Chapter L - Problems

Blinn College - Physics 2425 - Terry Honan

Problem L.1

Consider a pulse that in SI units has the shape

$$u = f(x) = \frac{8}{x^2+4}.$$

Write this as a function $u(x, t)$ that describes this pulse moving in the positive x direction with a speed of $3\, m/s$.

Solution to L.1

A pulse of shape $u = f(x)$ moving in the positive x-direction with speed v takes the form: $u = f(x - v\,t)$. Using this form of $f(x)$ with $v = 3\, m/s$ gives:

$$f(x) = \frac{8}{x^2+4} \implies u = \frac{8}{(x-3\,t)^2+4}.$$

Problem L.2

What are the speed and direction of a pulse on a string that (in SI units) has the form:

$$y(x, t) = 0.04\, e^{-\left(\frac{x+0.03\,t}{0.06}\right)^2}.$$

Solution to L.2

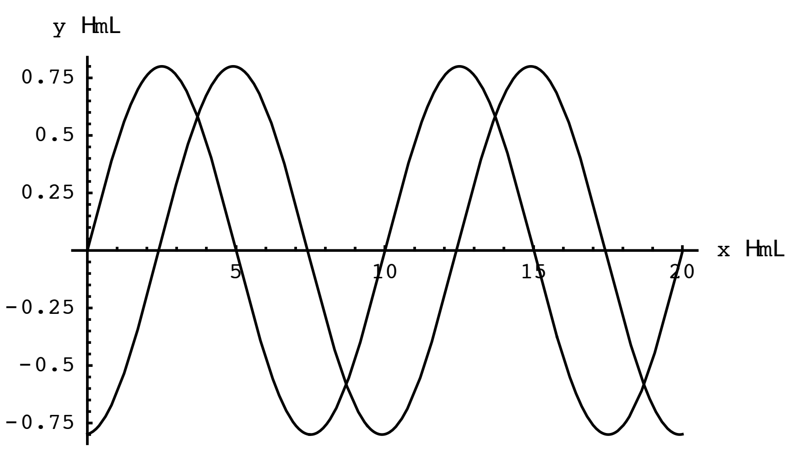
A wave of the form: $y(x, t) = f(x \mp v\,t)$ represents a pulse of shape $y = f(x)$ moving in the $\pm x$ direction with speed v. Here we have $y(x, t) = 0.04\, e^{-(x/0.06)^2}$ with $v = 0.03\, m/s$ moving in the negative x direction.

Problem L.3

A sinusoidal pulse on a string has the mathematical form $y(x, t) = (0.80\text{ m}) \sin\left[\frac{2\pi}{10}(x - 4\,t)\right]$. Plot the y vs. x graph at $t = 0$ s. By the time $t = 0.6$ s how much has the pulse shifted. On the same graph plot y vs. x at $t = 0.6$ s.

Solution to L.3

After $t = 0.6$ s the graph has shifted by $v\,t = 4 \times 0.6 = 2.4$ m.



Problem L.4

A string with a linear density of $\mu = 4\times10^{-3}$ kg/m is given a tension of 360 N. What is the speed of waves on this string?

Solution to L.4

The speed of waves on a stretched string is $v = \sqrt{T/\mu}$ where T is the tension in the string and μ is the linear density (mass/length) of the string. Here, $T = 360\,N$ and $\mu = 4\times10^{-3}$ kg/m giving $\mathrm{v} = 300\ \frac{\mathrm{m}}{\mathrm{s}}$.

Problem L.5

The elastic limit for steel is $S_{\max} = 2.7\times10^{9}\ \mathrm{N/m^2}$; this is the maximum force per area that steel under tension can withstand. $S_{\max}$ is the largest value that T/A, the tension per area, can have without a wire breaking. If the density of steel is $7860\ \mathrm{kg}/m^3$ then what is the largest speed a wave can travel down a steel wire?

Solution to L.5

The linear density μ (mass/length) is related to the volume density ρ (mass/volume) by $\lambda = \rho A$, where A is the cross-sectional area of the wire. This is easy to show: The volume of a wire of length L and area A is $L A$. Mass = $\rho\times$ Volume = $\rho L A$ and λ = Mass$/L = \rho A$.

The maximum stress $\mathrm{Stress}_{\max}$ gives the maximum tension: $T_{\max} = S_{\max}\times A$, which then gives the mixim um wave speed.

$$v_{\max} = \sqrt{\frac{T_{\max}}{\mu}} = \sqrt{\frac{S_{\max} A}{\rho A}} = \sqrt{\frac{S_{\max}}{\rho}} = \sqrt{\frac{2.7\times10^{9}}{7.86\times10^{3}}} = 586\ \frac{\mathrm{m}}{\mathrm{s}}$$

Problem L.6

A 30 m long copper wire with a 1.2 mm diameter is stretched to a tension of 200 N. How long does it take for a pulse to travel the length of the wire? The density of copper is $\rho = 8.92\times10^{3}\ \mathrm{kg/m^3}$.

Solution to L.6

The cross-sectional area of the wire is

$$\mathrm{A} = \pi\,\mathrm{r}^2 = \pi\times0.0006^2 = 1.1310\times10^{-6}\ \mathrm{m}^2.$$

We saw in the previous problem that the linear density μ of a wire with density ρ and cross-sectional area A is $\mu = \rho A$. The speed of a wave on a wire with tension T, cross-section A and density ρ is

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{\rho A}} = \sqrt{\frac{200}{8.92\times10^{3}\,A}} = 141\ \frac{\mathrm{m}}{\mathrm{s}}.$$

To get the time from the distance d and speed simply use $d = v\,t$ giving:

$$t = \frac{d}{v} = \frac{30}{v} = 0.213\ \mathrm{s}.$$

Problem L.7

A sinusoidal wave on a string has the form

$$y(x) = (15\ \mathrm{cm})\cos\left[\left(\tfrac{\pi}{20}\ \mathrm{cm}^{-1}\right)x - (16\,\pi\ \mathrm{s}^{-1})\,t\right].$$

(a) Plot the motion of the positon $x = 0$ as a function of time and find its period and frequency.
(b) What is the maximum speed of this point ($x = 0$) on the string?
(c) What are the wavelength and speed of this wave?

