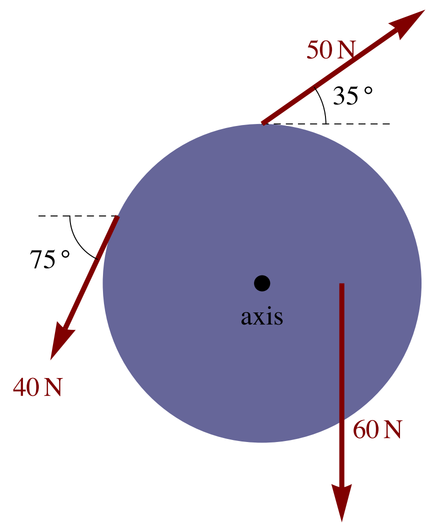
Chapter I - Problems

Blinn College - Physics 2425 - Terry Honan

Problem I.1

A uniform disk with a 12 kg mass and a 20 cm radius rotates about a perpendicular axis through the center. Three forces act on it as shown. The 50 N and 40 N forces act at the rim with the 40 N tangent to it, and the 60 N acts a distance of 10 cm from the axis as shown.

(a) What is the net torque on the disk?
(b) What is the angular acceleration of the disk?



Solution to I.1

(a) The torque about an axis is $\tau = r\,F_\perp = r_\perp\,F = r\,F\sin\theta$. Here there are two clockwise torques and one counterclockwise one. Take clockwise to be positive. For the torque of the 50 N force use $\tau = r\,F_\perp$, where $r = 0.20$ m and $F_\perp = 50\cos 35\,°$, It is clockwise so it ia a positive torque.

$$\tau_1 = +r\,F_\perp = +0.20 \times 50\cos 35\,° = +8.19152$$

The 60 N force is perpendicular to the radial distance of $r = 0.10$ m. It is also positive.

$$\tau_2 = +r\,F_\perp = r\,F = +0.10 \times 60 = +6$$

The third torque acts counterclockwise and is, with our convention, negative. It acts perpendicular to the radial direction, so the angle is 90° and the 75° is unimportant.

$$\tau_3 = -r\,F_\perp = -r\,F = -0.20 \times 40 = -8$$

The net towrque is the sum of the three.

$$\tau_{\text{net}} = \tau_1 + \tau_2 + \tau_3 = 6.1915 = 6.19\,N \cdot m$$

(b) We find the angular acceleration using the rotational second law. First we need the moment of inertia. It is a uniform disk about an axis through the center.

$$I = \frac{1}{2}\,M\,R^2 = \frac{1}{2}\,12 \times 0.2^2 = 0.24\,\text{kg} \cdot \text{m}^2$$

$$\tau_{\text{net}} = I\,\alpha \implies \alpha = \frac{\tau_{\text{net}}}{I} = \frac{6.1915}{0.24} = 25.8\,\frac{\text{rad}}{s^2}$$

Problem I.2

Consider Atwood's Machine with a massive pulley. m_1 begins on the floor, m_2 is initially at a height d, and they are connected over a pulley with a moment of inertia I and radius r. As usual, take $m_2 > m_1$. What is the speed of m_2 when it hits the floor?

Solution to I.2

The total energy in the system is the sum of the translational kinetic enegries of the two hanging masses, the rotational kinetic energy of the pulley and the gravitational potential energies of the hanging masses.

$$E = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \frac{1}{2} I \omega^2 + m_1 g\ y_1 + m_2 g\ y_2$$

The speeds are related by $v = v_1 = v_2 = R\,\omega$.

