Chapter H - Problems

Blinn College - Physics 2425 - Terry Honan

Problem H.1

A wheel rotates from rest to 12 rad/s in 3 s. Assume the angular acceleration is constant.
(a) What is the magnitude of the wheel's angular acceleration?
(b) Through what angle (in radians) does the wheel rotate?

Solution to H.1

(a) $\omega = \omega_0 + \alpha\,\mathrm{t} \implies 12 = 0 + \alpha\,3 \implies \alpha = 4\,\frac{\mathrm{rad}}{\mathrm{s}}$

(b) $\Delta\theta = \frac{1}{2}(\omega_0 + \omega)\,\mathrm{t} = \frac{1}{2}(0 + 12)\,3 = 18\,\mathrm{rad}$

Problem H.2

(a) What is the angular velocity of the Earth in its orbit about the Sun.
(b) What is the angular velocity of the Moon in its orbit about the Earth.

Solution to H.2

A constant angular velocity is related to the period T by $\omega = \frac{2\pi}{T}$.

(a) $T = 1\,\text{year} = 365.24\times 24\times 3600 = 3.1557\times 10^7\,\mathrm{s} \implies \omega = 1.99\times 10^{-7}\,\frac{\mathrm{rad}}{\mathrm{s}}$

(b) $T = 27.32\,\text{days} = 27.32\times 24\times 3600 = 2.36\quad 05\times 10^6\,\mathrm{s} \implies \omega = 2.66\times 10^{-6}\,\frac{\mathrm{rad}}{\mathrm{s}}$

Problem H.3

A grinding wheel rotating at 100 rev/min is turned off. It slows with a constant angular acceleration of 2 rad/s.
(a) How long does it take the wheel to stop?
(b) Through what angle, in radians, does it turn while slowing?

Solution to H.3

$\omega = 0$ and $\omega_0 = 100\,\frac{\mathrm{rev}}{\mathrm{min}}\times\frac{2\pi\,\mathrm{rad}}{\mathrm{rev}}\times\frac{1\,\mathrm{min}}{60\,\mathrm{s}} = 10.472\,\frac{\mathrm{rad}}{\mathrm{s}}$

(a) $\omega = \omega_0 + \alpha\,t \implies 0 = 10.472 - 2\,t \implies 5.24\,\mathrm{s}$

(b) $\omega^2 - \omega_0^2 = 2\,\alpha\,\Delta\theta \implies \Delta\theta = \frac{0 - 10.472^2}{2\times 2} = 27.4\,\mathrm{rad}$

Problem H.4

A car races around a circular track of radius 250 m at a constant speed of 45 m/s.
(a) What is its angular velocity?
(b) What is the magnitude and direction of its acceleration?

Solution to H.4

$R = 250$ m and $v = 45\, m$

(a) $v = r\,\omega \implies \omega = \frac{v}{R} = 0.18\ \frac{\text{rad}}{\text{s}}$

(b) $a_c = \omega^2\, r = \frac{v^2}{R} = 8.1\ \frac{\text{m}}{\text{s}^2}$ The direction of the acceleration is centripetal, meaning that is directed toward the center.

Problem H.5

By rotating through 1.25 rev of a 1 m radius arc, a discus thrower uniformly accelerates a discus from rest to 25 m/s.
(a) What is the final angular velocity of the discus?
(b) What is its angular acceleration?
(c) What is the total time of acceleration?

Solution to H.5

(a) $v = r\,\omega \implies \omega = \frac{v}{r} = \frac{25}{1} = 25\ \frac{\text{rad}}{\text{s}}$

(b) $\omega^2 - \omega_0^2 = 2\,\alpha\,\Delta\theta \implies 25^2 - 0 = 2\,\alpha\,(2\,\pi \times 1.25) \implies \alpha = 39.8\ \frac{\text{rad}}{\text{s}}$

(c) $\Delta\theta = \frac{1}{2}\,(\omega_0 + \omega)\,t \implies 2\,\pi \times 1.25 = \frac{1}{2}\,(0 + 25)\,t \implies t = 0.628$ s

Problem H.6

Assume that a baseball is a uniform sphere of radius 3.80 cm. If it moves at a speed of 38 m/s (the speed of the center of mass) and rotates at 125 rad/s then what is the ratio of the rotational kinetic energy to the translational kinetic energy.

