Chapter N - Problems

Blinn College - Physics 2425 - Terry Honan

Problem N.1

30 g of water is placed in a 2.5 liter pressure cooker and heated up to 350 °C. What is the pressure inside the cooker assuming no gas escapes and neglecting the gas that was initially inside it?

Solution to N.1

The mass of a molecule in u gives the mass of a mole in g. (This is because one mole is N_A, Avogadros number, of molecules and N_A u = 1 g.)

$$m_{H_2O} = 2\,\mathrm{m_H} + m_O = 2\times 1\,u + 16\,u = 18\,u \implies m_{\text{mole}} = 18\,\mathrm{g}$$

The number of moles is:

$$m_{\text{tot}} = n\,m_{\text{mole}} \implies n = \frac{m_{\text{tot}}}{m_{\text{mole}}} = \frac{30}{18} = 1.667\text{ moles}$$

The ideal gas law gives the pressure.

$$P = \frac{n\,R\,T}{V} = \frac{n\;8.31\,(273+350)}{2.5\times 10^{-3}} = 3.45\times 10^{6}\,\mathrm{Pa}$$

Problem N.2

A spherical weather balloon can expand to a volume with radius 25 m when it is at the extreme height where the pressure is 0.04 atm and where the temperature is −70 °C. What is its radius when it is near the ground at 1 atm and 25 °C?

Solution to N.2

The volume of a sphere is proportional to its radius.

$$V = \frac{4}{3}\pi\,r^3 \implies \frac{V_2}{V_1} = \left(\frac{r_2}{r_1}\right)^3.$$

The ideal gas law, $P\,V = n\,R\,T$ with a constant n gives

$$\frac{P_2}{P_1}\,\frac{V_2}{V_1} = \frac{T_2}{T_1} \implies \frac{P_2}{P_1}\left(\frac{r_2}{r_1}\right)^3 = \frac{T_2}{T_1} \implies \frac{1}{0.04}\left(\frac{r_2}{25}\right)^3 = \frac{273+25}{273-70} \implies r_2 = 9.72\,m$$

Problem N.3

A tank contains some unknown gas that may be treated as ideal. When the tank contains 8 kg of gas, the pressure gauge on the tank reads 30 atm. After gas is let out of the tank and the temperature is allowed to return to its earlier value the gauge reads 12 atm. What is the mass of gas left in the tank? Note that a pressure gauge reads the pressure difference between the internal pressure and 1 atm so a gauge pressure of 30 atm means the absolute pressure inside is 31 atm.

Solution to N.3

Gauge pressures of 30 atm and 12 atm correspond to absolute pressures of 31 atm and 13 atm. The type of gas is unimportant. There is a proportionality between the number of moles and the mass of the gas and that allows us to equate their ratios.

$$m_{\text{tot}} = n\;m_{\text{mole}} \implies \frac{n_2}{n_1} = \frac{m_2}{m_1}$$

Here we have the ideal gas law $P\,V = n\,R\,T$ with a constant V and T. This gives:

$$\frac{P_2}{P_1} = \frac{n_2}{n_1} \implies \frac{P_2}{P_1} = \frac{m_2}{m_1} \implies \frac{13}{31} = \frac{m_2}{8} \implies m_2 = 3.35\,\mathrm{kg}$$

Problem N.4

In a laboratory pressures as low as 10^{-9} Pa can be reached. For such a vacuum how many molecules are in a cm^3at 20 °C?

Solution to N.4

$$N = \frac{P\,V}{k_B\,T} = \frac{10^{-9}\times(10^{-2})^3}{1.38\times10^{-23}\,(273+20)} = 2.47\times10^5$$

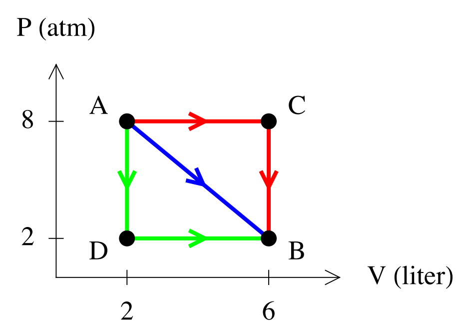
Problem N.5

STP, standard temperature and temperature, refers to a pressure of 1 atm and a temperature of 0 °C. How many liters does one mole of an ideal gas at STP occupy?

Solution to N.5

$$V = \frac{n\,R\,T}{P} = \frac{1\times8.31\times273}{1.013\times10^5} = 0.0224\,m^3 = 22.4\text{ liter}$$

Problem N.6



Use the PV-diagram above to answer the following.
(a) What is W_{ACB}, the work from A to C to B?
(b) What is W_{ADB}, the work from A to D to B?
(c) What is W_{AB}, the work from A to B along the direct path?
(d) What is W_{BA}, the work from B to A along the direct path?
(e) What is W_{ABDA}, the work for a cycle?
(f) What is W_{ABCA}, the work for a cycle?