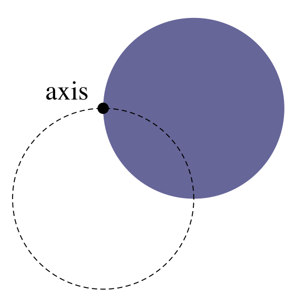
$$E = \frac{1}{2}\left(m_1 + m_2 + I/r^2\right) v^2 + m_1 g\ y_1 + m_2 g\ y_2$$

$$E_i = E_f \implies 0 + 0 + m_2\,\mathrm{g}\ d = \frac{1}{2}\left(m_1 + m_2 + I/r^2\right) v^2 + m_1 g\, d + 0$$

$$\implies v = \sqrt{\frac{2\,(m_2 - m_2)\, g\, d}{m_1 + m_2 + I/r^2}}$$

Problem I.3

A disk swings without friction about a perpendicular axis at its rim. It is released from a postion shown as a solid disk.
(a) What is the angular velocity of the disk when it swings through the bottom position, shown as dashed.
(b) What is the speed of the bottom point on the disk at the bottom position of the disk?
(c) Repeat parts (a) and (b) replacing the disk with a hoop.



Solution to I.3

(a) Energy is conserved here.

$$E = \frac{1}{2} I \omega^2 + m\, g\, y_{\mathrm{cm}}$$

The position of the center of mass falls a total distance of R.

$$E_i = E_f \implies 0 + m\, g\, R = \frac{1}{2} I \omega^2 + 0$$

Use the parallel axis theorem to get the moment about the rim.

$$I = I_{\mathrm{cm}} + M\, D^2 = \frac{1}{2} m R^2 + m R^2 = \frac{3}{2} m R^2$$

We can now find ω.

$$\omega = \sqrt{\frac{2\,m\,g\,R}{I}} = \sqrt{\frac{4\,g}{3\,R}}$$

(b) To get the speed use $v = r\,\omega$. Here r, the perpendicular distance of the point from the axis, is $2\,R$.

$$v = r\,\omega = 2\,R\,\sqrt{\frac{4\,g}{3\,R}} = \sqrt{\frac{16\,g\,R}{3}}$$

(c) If there is a hoop instead of a disk then $I_{\rm cm} = M\,R^2$

$$I = I_{\rm cm} + M\,D^2 = m\,R^2 + m\,R^2 = 2\,m\,R^2 \implies \omega = \sqrt{\frac{2\,m\,g\,R}{I}} = \sqrt{\frac{g}{R}}$$

$$v = r\,\omega = 2\,R\,\sqrt{\frac{g}{R}} = \sqrt{4\,g\,R}$$

Problem I.4

A pulley is a uniiform disk of mass M and radius R. It is in a vertical orientation with a horizontal axis. A string is attached to the rim of the pulley and wrapped many times around it. The free end of the string is connected to a hanging mass m. The hanging mass is released from rest from a height d above a floor.
(a) What is the tension in the string and what is the acceleration of the haning mass?
(b) Using kinematics find the speed of the mass when it hits the floor.
(c) Use conservation of energy to find the speed of the mass when it hits the floor.

Solution to I.4

(a) On the hanging weight:

$$F_{\rm net} = m\,a \implies m\,g - T = m\,a$$

Since the pulley is a uniform disk: $I = \frac{1}{2}\,M\,R^2$. On the pulley:

$$\tau_{\rm net} = I\,\alpha \implies R\,T = \frac{1}{2}\,M\,R^2\,\alpha$$

The angular acceleration of the pulley is related to the linear acceleration fo the string and hanging weight by $\alpha = a/R$.
This gives

$$T = \frac{M}{2}\,a.$$

Adding the two expressions gives the acceleration

$$m\,g - T = m\,a \text{ and } T = \frac{M}{2}\,a \implies m\,g = (m + M/2)\,a \implies a = \frac{m}{m+M/2}\,g$$

We can now solve for the tension

$$T = \frac{M}{2}\,a \text{ and } a = \frac{m}{m+M/2}\,g \implies T = \frac{m\,M}{2\,m+M}\,g$$

(b) To get the speed from the acceleration and distance d using kinematics:

$$v^2 - v_0^2 = 2\,a\,\Delta x \implies v = \sqrt{2\,a\,d} = \sqrt{\frac{2\,m\,g\,d}{m+M/2}}$$

