

Physics 2326 - Formula List

Integrals

$$\int u^n du = \frac{u^{n+1}}{n+1} + C \quad (n \neq -1) \quad \int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin\left(\frac{u}{a}\right) + C$$

$$\int \frac{du}{u} = \ln|u| + C \quad \int \frac{du}{\sqrt{u^2 + a^2}} = \ln\left(u + \sqrt{u^2 + a^2}\right) + C$$

$$\int e^{au} du = \frac{1}{a} e^{au} + C \quad \int \frac{du}{u^2 + a^2} = \frac{1}{a} \arctan\left(\frac{u}{a}\right) + C$$

$$\int \sin(au) du = -\frac{1}{a} \cos(au) + C \quad \int \frac{du}{(u^2 + a^2)^{3/2}} = \frac{1}{a^2} \frac{u}{\sqrt{u^2 + a^2}} + C$$

$$\int \cos(au) du = \frac{1}{a} \sin(au) + C \quad \int \frac{u du}{(u^2 + a^2)^{3/2}} = -\frac{1}{\sqrt{u^2 + a^2}} + C$$

Formulas from Geometry

Sphere: Volume = $\frac{4}{3} \pi r^3$, Area = $4 \pi r^2$

Circle: Area = πr^2 , Circumference = $2 \pi r$

Right Cylinder: Volume = Area \times height

Coulomb's Law

$F = k_e \frac{|Q_1||Q_2|}{r^2}$ (magnitude of force)

$\vec{F}_{21} = k_e Q_1 Q_2 \frac{\hat{r}_{12}}{r_{12}^2}$ where $\frac{\hat{r}_{12}}{r_{12}^2} = \frac{\vec{r}_{12}}{r_{12}^3}$

and where \hat{r}_{12} is the vector from Q_1 to Q_2 .

Charge quantization: $Q = n e$, n is an integer.

Electric Field

$\vec{E} = \vec{F}/q_0$ or $\vec{E} = \lim_{q_0 \rightarrow 0} \vec{F}/q_0$, $\vec{F} = Q \vec{E}$ (force on Q)

Point Charge: $\vec{E} = k_e Q \frac{\hat{r}}{r^2} = k_e Q \frac{\vec{r}}{r^3}$, $E = k_e \frac{|Q|}{r^2}$

Discrete: $\vec{E} = k_e \sum_i Q_i \frac{\hat{r}_i}{r_i^2} = k_e \sum_i Q_i \frac{\vec{r}_i}{r_i^3}$

Continuous: $\vec{E} = k_e \int \frac{\hat{r}}{r^2} dq = k_e \int \frac{\vec{r}}{r^3} dq$

$\vec{E} = k_e Q \frac{z_0}{(R^2 + z_0^2)^{3/2}} \hat{z}$ (z_0 from center of uniform ring)

Electric Flux

$\Phi = \int \vec{E} \cdot d\vec{A}$, uniform \vec{E} and flat surface: $\Phi = \vec{E} \cdot \vec{A}$

Dot Product: $\vec{A} \cdot \vec{B} = AB \cos \theta = A_x B_x + A_y B_y + A_z B_z$

Gauss's Law

$\oint \vec{E} \cdot d\vec{A} = \frac{1}{\epsilon_0} Q_{\text{enclosed}}$

Potential and Potential Energy

$V = U/q_0$, $\Delta U = Q \Delta V$

For a charge distribution: $U = k_e \sum_{i < j} \frac{Q_i Q_j}{r_{ij}}$

Conservation of Energy: $K_i + U_i = K_f + U_f$, where $K = \frac{1}{2} m v^2$

Potential due to Charges

Point Charge: $V = k_e \frac{Q}{r}$

Discrete: $V = k_e \sum_i \frac{Q_i}{r_i}$, Continuous: $V = k_e \int \frac{dq}{r}$

Potential and Electric Field

$\Delta V = -\int \vec{E} \cdot d\vec{r}$, $\Delta V = -\vec{E} \cdot \Delta \vec{r}$ (uniform \vec{E})

$E_x = -\frac{\partial V}{\partial x}$ and for y and z , also $E_r = -\frac{\partial V}{\partial r}$

Conductors in Electrostatics

At surface: $\vec{E} \perp$ surface and $E = \sigma/\epsilon_0$

Inside: $\vec{E} = \vec{0}$, voltage is const., no excess charge.

Capacitance

$Q = C V$, C is the capacitance.

$C = \kappa C_0$, where $C_0 =$ empty cap. and dielectric const. = $\kappa \geq 1$

$C_0 = \frac{\epsilon_0 A}{d}$ (|| plate), $C_0 = \frac{1}{k_e \left(\frac{1}{a} - \frac{1}{b}\right)}$ (sph.), $C_0 = \frac{2 \pi \epsilon_0 \ell}{\ln(b/a)}$ (cyl.)

Energy

$U = \frac{1}{2} C V^2 = \frac{Q^2}{2C} = \frac{1}{2} Q V$ (energy in a cap.)

