Chapter K - Problems

Blinn College - Physics 2425 - Terry Honan

Problem K.1

The position as a function of time for a particle in simple harmonic motion is

$$x(t) = (4\,\text{cm})\ \cos\left[\left(3\,\pi\,\text{s}^{-1}\right)\ t + \pi\right].$$

(a) What are the period and frequency?
(b) What is the amplitude of the oscillation?
(c) What is the phase angle?
(d) What is the maximum speed and maximum acceleration?
(e) At $t = 0.25$ s what is the position of the particle?
(f) Suppose this describes the postion of a 0.6 kg mass at the end of a spring. What is the spring constant?

Solution to K.1

The general form for simple harmonic motion is $x(t) = A\cos(\omega\,t + \phi)$.

Here we have $x(t) = (4\,\text{cm})\ \cos\left[\left(3\,\pi\,\text{s}^{-1}\right)\ t + \pi\right) \implies A = 4\,\text{cm}\,,\ \omega = 3\,\pi\,\frac{\text{rad}}{\text{s}}$ and $\phi = \pi$

(a) $T = \frac{2\,\pi}{\omega} = \frac{2}{3}\,\text{s}$ and $f = \frac{\omega}{2\,\pi} = 1.5\,\text{Hz}$

(b) $A = 4\,\text{cm}$

(c) $v_{\text{max}} = \omega\,A = 3\,\pi\,(0.04) = 0,\,377\,m/s$ and $a_{\text{max}} = \omega^2\,A = (3\,\pi)^2\,0.04 = 3.55\,\text{m}/\text{s}^2$

(d) $\phi = \pi$

(e) $x(0.25\,s) = 4\ \cos(3\,\pi \times 0.25\ + \pi) = 2.83\,\text{cm}$

(Note that your calculator must be in the radians mode to evaluate the above expression.)

(f) For a mass/spring system: $\omega = \sqrt{\frac{k}{m}} \implies k = m\,\omega^2 = 53.3\,N/m$

Problem K.2

When a mass is hung from a spring is stretches it by 15 cm. What is the period of the oscillations of this system?

Solution to K.2

The period of a mass spring system is:

$$\omega = \sqrt{\frac{k}{m}} \text{ and } T = \frac{2\,\pi}{\omega} = 2\,\pi\,\sqrt{\frac{m}{k}}\,.$$

The distance a hanging mass stretched a spring is x_{eq}.

$$k\,x_{\text{eq}} = m\,g \implies \frac{m}{k} = \frac{x_{\text{eq}}}{g} \implies T = 2\,\pi\,\sqrt{\frac{m}{k}} = 2\,\pi\,\sqrt{\frac{x_{\text{eq}}}{g}} = 2\,\pi\,\sqrt{\frac{0.15}{9.8}} = 0.777\,\text{s}$$

Problem K.3

A .5 kg mass oscillates witn an amplitude of 10 cm at the end of a spring with spring constant of 8 N/m
(a) What are the maximum speed and acceleration?
(b) What are the speed and acceleration of the mass when it is 6 cm from the equilibrium position?
(c) How long does it take for the mass to move from eqilibrium to 6 cm from equilibrium?

Solution to K.3

$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{8}{0.5}} = 4$

(a) $v_{max} = \omega A = 4 \times 0.1 = 0.4 \frac{\text{m}}{\text{s}}$ and $a_{max} = \omega^2 A = 4^2 \times 0.1 = 1.6 \frac{\text{m}}{\text{s}^2}$

(b) $v = \omega \sqrt{A^2 - x^2} = 4 \sqrt{0.1^2 - 0.06^2} = 0.32 \frac{\text{m}}{\text{s}}$ and $a = \omega^2 x = 4^2 \times 0.06 = 0.96 \frac{\text{m}}{s^2}$

Note that $a = -\omega^2 x$, but the sign has been neglected because only the distance from the center $|x|$ is given and not the sign of x.