(c) We can get the same result using conservation of energy. There is kinetic energy in the hanging mass and in the rotating pulley. There is potential energy in the hanging mass. The angular velocity of the pulley is related to the linear velocity of the hanging mass by $v = R\,\omega$

$$E = \frac{1}{2}\,m\,v^2 + \frac{1}{2}\,I\,\omega^2 + m\,g\,y_{\rm cm} = \frac{1}{2}\,m\,v^2 + \frac{1}{2}\left(\frac{1}{2}\,M\,R^2\right)\left(\frac{v}{R}\right)^2 + m\,g\,y$$

$$\Longrightarrow E = \frac{1}{2}\left(m + \frac{1}{2} M\right) v^2 + m\,g\,y$$

$$E_i = E_f \Longrightarrow 0 + m\,g\,d = \frac{1}{2}\left(m + \frac{1}{2} M\right) v^2 + 0 \Longrightarrow v = \sqrt{\frac{2\,m\,g\,d}{m+M/2}}$$

Problem I.5

A 8 kg uniform solid cylinder rolls without slipping on a horizontal surface. When its speed is 10 m/s (the speed of the center of mass) what are
(a) the translational kinetic energy,
(b) the rotational kinetic energy and
(c) the total kinetic energy.

Solution to I.5

(a) $K_{\text{trans}} = \frac{1}{2} m\,v^2 = \frac{1}{2} 8 \times 10^2 = 400\text{ J}$

(b) $K_{\text{rot}} = \frac{1}{2} I\,\omega^2 = \frac{1}{2}\left(\frac{1}{2} m^2 R^2\right)\left(\frac{v}{R}\right)^2 = \frac{1}{4} m\,v^2 = 200\text{ J}$

(c) $K_{\text{tot}} = K_{\text{trans}} + K_{\text{rot}} = 600\text{ J}$

Problem I.6

Consider a bowling ball of mass M. If it is a uniform sphere, then what is its total kinetic energy when it rolls down a lane with a linear speed of v. Give a value when $M = 4$ kg and $v = 10\ m/s$.

Solution to I.6

We do not need the raduis here. For a uniform sphere $I = \frac{2}{5} M\,R^2$.

$$K_{\text{trans}} = \frac{1}{2} M\,v^2$$

$$K_{\text{rot}} = \frac{1}{2} I\,\omega^2 = \frac{1}{2}\left(\frac{2}{5} M^2 R^2\right)\left(\frac{v}{R}\right)^2 = \frac{1}{5} M\,v^2$$

$$K_{\text{tot}} = K_{\text{trans}} + K_{\text{rot}} = \frac{7}{10} M\,v^2$$

$$M = 4\text{ kg and } v = 10\,\frac{\text{m}}{\text{s}^2} \Longrightarrow K_{\text{tot}} = 280\text{ J}$$

Problem I.7

$$\vec{A} = \left(-3\,\hat{x} + 4\,\hat{y}\right) \text{ and } \vec{B} = \left(2\,\hat{x} + 3\,\hat{y}\right)$$

Using the vectors $\vec{A}$ and $\vec{B}$ evaluate
(a) $\vec{A} \times \vec{B}$ and
(b) The angle between $\vec{A}$ and $\vec{B}$.

Solution to I.7

$\vec{A} \times \vec{B} = \left(-3\,\hat{x} + 4\,\hat{y}\right) \times \left(2\,\hat{x} + 3\,\hat{y}\right) = (-3)\,(3)\,\hat{x} \times \hat{y} + (4)\,(2)\,\hat{y} \times \hat{x} = -9\,\hat{z} + 8\left(-\hat{z}\right) = -17\,\hat{z}$

To find the angle between two vectors use $\cos\theta = \frac{\vec{A}\cdot\vec{B}}{A\,B} = \frac{6}{5\sqrt{13}} \implies \theta = 70.6\,°$

Note that using the formula $\|\vec{A}\times\vec{B}\| = A\,B\sin\theta$ will not directly give the correct answer for the angle when it is larger than $90\,°$.

Problem I.8

If two vectors satisfy $\|\vec{A}\times\vec{B}\| = \vec{A}\cdot\vec{B}$ then what is the angle beween $\vec{A}$ and $\vec{B}$?