Solution to N.6

Since thermodynamic work is given by $W = \int P\,dV$ it can be interpreted as the area under a PV-diagram. (It is the area when $\Delta V > 0$ and the negative of the area when $\Delta V < 0$) The area is in units of atm·liter; we can easily convert this to J.

$$1\text{ atm}\cdot\text{liter} = 1.013\times10^5\text{ Pa}\cdot10^{-3}\text{ m}^3 = 101.3\text{ J}$$

(a) We can write this as a sum of two segments, $W_{ACB} = W_{AC} + W_{CB}$. The area under the second term is zero. The first gives a rectangle.

$$W_{ACB} = W_{AC} + W_{CB} = (8\times4+0)\text{ atm}\cdot\text{liter} = 3240\,J$$

(b) We can also write this as a sum of two segments, $W_{ADB} = W_{AD} + W_{DB}$. Here, the area under the first term is zero and the second gives a rectangle.

$$W_{ADB} = W_{AD} + W_{DB} = (0+2\times4)\text{ atm}\cdot\text{liter} = 810\,J$$

(c) The reagion under the direct path is a trapezoid. Its area gives

$$W_{AB} = \frac{1}{2}(8+2)\times4\text{ atm}\cdot\text{liter} = 2026\,J$$

(d) From B to A has $\Delta V < 0$ so $W = -$Area.

$$W_{BA} = -W_{AB} = -2026\text{ J}$$

(e) The work for a cycle is $\pm$Area Enclosed. If the cycle is clockwise then the positive contribution dominates the negative and the + sign is used. If counterclockwise the negative dominates and the work is negative. For W_{ABDA} the contour is clockwise. The region is a triangle.

$$W_{ABDA} = +\frac{1}{2}\,4\times 6\text{ atm}\cdot\text{liter} = 1216\text{ J}$$

(f) For W_{ABCA} we have a counter-clockwise contour around a triangle.

$$W_{ABCA} = -\frac{1}{2}\,4\times 6\text{ atm}\cdot\text{liter} = -1216\text{ J}$$

Problem N.7

An ideal gas is compressed from 5 liter to 2 liter at a constant pressure of 12 atm, If the internal energy of the gas decreases by 2000 J then what is the heat added to the gas during this process?

Solution to N.7

For an isobaric (constant pressure) process the work is $W = P\,\Delta V$.

$$W = P\,\Delta V = 12\times(-3)\text{ atm}\cdot\text{liter} = -3646.8\text{ J}$$

The change in internal energy is $\Delta U = -2000$ J. The first law of thermodynamics gives the heat Q.

$$\Delta U = Q - W \implies Q = \Delta U + W = -2000 - 3646.8 = -5650\text{ J}$$

Note that the negative sign implies that heat is flowing out of the gas.

Problem N.8

Referring to the diagram for **Problem M.6**. What is the heat added to the system for the *ABDA* cycle, for the *ABCA* cycle?

Solution to N.8

For a cycle we have $\Delta U = 0$, so $\Delta U = Q - W$ implies that $Q = W$.

$$Q_{ABDA} = W_{ABDA} = +\frac{1}{2}\,4\times 6\text{ atm}\cdot\text{liter} = 1216\text{ J}$$

$$Q_{ABCA} = W_{ABCA} = -\frac{1}{2}\,4\times 6\text{ atm}\cdot\text{liter} = -1216\text{ J}$$

Problem N.9

Some quantity of an ideal gas does 6000 J of work as expands isothermally at 20 °C to a final pressure and volume of 1.2 atm and 40 liter.
(a) What are the initial values of the pressure and volume?
(b) How many moles are in the gas?

Solution to N.9

(a) For an isothermal expansion of an ideal gas we have $W = n\,R\,T\ln\left(V_f/V_i\right)$. Using the ideal gas law $P\,V = n\,R\,T$ we get $W = P_f\,V_f\ln\left(V_f/V_i\right)$. (Note that for isothermal expansion $P_f\,V_f = P_i\,V_i$.)

$$W = P_f\,V_f\ln\frac{V_f}{V_i} \implies \frac{V_f}{V_i} = \exp\left(\frac{W}{P_f\,V_f}\right)$$

$$\implies \frac{40\text{ liter}}{V_i} = \exp\left(\frac{6000}{(1.2\times 1.013\times 10^5)\times 0.040}\right) \implies V_i = 11.6455 = 11.6\text{ liter}$$

Note that this result is independent of the value of T..

$$P_f\,V_f = P_i\,V_i \implies 1.2\times 40 = P_i\times 9.0987 \implies P_i = 5.28\text{ atm}$$

(b) We need n to get the temperature using the ideal gas law

$$P\,V = n\,R\,T \implies n = \frac{\left(1.2\times 1.013\times 10^{5}\right)\times 0.040}{8.31\times 293} = 2.00\ \text{moles}$$