm}} = \frac{m_H\,x_H + m_{\text{Cl}}\,x_{\text{Cl}}}{m_H + m_{\text{Cl}}} = \frac{0 + 35\ \text{m}\times 1.3\times 10^{-10}}{\text{m} + 35\ \text{m}} = 1.26\times 10^{-10}\ \text{m}$$

Problem G.7

A small block of mass m sits at the top of a triangular wedge of mass M. The wedge has a base of width L and slides without friction on a horizontal surface. If the block is released from rest from the top of the wedge, then how far has the wedge moved d, when the block has reached the bottom of the wedge? Neglect the size of the small mass compared to the wedge.



Solution to G.7

The fact that there is no friction implies that there is no net horizontal force and thus the horizontal component of the center of mass' acceleration is zero. Since the

$$F_{\text{net,hor}} = 0 \implies a_{\text{cm,hor}} = 0 \implies v_{\text{cm,hor}} = 0 \implies x_{\text{cm,hor}} = \text{constant}$$

Take the horizontal positions of m and M to be x and X respectively. We do not need to find the actual position of the center of mass of the wedge. The position of the center of mass will shift by the same distance as the whole wedge. Thus $\Delta X = d$.

$$0 = m\,\Delta \mathrm{x} + M\,\Delta X = m\,(d - L) + M\,d \implies d = \frac{m}{M+m}\,L$$

Problem G.8

A 75 kg man stands on a frozen ice rink (taken to be frictionless) next to a wall. He throws a 0.5 kg ball at 12 $\frac{\mathrm{m}}{\mathrm{s}}$ toward the wall and catches it after it rebounds elastically (at the same speed.) Ignoring the projectile motion of the ball, how fast is he moving after he catches it.

Solution to G.8

$$M = 75\,\text{kg},\ m = 0.5\,\text{kg and } v = 12\,\frac{\text{m}}{\text{s}}$$

Conserving momentum when the ball is thrown gives

$$0 = M\,V - m\,v \implies V = \frac{m}{M}\,v\,.$$

The motion of the ball changes direction when it bounces. Conserving momentum when the ball is caught gives

$$M\,V + m\,v = (M + m)\,V' \implies V' = \frac{2\,m}{M+m}\,v = 0.159\,\frac{\text{m}}{\text{s}}.$$

Problem G.9

A 40 gram bullet is fired into a stationary 2 kg block. The bullet embeds in the block and they both move off at $5\,m/s$. What is the velocity of the bullet before hitting the block?

Solution to G.9

$$m_1\,v_{1i} + m_2\,v_{2i} = (m_1 + m_2)\,v_f \implies 0.040\times v_{1i} + 2\times 0 = 2.04\times 5$$
$$\implies v_{1\,\mathrm{i}} = \frac{2.04\times 5}{0.040} = 255\,\frac{\mathrm{m}}{\mathrm{s}}$$

Problem G.10

Two masses m and $3\,m$ move toward each other (in opposite directions) at the same speed v. If they collide elastically and bounce so thaat they stay in the same line of motion, then what are both final velocities?

Solution to G.10

$$m_1\,v_{1i} + m_2\,v_{2i} = m_1\,v_{1f} + m_2\,v_{2f} \implies m\,v + 3\,m\times(-v) = m\,v_{1f} + 3\,m\,v_{2f}$$
$$\implies -2\,v = v_{1f} + 3\,v_{2f}$$

In an elastic collision kinetic energy is also conserved. We will use the trick, discussed in the notes, to replace the conservation of kinetic energy equation with a simpler linear one.

$$\frac{1}{2}\,m_1\,v_{1i}^2 + \frac{1}{2}\,m_2\,v_{2i}^2 = \frac{1}{2}\,m_1\,v_{1f}^2 + \frac{1}{2}\,m_2\,v_{2f}^2 \implies v_{1i} + v_{1f} = v_{2i} + \mathrm{v}_{2f}$$
$$\implies v + v_{1f} = -v + v_{2f} \implies 2\,v + v_{1f} = v_{2f}$$

Solving gives:

$$-2\,v = v_{1f} + 3\left(2\,v + v_{1f}\right) \implies v_{1f} = -2\,v \text{ and}$$

$$v_{2f} = 2\,v + v_{1f} \implies v_{2f} = 0.$$

Problem G.11

A bullet of mass m_1 is fired into a stationary block of mass m_2. The bullet embeds in the block and they both slide on a horizontal surface with a coefficient of friction μ. The bullet and block both slide a distance d before stopping. What is the speed of the bullet *before* it hits the block?

Solution to G.11

After the bullet embeds in the block we can relate the coefficient of kinetic friction and the distance to the initial speed. One way of doing this is to use the work-energy theorem. The net work is the work done by friction. Since the normal force is $m\,g$ the friction force is $\mu\,m\,g$.

$$W_{\text{net}} = \Delta K \implies -(\mu\,m\,g)\,d = \frac{m}{2}\left(0 - v^2\right) \implies v = \sqrt{2\,\mu\,g\,d}$$

This is the speed of the bullet-block combination just after the bullet embeds. (Another way to get this is to use $a = -\mu g$ and solve for v using kinematics.) m in the above expression is the total mass $m = m_1 + m_2$; note that it cancels.

The speed of the bullet before hitting the block can be related to the v above using conservation of momentum during the totally inelastic collision.

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f \implies m_1 v_0 + m_2 0 = (m_1 + m_2) v \implies v_0 = \frac{m_1+m_2}{m_1} v = \frac{m_1+m_2}{m_1} \sqrt{2 \mu g d}$$

Problem G.12

A 10 kg mass moving in the x direction at $5\, m/s$ collides with a 6 kg mass moving in the negative y direction at $20\, m/s$. If the two masses stick together then what is their final velocity after the collision?

Solution to G.12

$$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = (m_1 + m_2) \vec{v}_f \implies 10 \langle 5, 0 \rangle + 6 \langle 0, -20 \rangle = (16) \vec{v}_f \implies \vec{v}_f = \langle 3.13, -7.5 \rangle \frac{\text{m}}{\text{s}}$$

Problem G.13

A particle of mass m moving with speed v collides elastically with an identical particle initially at rest. Show that after the collision the velocities of the two particles are perpendicular.

Solution to G.13

Conservation of momentum for equal masses ($m_1 = m_2$) with m_2 initially at rest gives:

$$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f} \implies \vec{v}_{1i} = \vec{v}_{1f} + \vec{v}_{2f}$$
$$\implies v_{1i}^2 = \vec{v}_{1i} \cdot \vec{v}_{1i} = \vec{v}_{1f} \cdot \vec{v}_{1f} + \vec{v}_{2f} \cdot \vec{v}_{2f} + 2 \vec{v}_{1f} \cdot \vec{v}_{2f}$$
$$\implies v_{1i}^2 = v_{1f}^2 + v_{2f}^2 + 2 \vec{v}_{1f} \cdot \vec{v}_{2f}$$

Conservation of energy for this case gives:

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \implies v_{1i}^2 = v_{1f}^2 + v_{2f}^2$$

Equating the two expressions gives: $\vec{v}_{1f} \cdot \vec{v}_{2f} = 0$, which implies that the two final velocities are perpendicular.