Solution to I.8

$\|\vec{A}\times\vec{B}\| = \vec{A}\cdot\vec{B}$ inplies that $\theta = 45\,°$, since $\|\vec{A}\times\vec{B}\| = A\,B\sin\theta$ and $\vec{A}\cdot\vec{B} = A\,B\cos\theta$, and since $\sin\theta = \cos\theta$ has $\theta = 45\,°$ as its solution.

Problem I.9

The position vector as a function of time for a 2 kg particle is given by $\vec{r}(t) = 6\,\hat{x} + 5\,t\,\hat{y}$ in SI units. What is the angular momentum as a function of time?

Solution to I.9

$\vec{r} = 6\,\hat{x} + 5\,t\,\hat{y} \implies \vec{v} = \dot{\vec{r}} = 5\,\hat{y}$

$\vec{L} = \vec{r}\times m\,\vec{v} = 2\left(6\,\hat{x} + 5\,t\,\hat{y}\right)\times 5\,\hat{y} = 60\,\frac{\text{kg}\,m^2}{\text{s}}\,\hat{z}$

Problem I.10

A projectile is shot from the origin with an initial speed v_0 at an angle θ above horizontal.
(a) What is the angular momentum of the projectile at the origin?
(b) What is the angular momentum when at the highest position?
(c) What is the angular momentum just before it hits the level ground?
(d) What torque is responsible for this change in angular momentum?

Solution to I.10

$\vec{L} = \vec{r}\times m\,\vec{v}$ points inward, relative to the page. This inward component of the angular momentum can be written $L = r\,p\sin\theta = r\,p_\perp = r_\perp\,p$.

(a) At the origin: $L = 0$ since $r = 0$.

(b) At the top of the arc the velocity is horizontal so $r_\perp$ is the vertical part or r, which is just the maximum height of a projectile. This gives: $r_\perp = y_{max} = \frac{v_{0\,y}^2}{2\,g} = \frac{v_0^2\sin^2\theta}{2\,g}$.

$p = m\,v = m\,v_x = m\,v_0\cos\theta \implies L = r_\perp\,p = \frac{m\,v_0^3\sin^2\theta\cos\theta}{2\,g}$

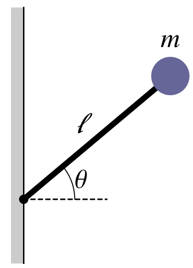
(c) When it hits the ground $\vec{r}$ is horizontal. r is then the range R for a projectile. There the downward component of the velocity is the same as the initial upward component. $p_\perp = m\,v_0\sin\theta$. This gives

$$L = r\,p_\perp = R \cdot m\,v_0 \sin\theta = \left(\frac{v_0^2}{g}\sin 2\theta\right) m\,v_0 \sin\theta\,.$$

(d) The net torque is due to gravity and points into the page

Problem I.11

At $t = 0$ a ball of mass m is dropped from the top end of a rod of lenght ℓ at an angle of θ above horizontal, as shown. What is the angular momentum as a function of time about the base of the rod.



Solution to I.11

The direction of $\vec{L} = \vec{r} \times m\,\vec{v}$ is, relative to the page, pointing inward. For this inward component of the angular momentum we can write

$$L = r\,p \sin\theta = r\,p_\perp = r_\perp\,p\,.$$

In this case since the momentum is vertical then $r_\perp$ is the horizontal part of $\vec{r}$, which is $\ell \cos\theta$. The speed as a function of time for a dropped ball is $v = g\,t$. This gives $L = r_\perp\,p = r_\perp\,m\,v = m\,\ell\,g\,t\ \cos\theta$.

Problem I.12

A horizontal platter with moment of inertia I rotates without friction with an angular velocity ω_0. A mass m is dropped onto the platter. It lands and sticks to the platter a distance d from the axis.

(a) What is the angular velocity after the mass sticks to the platter?

(b) What is K_f / K_i, the ratio of the final kinetic energy to the initial kinetic energy?