Solution to L.7

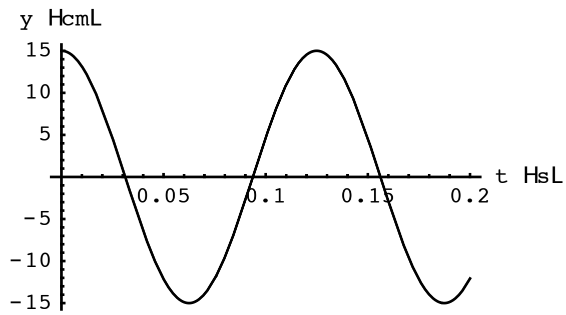
(a) At $x = 0$ we have

$$y(x) = A\cos(\omega\ t),\ \text{where}\ A = 15\,\text{cm}\ \text{and}\ \omega = 16\,\pi\,\text{s}^{-1}.$$

The period T and frequency f are

$$T = \frac{2\pi}{\omega} = \frac{1}{8} = 0.125\,\text{s}\ \text{and}\ f = \frac{\omega}{2\pi} = \frac{1}{T} = 8\,\text{Hz}.$$

With this we can plot the function.



(b) This motion is an example of simple harmonic motion, where the maximum speed is given by

$$v_{\text{max}} = \omega\,A = 16\,\pi \times 15 = 240\,\pi = 754\ \frac{cm}{\text{s}}.$$

(c) The wave number k of this wave gives the wavelength.

$$k = \frac{\pi}{20}\,\text{cm}^{-1} \implies \lambda = \frac{2\pi}{k} = 40\,\text{cm}$$

The speed can be found by using $v = f\,\lambda$ or directly in term of what is giving by using

$$v = \frac{\omega}{k} = \frac{16\pi}{\pi/20} = 320\ \frac{\text{m}}{\text{s}}.$$

Problem L.8

As a sinusoidal wave passes, a point on a string makes 50 complete vibrations is 20 s. In the same time a crest (maximun) of the wave moves a distance of 4 m. What is the frequency, speed and wavelength of this wave?

Solution to L.8

50 vibrations in 20s gives a frequency of

$$f = \frac{50}{20} = 2.5\,\text{Hz}\,.$$

4 m in 20 s gives the speed.

$$v = \frac{4}{20} = 0.2\ \frac{\text{m}}{\text{s}}$$

We can now find the wavelength.

$$f\,\lambda = v \implies \lambda = \frac{v}{f} = \frac{0.2}{2.5} = 0.08\,\text{m}$$

Problem L.9

A 15 m length of rope has a mass of 0.6 kg and is given a tension of 500 N. What power is required to put a wave with an amplitude of 20 cm and a frequency of 3 Hz?

Solution to L.9

The linear density is $\mu = M/L = 0.6/15 = 0.04\,\text{kg/m}$. The speed of the wave is

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{500}{0.04}} = 111.80\, m/s$$

The angular frequency follows from the frequency.

$$\omega = 2\,\pi\, f = 2\,\pi\, 3 = 18.850\, s^{-1}$$

Using $A = 0.20$ m we can now find the power transmitted down a string.

$$\mathcal{P} = \frac{1}{2}\,\mu\,\omega^2\,A^2\,\text{v} = \frac{1}{2}\,0.04\,\times 1.8850^2\,(0.20)^2\,111.80\; = 31.8\text{ W}.$$

Problem L.10

A wave of the form

$$y(x,\,t) = (0.12\text{ m})\sin[(0.8\text{ m}^{-1})\,x + (40\text{ s}^{-1})\,t]$$

travels down a string with a linear density of 8 g/m.

(a) What is the speed of the wave and in what direction is it moving?
(b) What are the wavelength and frequency of this wave?
(c) What is the tension in the string?
(d) What is the power transmitted by this wave?

Solution to L.10

Since the general form is $y(x,\,t) = A\sin(k\,x \mp \omega\,t - \phi)$, we can conclude that: $A = 0.12\,m$, $\text{k} = 0.8\text{ m}^{-1}$ and $\omega = 40\,s^{-1}$. We are also given that $\mu = 0.008\,\frac{\text{kg}}{\text{m}}$.

(a) The positive sign before ω implies it is moving in the negative x direction. The speed is

$$v = \frac{\omega}{k} = \frac{40}{0.8} = 50\,\frac{\text{m}}{\text{s}}$$

(b) The wavelength and frequency come from the wave number and angular freauency.

$$\lambda = \frac{2\,\pi}{k} = \frac{2\,\pi}{0.8} = 7.85\text{ m} \;\text{ and }\; f = \frac{\omega}{2\,\pi} = \frac{40}{2\,\pi} = 6.37\text{ Hz}$$

(c) The speed and linear density give the tension.

$$v = \sqrt{\frac{T}{\mu}} \;\Longrightarrow\; T = v^2\,\mu = 50^2 \times 0.008 = 20\text{ N}$$

(d) The power transmitted is

$$\mathcal{P} = \frac{1}{2}\,\mu\,\omega^2\,A^2\,\text{v} = \frac{1}{2}\,0.008\,\times 40^2\,(0.12)^2\,50\; = 4.61\text{ W}.$$