Solution to H.6

For a uniform solid sphere $I = \frac{2}{5}\,\text{M}\,R^2$

$$K_{\text{rot}} = \frac{1}{2}\,I\,\omega^2 = \frac{1}{2}\,\frac{2}{5}\,M\,R^2\,\omega^2 = \frac{1}{5}\,M\,R^2\,\omega^2$$

$$\frac{K_{\text{rot}}}{K_{\text{trans}}} = \frac{\frac{1}{5}\,M\,R^2\,\omega^2}{\frac{1}{2}\,M\,v^2} = \frac{2\,R^2\,\omega^2}{5\,v^2}$$

Insert the numbers: $R = 0.038\,m$, $v = 38\ \frac{\text{m}}{\text{s}}$ and $\omega = 125\ \frac{\text{rad}}{\text{s}}$ giving $\frac{K_{\text{rot}}}{K_{\text{trans}}} = 0.00625$.

Problem H.7

Three masses glued onto a light rigid sheet in the xy plane. The masses and positions are:

4 kg at (−3 m, 5 m), 6 kg at (4 m, 0) and 8 kg at (0, −3 m).

(a) Suppose this sheet is rotated about the y axis at an angular velocity of 12 rad/s. What is its total rotational kinetic energy?

(b) Suppose it rotates about the z axis at 12 rad/s. What is its rotational kinetic energy?

Solution to H.7

(a) For a discrete distribution of mass: $I = \sum_i m_i r_i^2$ where r_i is the perpendicular distance of the mass m_i from the axis. If the axis is the y axis then $r_i = |x_i|$

$$I = \sum_i m_i r_i^2 = \sum_i m_i x_i^2 = 4\times 3^2 + 6\times 4^2 + 8\times 0 = 132\ \text{km}\cdot m^2$$

The kinetic energy is

$$K = \frac{1}{2} I \omega^2 = \frac{1}{2} 132\times 12^2 = 9500\ \text{J}.$$

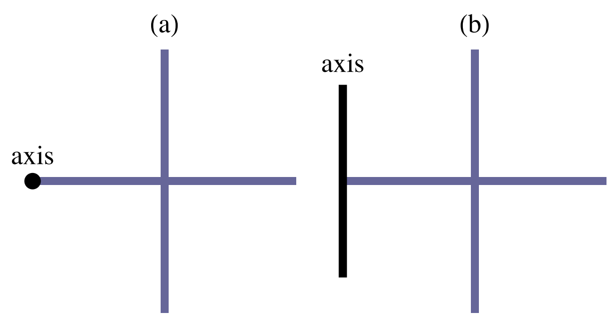
(b) If the axis is the z-axis then $r_i = \sqrt{x_i^2 + y_i^2}$.

$$I = \sum_i m_i r_i^2 = \sum_i m_i \left(x_i^2 + y_i^2\right) = 4\times\left(3^2 + 5^2\right) + 6\times\left(4^2 + 0\right) + 8\times\left(0 + 3^2\right) = 304\ \text{km}\cdot m^2$$

$$K = \frac{1}{2} I \omega^2 = \frac{1}{2} 304\times 12^2 = 21\,900\ \text{J}.$$

Problem H.8

Two identical thin rods of length L and mass M are perpendicular and joined at their center. What is the moment of inertia about an axis at the end of one rod that is:
(a) perpendicular to the plane,
(b) parallel to the other rod.



Solution to H.8

The rod connected to the axis has, in both cases, a moment of

$$I_1 = \frac{1}{3} M L^2.$$

The other rod's moment varies.

(a) Use the parallel axis theorem to find the other moment. The distance from the center of mass to the axis is $D = L/2$.

$$I_2 = I_{\text{cm}} + M D^2 = \frac{1}{12} M L^2 + M \left(\frac{L}{2}\right)^2 = \frac{1}{3} M L^2.$$

The total moment is the sum.