Solution to I.12

(a) Angular momentum is conserved here. For a rigid body rotating about a fixed axis $L = I\,\omega$. The moment of inertia of the mass m a distance d from the axis is $m\,d^2$.

$$I\,\omega_0 + 0 = \left(I + m\,d^2\right)\omega \implies \omega = \frac{I}{I+m\,d^2}\,\omega_0$$

(b) The kinetic energy for a rotating rigid body is $K = \frac{1}{2} I\,\omega^2$. Using $L = I\,\omega$ we can write the angular momentum in terms of the angular momentum. This is useful here because the initial and final angular momenta are equal.

$$K_i = \frac{L^2}{2\,I} \text{ and } K_f = \frac{L^2}{2\left(I+m\,d^2\right)} \implies \frac{K_f}{K_i} = \frac{I}{I+m\,d^2}$$

Problem I.13

Junior stands at the rim of a stationary horizontal turntable. Junior starts to walk clockwise at a speed of $2\,m/s$ relative to the ground. The turntable has a radius of 3 m and a moment of inertia of 300 $\text{kg}\cdot m^2$. If Junior has a mass of 45 kg then what is the angular velocity and sense of rotation of the turntable while Junior is walking?

Solution to I.13

(a) Angular momentum is conserved. Initially nothing is moving, so $L_i = 0$ The angular momentum of a particle about some asix is $L = r_\perp m v$. Choose the clockwise direction of walking to be the positive direction.

$$0 = L_i = L_f = r m v + I \omega$$
$$\implies \omega = -\frac{r m v}{I} = -\frac{3 \times 45 \times 2}{300} = -0.9 \frac{\text{rad}}{\text{s}}$$

The negative sign indicates that the turntable rotates counterclockwise.

(b) Since the system is initially at rest, the work done by Junior is the final kinetic energy.

$$\text{W} = \text{K}_{f,\text{total}} = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2 = \frac{1}{2} 45 \times 2^2 + \frac{1}{2} 300 \times 0.9^2 = 212 \text{ J}$$

Problem I.14

A thin rod of mass M and length ℓ is free to rotate in a horizontal plane about an axis at one end without friction. A bullet of mass m is shot perpendicuarly into the rod, initially at rest. It hits and embeds in the rod a distance d from the axis. What is the final angular velocity of the rod after the bullet embeds?

Solution to I.14

Angular momentum is conserved. The moment of the rotating rod is $I = \frac{1}{3} M \ell^2$. The angular momentum of a particle is

$$L = r p_\perp = r p \sin\theta = r_\perp p$$

The perpendicular part of r is $r_\perp = d$. The final moment of inertia, including the bullet is $\frac{1}{3} M \ell^2 + m d^2$.

$$L_i = L_f \implies 0 + d m v = \left(\frac{1}{3} M \ell^2 + m d^2\right) \omega \implies \omega = \frac{d m v}{\frac{1}{3} M \ell^2 + m d^2}$$

Problem I.15

A person holds a baseball bat in a horizontal position with one hand with the hand at the base of the bat. The bat has a weight of 9 N and the center of mass is 55 cm from the hand. That is the force of the hand on the bat and what is the torque of the hand on the bat?

Solution to I.15

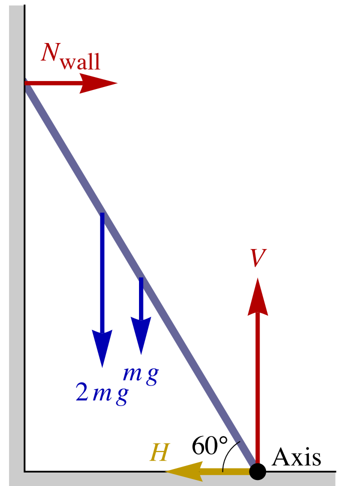
The force of the hand on the bat must be the same as the bat's weight of 9 N, since $F_{\text{net}} = 0$

$$0 = \tau_{\text{net}} = \tau_{\text{hand}} + \tau_{\text{gravity}} = \tau_{\text{hand}} - 0.55 \times 9 \implies \tau_{\text{hand}} = 4.95\, N \cdot m$$

Problem I.16

A uniform ladder leans against a frictionless wall. The ladder makes an angle of 60° with the floor and the coefficient of static friction between the ladder and floor is 0.40. Suppose a painter whose mass is twice that of the ladder climbs the ladder. How far along the ladder can the painter climb without the base of the ladder slipping? Give your answer as a fraction of the length of the ladder.

