Chapter O - Problems

Blinn College - Physics 2425 - Terry Honan

Problem O.1

What is the change in entropy of 3 kg of water initially at 20 °C when it is cooled and frozen resulting in ice at 0°C?

Solution to O.1

This is a two step problem. $\Delta S_1$is the change in entropy when the water is brought from 20 °C to 0 °C and ΔS_2 is the change when the water at 0 °C freezes to 0 °C. When water freezes to ice heat is removed so $Q = -m L_f$. The sign of the entropy of the water to water case follows from the values of the temperatures. The temperature must be in Kelvin in all cases here.

$$\Delta S = \Delta S_1 + \Delta S_2 = m\, c \ln \frac{T_f}{T_i} + \frac{Q}{T} = m\, c_{\text{water}} \ln \frac{T_f}{T_i} + \frac{-m\, L_f}{T}$$

$$= 3 \times 4186 \ln \frac{273}{293} + \frac{-3 \times 3.33 \times 10^5}{273} = -887.9 - 3659.3 = -4550\, J / K$$

Problem O.2

What is the total change in entropy when 3000 J of heat flows into a house at 20 °C from the warm exterior at 40 °C?

Solution to O.2

The entropy of the exterior decreases, since heat flows out of it, and the entropy of the interior increases since heat flows into it.

$$\Delta S_{\text{tot}} = \Delta S_{\text{outside}} + \Delta S_{\text{inside}} = \frac{-Q}{T_{\text{outside}}} + \frac{Q}{T_{\text{inside}}} = \frac{-3000}{313} + \frac{3000}{293}$$
$$= -9.585 + 10.239 = 0.654\, J / K$$

The second law of thermodynamics says that the total change in entropy cannot be negative for any thermally isolated system. A system is thermally isolated if no heat flow into or out of it.

Problem O.3

What is the total change in entropy when a 20 g ice cube at 0 °C is dropped into the Gulf of Mexico, which is at a temperature 15 °C? It is *quite* safe to assume that the temperature change of the Gulf is negligible.

Solution to O.3

The heat that flows into the ice cube comes from the Gulf. This heat Q first melts the ice and then raises its temperature from 0 °C to 15 °C.

$$Q = m\, L_f + m\, c\, \Delta T = 0.02 \times 3.33 \times 10^5 + 0.02 \times 4186 \times 15 = 6660.00 + 1255.80 = 7915.80$$

Since this heat flows out of the Gulf, which is at constant temperature, its entrop change is

$$\Delta S_{\text{Gulf}} = \frac{-Q}{T} = \frac{-Q}{288} = -27.485\, J / K$$

The entropy change of the ice involves two terms, the first to melt it and the second to raise its temperature.

$$\Delta S_{\text{ice}} = \Delta S_1 + \Delta S_2 = \frac{+m\, L_f}{T} + m\, c \ln \frac{T_f}{T_i}$$
$$= \frac{0.02 \times 3.33 \times 10^5}{273} + 0.02 \times 4186 \ln \frac{288}{273} = 24.396 + 4.478 = 28.874\, J / K$$

The total change is the sum of the two.

$$\Delta S_{\text{tot}} = \Delta S_{\text{Gulf}} + \Delta S_{\text{ice}} = 1.39\, J / K$$

Problem O.4

A heat engine absorbs 6000 J of heat from a hot reservoir and expels 4100 J to a cold reservoir at 30 °C.
(a) What is the efficiency of this engine?
(b) What is the work done by the engine?
(c) What is the smallest temperature the hot reservoir could have?

Solution to O.4

(a) $e = 1 - \frac{Q_c}{Q_H} = 1 - \frac{4100}{6000} = 0.31667 = 31.7\ \%$

(b) $Q_H = Q_C + W \implies W = Q_H - Q_C = 6000 - 4100 = 1900\ \text{J}$

(c) The efficiency has an upper limit of $e_{\text{max}} = 1 - \frac{T_C}{T_H}$ where T must be in Kelvin.

$$e \leq e_{\text{max}} = 1 - \frac{T_C}{T_H} \implies T_H = \frac{T_C}{1-e} = \frac{303}{1-0.31667} = 443.4\ \text{K} = 170.4\ °\text{C}$$

Problem O.5

A Carnot engine that operates between a 500 K hot reservoir and a 300 K cold reservoir has a power output of 6000 W. What is the efficiency of this heat engine? At what rate does this engine absorb heat?

Solution to O.5

A Carnot engine is a theoretical heat engine of maximum efficiency.

$$e = e_c = 1 - \frac{T_C}{T_H} = 1 - \frac{300}{500} = 0.40 = 40\ \%$$

The power output is the work per time.

$$e = \frac{W}{Q_H} = \frac{W/t}{Q_H/t} = \frac{\mathcal{P}}{Q_H/t} \implies Q_h/t = \frac{\mathcal{P}}{e} = \frac{6000}{0.40} = 15\,000\ \text{W}$$