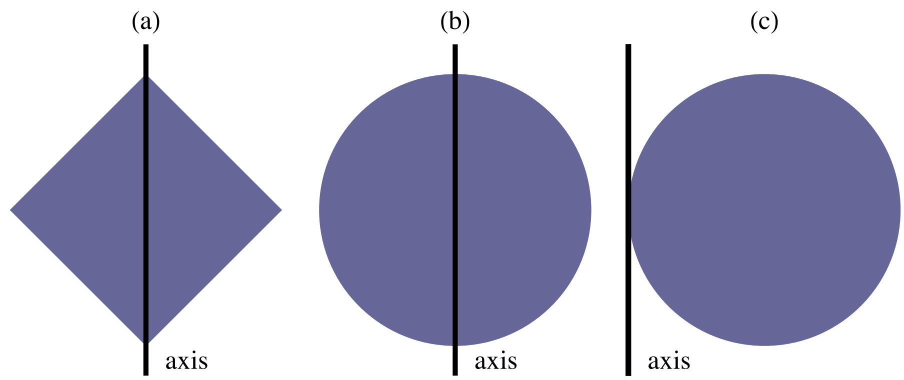
$$I = I_1 + I_2 = \frac{2}{3} M L^2.$$

(b) In this case all the mass of the second rod is a distance $L/2$ rom the axis. It's moment is

$$I_2 = M \left(\frac{L}{2}\right)^2 = \frac{1}{4} M L^2. \implies I = I_1 + I_2 = \left(\frac{1}{3} + \frac{1}{4}\right) M L^2 = \frac{7}{12} M L^2$$

Problem H.9

What are the moments of inertia of the following flat uniform bodies of mass M about the axis mentioned?
(a) an $L\times L$ square about a diagonal
(b) a circle of radius R about a diameter
(c) a circle of radius R about a tangent



Solution to H.9

(a) The moment of a flat $a\times b$ rectangle of mass M about a perpendicular axis through the center is $I = \frac{1}{12} M \left(a^2 + b^2\right)$. This follows from the perpendicular axis theorem. (For a flat body in xy-plane, $I_z = I_x + I_y$) Here take the x and y axes to axes through the center that are parallel to the sides. For I_x and I_y the moment is equivalent to a uniform rod about a perpendicular axis through the center.

It follows that for a square $I_z = \frac{1}{6} M L^2$. Now take the x and y axes to be the diagonals. By symmetry $I_x = I_y = I$ and by the perpendicular axis theorem

$$I_z = I_x + I_y \implies I = \frac{1}{2} I_z = \frac{1}{12} M L^2$$

(b) $I_z = \frac{1}{2} M R^2$. Now take the x and y axes to be the perpendicular diameters.. By symmetry $I_x = I_y = I$ and by the perpendicular axis theorem

$$I_z = I_x + I_y \implies I = \frac{1}{2} I_z = \frac{1}{4} M R^2$$

(c) Use the parallel axis theorem to find this moment in terms of the previous. . The distance from the center of mass to the axis is $D = R$. The axis parallel to the tangent through the center is the diameter, so I_{cm} is the answer to part (b).

$$I = I_{\text{cm}} + M D^2 = \frac{1}{4} M R^2 + M R^2 = \frac{5}{4} M L^2.$$

Problem H.10

(a) A uniform solid cylinder of mass m and radius R rotates about an axis at its rim parallel to the central axis. What is its moment of inertia.
(b) What is the moment of inertia of a uniform solid sphere of mass m and radius R about an axis tangent to its surface.
(c) What is the moment of inertia of a uniform thin-shelled hollow sphere of mass m and radius R about an axis tangent to its surface.

Solution to H.10

For all three cases we use the parallel axis theorem.

(a) $I = I_{\text{cm}} + M\,D^2 = \frac{1}{2}\,m\,R^2 + m\,R^2 = \frac{3}{2}\,m\,R^2$.

(b) $I = I_{\text{cm}} + M\,D^2 = \frac{2}{5}\,m\,R^2 + m\,R^2 = \frac{7}{5}\,m\,R^2$.

(c) $I = I_{\text{cm}} + M\,D^2 = \frac{2}{3}\,m\,R^2 + m\,R^2 = \frac{5}{3}\,m\,R^2$.