Solution to I.I6



Take L to be thje length of the ladder and m to be its mass.

The distance of the painter from the base of the ladder is $x\,L$. We are looking for x, the fraction of the distance up the ladder. Choose the positive direction to be counterclockwise and choose the base of the ladder to be the origin.

The torque of the weight of the ladder is: $\tau = +r_\perp F = \left(\frac{L}{2}\cos 60°\right) m\,g$

The torque of the weight of the painter is: $\tau = +r_\perp F = (x\,L\cos 60°)\,2\,m\,g$

The torque of the wall's normal force is: $\tau = -r_\perp F = -L\sin 60°\ \mathrm{N_{wall}}$

$0 = \tau_{net} = \left(\frac{L}{2}\cos 60°\right) m\,g + (x\,L\cos 60.°)\,2\,m\,g - L\sin 60°\ N_{wall}$

This gives $N_{wall} = \frac{m\,g}{\tan 60°}\left(\frac{1}{2} + 2\,x\right)$

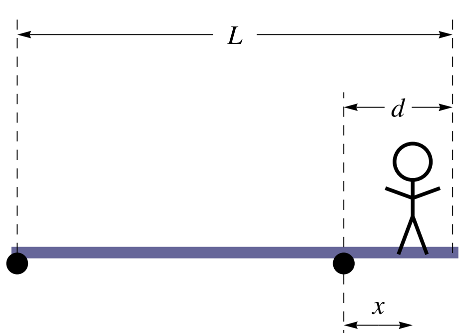
To find V, the vertical component of the floor's force on the ladder: $F_{net,ver} = 0 \implies V = m\,g + 2\,m\,g = 3\,m\,g$

To find H, the horizontal component of the floor's force on the ladder: $F_{net,hor} = 0 \implies H = N_{wall} = \frac{m\,g}{\tan 60°}\left(\frac{1}{2} + 2\,x\right)$

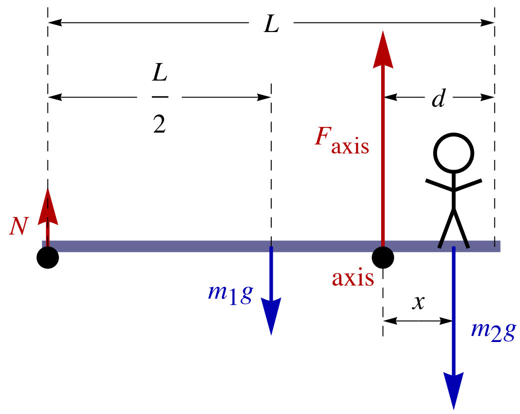
The maximum value of x can be found using the static friction inequality, $f_s = \mu_s \mathrm{N_{floor}}$. When $x = x_{max}$ the inequality becomes an equality. Note that $H = f_s$ and $V = N_{floor}$. This gives: $\frac{m\,g}{\tan 60°}\left(\frac{1}{2} + 2\,x\right) \le 0.4\times 3\,m\,g \implies x_{max} = \frac{1}{2}\left(0.4\times 3\tan 60° - \frac{1}{2}\right) = 0.789$

Problem I.17

A scaffold with a length L and mass $m_1$is supported at two positons, at the left end and a distance d from the right end as shown in red. A painter of mass m_2 stands a distance of x from the support on the right. If the painter stands too far to the right the scaffold will tilt and fall. What is the largest value of x where the painter can stand?



Solution to I.17



The scaffold supports (shown as red dots) are at the left end of the plank and a distance d from the right end. Take the right scaffold support as the axis and take the positive direction to be clockwise rotations.

There is a large upward force at the axis F_{axis} but this force doesn't contribute a torque. Generally, any force acting at the axis produces no torque.