(c) To start the motion at $x = 0$ at $t = 0$ we get

$$x = A \sin \omega t \implies t = \frac{1}{\omega} \sin^{-1}\left(\frac{x}{A}\right) = \frac{1}{4} \sin^{-1}\left(\frac{0.06}{0.10}\right) = 0.161 \text{ s}$$

Problem K.4

A 7 kg mass hanging at the end of a spring with a 2.6 s period. What is the spring constant of the spring?

Solution to K.4

$$\sqrt{\frac{k}{m}} = \omega = \frac{2\pi}{T} \implies \sqrt{\frac{k}{7}} = \frac{2\pi}{2.6} \implies k = 40.9 \frac{\text{N}}{\text{m}}$$

Problem K.5

The bumper on a 1000 kg car is tested by driving it into a brick wall. The bumper is equivalent to a spring with spring constant 5×10^6 N/m and it compresses 3.16 cm to stop the car. Assuming all the energy is absorbed by the bumper elastically, what is the car's speed before it hit?

Solution to K.5

$$E = \frac{1}{2} m v^2 + \frac{1}{2} k x^2$$

$$E_i = E_f \implies 1000 \times v^2 + 0 = 0 + \left(5 \times 10^6\right) \times 0.0316^2 \implies v = 2.23 \frac{\text{m}}{\text{s}}$$

Problem K.6

When a particle move is simple marmonic motion with a 10 cm amplitude, at what distance from equilibrium will the particle have one half its maximum speed?

Solution to K.6

$$v = \omega \sqrt{A^2 - x^2} \quad \text{and} \quad v_{max} = \omega A$$

$$v = \frac{1}{2} v_{max} \implies \omega \sqrt{A^2 - x^2} = \frac{1}{2} \omega \mathrm{A} \implies A^2 - \mathrm{x}^2 = \frac{1}{4} A^2 \implies x = \pm \frac{\sqrt{3}}{2} A = \pm \frac{\sqrt{3}}{2} 10 = \pm 8.66 \,\text{cm}$$

Problem K.7

A simple pendulum has a period of 2.5s on Earth. What would the period of this pendulum be if it were moved to the moon, where the acceleration due to gravity is $1.67 \frac{\mathrm{m}}{s^2}$?

Solution to K.7

$$T = 2\pi \sqrt{\frac{L}{g}} \implies \frac{T_2}{T_1} = \frac{1}{\sqrt{g_2/g_1}} \implies T_2 = \frac{T_1}{\sqrt{g_2/g_1}} = \frac{2.5}{\sqrt{1.67/9.80}} = 6.06 \,\text{s}$$

Problem K.8

A uniform solid sphere with a 10 cm radius swings at the end of a 15 cm light rigid rod. (The center of the sphere is 25 cm from the axis.)
(a) What is the period of small oscillations?
(b) Compare this with a simple pendulum with a point mass 25 cm from the axis. Find the period of the simple pendulum? What percent error would be introduced approximating the physical pendulum with the simple one?

Solution to K.8

(a) For a physical pendulum

$$\omega = \sqrt{\frac{m\,g\,d}{I}} \implies T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{I}{m\,g\,d}}$$

Use the parallel axis theorem to find the moment of inertia.

$$I = I_{cm} + m\,D^2 = \frac{2}{5} m\,R^2 + m\,D^2 = m\left(\frac{2}{5} 0.10^2 + 0.25^2\right) = m \times 0.0665$$

$$T = 2\pi \sqrt{\frac{I}{m\,g\,d}} = 2\pi \sqrt{\frac{m\,0.0665}{m\,9.8 \times 0.25}} = 1.0352 = 1.04 \,\text{s}$$

For a simple pendulum of length 0.25 m: $T = 2\pi \sqrt{\frac{L}{g}} = 2\pi \sqrt{\frac{0.25}{9.8}} = 1.0035 = 1.00\,s$.

Thus, it is too small by $\frac{|1.0035 - 1.0035|}{1.0035} = 0.0305 = 3.05\,\%$.