We are trying to find the maximum value of x, where x is the distance of the painter from the axis, the right scaffold support. The torque due to the painter is a positive torque with the value

$$\tau_{painter} = r\,F_\perp = r\,F = +x\,m_2\,g\,.$$

The center of the plank is $L/2 - d$ from the axis. This will create a negative torque.

$$\tau_{plank} = -r\,F_\perp = -r\,F = -\left(\frac{L}{2} - d\right) m_1\,g$$

There is another torque due to the normal force N at the scaffold support at the left end of the plank. This is a distance of $L - d$ from the axis. The torque due to this is positive

$$\tau_N = r\,F_\perp = r\,F = (L - d)\,N\,.$$

$$0 = \tau_{net} = +x\,m_2\,g - \left(\frac{L}{2} - d\right) m_1\,g + (L - d)\,N$$

The normal force N cannot be negative. This gives the condition for $x = x_{max}$.

$$N = 0 \implies x_{max} = \left(\frac{L}{2} - d\right) \frac{m_1}{m_2}$$

Problem I.18

The wheel base (the distance between axles) of a 1500 kg car is 3 m. If the center of mass of the car is 1.20 m behind the front axle, then what is the force on the ground of each wheel of the car?

Solution to I.18

There are three vertical forces acting on the car. N_{front} is the total force of the road pushing upward on the front tires, N_{rear} is the total force of the road pushing upward on the rear tires and $W = 1500 \times 9.8 = 14\,700\,N$ is the weight of the car acting dowward at the center of mass. Since the forces are all vertical we can write the torques as $r_\perp$ F, where $r_\perp$ is the horizontal part of the distance from the axis.

$$0 = F_{net} \implies N_{front} + N_{rear} = W$$

Use the arbitrariness in the choice of axis to simplify the problem by eliminating one of the unknowns from the torque equation. Take the front axle to be the axis.

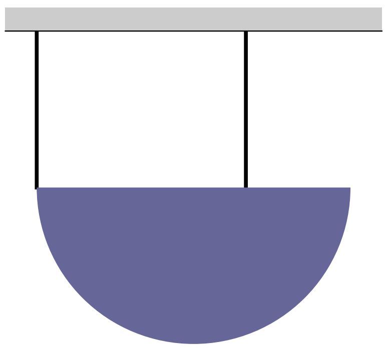
$$0 = \tau_{\text{net}} \implies 0 + 3\,N_{\text{rear}} - 1.2\,W$$

This gives $N_{\text{rear}} = 0.4\,W = 5880\,N$ and $N_{\text{front}} = 0.6\,W = 8820\,N$

The force on each tire is half the forces above. $F_{\text{front}} = 4410\,\text{N}$ and $F_{\text{rear}} = 2940\,N$.

Problem I.19

A sign consists of a uniform half circular disk. It is supported by two vertical ropes as shown. One is on the left side of the sign. The distance between the ropes is 2/3 the width of the sign. What fraction of the sign's weight is supported by each rope?



Solution to I.19

Take the weight of the sign to be W and the width to be L. There are three forces acting on the sign T_L, T_R and W. The weight acts on the center of mass. It is a somewhat difficult matter to find the vertical position of the center of mass but, fortunately, this isn't necessary; only the horizontal position is needed and that is along the center line of the sign. Take the axis to be at the left end and take clockwise to be the positive sense of rotation. The torque due to the weight is in the positive sense with the value

$$\tau_{\text{gravity}} = r_\perp\,F = \frac{L}{2}\,W$$

where $L/2$ is the horizontal part of the vector from the axis to the center of mass. The rope on the left produces no torque, since it acts on our axis. The torque due to the rope on the right is

$$\tau_R = r_\perp\,F = -\left(\frac{2}{3}\,L\right)T_R$$

where the negative sign reflects the counterclockwise sense of this torque.

$$0 = \tau_{\text{net}} = \tau_{\text{gravity}} + \tau_R = \frac{L}{2}\,W - \left(\frac{2}{3}\,L\right)T_R \implies T_R = \frac{3}{4}\,W$$

$$0 = F_{\text{net}} \implies T_L + T_R = W \implies T_R = \frac{1}{4}\,W$$