$u = \frac{\epsilon_0}{2} E^2 = \frac{\text{Energy}}{\text{Volume}}$ (energy density in a field)

Electric Dipoles

Dipole Moment: $\vec{p} = Q \vec{d}$, \vec{d} from $-Q$ to Q

Torque: $\vec{\tau} = \vec{p} \times \vec{E}$, $\tau = p E \sin \theta$, Pot. Energy: $U = -\vec{p} \cdot \vec{E}$

Current and Current Density

$I = \frac{dQ}{dt}$ (current through surface)

\vec{J} is the current density. $I = \int_{\text{surface}} \vec{J} \cdot d\vec{A}$, $I = J A$ (for a wire)

Drift Velocity

$I = n |q| v_d A$, $\vec{J} = n q \vec{v}_d$

$n = \frac{\# \text{ of charge carriers}}{\text{vol.}}$, $q =$ charge of charge carriers, $\vec{v}_d =$ drift vel.

Ohm's Law

$V = I R$, $R = \frac{\rho L}{A}$, $\vec{J} = \sigma \vec{E}$

$\sigma =$ conductivity, $\rho = \frac{1}{\sigma} =$ resistivity

Temperature Dependence

$\Delta T = T - T_0$

$\Delta R = \alpha R_0 \Delta T$, $R = R_0(1 + \alpha \Delta T)$, $\Delta \rho = \alpha \rho_0 \Delta T$, $\rho = \rho_0(1 + \alpha \Delta T)$

Power

$\mathcal{P} = V I$, For a resistor: $\mathcal{P} = V I = I^2 R = \frac{V^2}{R}$

Combinations of Resistors

Series: $R_{\text{eq}} = R_1 + R_2 + \dots$, Parallel: $R_{\text{eq}} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \dots\right)^{-1}$

Node Reduction: $R'_{ij} = \frac{R_i R_j}{R_{ij}}$, where $R_{ii} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1}$

Combinations of Capacitors

Series: $C_{\text{eq}} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \dots\right)^{-1}$, Parallel: $C_{\text{eq}} = C_1 + C_2 + \dots$

Kirchhoff's Rules

Junctions: $\sum I_{\text{in}} = \sum I_{\text{out}}$, Loops: $0 = \sum \Delta V$

Cross or Vector Product

$\vec{A} \times \vec{B} = \hat{u} AB \sin \theta$, right hand rule $\Rightarrow \hat{u}$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{x} \begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} - \hat{y} \begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} + \hat{z} \begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix}$$

Magnetic Force on Particle

$\vec{F} = Q \vec{v} \times \vec{B}$

$\vec{v} \perp$ to uniform $\vec{B} \Rightarrow$ circle with $r = \frac{m v}{|Q| B}$

Magnetic Force on Wire

$\vec{F} = I \int d\vec{r} \times \vec{B}$

$\vec{F} = I \vec{l} \times \vec{B}$ (straight segment, uniform field)

$V_{\text{Hall}} = v_d B L$ (Hall Voltage)

Magnetic Dipoles

Dipole Moment: $\vec{\mu} = N I \vec{A}$ (N is # of turns)

Torque: $\vec{\tau} = \vec{\mu} \times \vec{B}$, $\tau = \mu B \sin \theta$, Pot. Energy: $U = -\vec{\mu} \cdot \vec{B}$

Biot-Savart Law

$\vec{B} = \frac{\mu_0 I}{4 \pi} \int d\vec{s} \times \frac{\hat{r}}{r^2}$

$\vec{B} = \hat{z} \frac{\mu_0 I}{4 \pi R} \theta$ (at center of arc in xy -plane)

$\vec{B} = \hat{z} \frac{\mu_0 I}{2} \frac{R^2}{(R^2 + z^2)^{3/2}}$ (z from center of circle)

$B = \frac{\mu_0 I}{4 \pi a} (\sin \phi_2 - \sin \phi_1) = \frac{\mu_0 I}{4 \pi a} \left(\frac{x_2}{\sqrt{x_2^2 + a^2}} - \frac{x_1}{\sqrt{x_1^2 + a^2}} \right)$ (segment)

$B = \frac{\mu_0 I}{2 \pi r}$ (distance r from long wire)

■ **Ampere's Law** $\oint \vec{B} \cdot d\vec{r} = \mu_0 I_{\text{enclosed}}$
 $B = \mu_0 n I$, $n = \frac{\# \text{ of turns}}{\text{length}}$ (inside long solenoid)

■ **Magnetic Flux** $\Phi_m = \int \vec{B} \cdot d\vec{A}$
 For uniform field and flat surface: $\Phi = \vec{B} \cdot \vec{A}$

■ **Faraday's Law** $\mathcal{E} = -N \frac{d\Phi}{dt}$, $\mathcal{E}_{\text{ave}} = -N \frac{\Delta\Phi}{\Delta t}$
 AC Generator: $\Phi = B A \cos \omega t \Rightarrow \mathcal{E}(t) = N B A \omega \sin \omega t$

■ **Motional EMF**
 moving rod: $\mathcal{E} = B \ell v$ ($\vec{B} \perp \vec{v} \perp \text{rod}$)
 rotating rod: $\mathcal{E} = \frac{1}{2} B \ell^2 \omega$ ($\vec{B} \parallel \text{axis} \perp \text{rod}$)

■ **Lenz's Law** (Choose + dir. and make a table of signs.)
 Sign Φ is from dir. of field through loop.
 $\frac{d\Phi}{dt}$ is same as (opposite to) sign of Φ when incr. (decr.).
 Φ_{induced} sign is opposite to sign for $\frac{d\Phi}{dt}$.
 Get direction of \mathcal{E} or I by Right Hand Rule.

■ **Maxwell's Equations**
 $\oint \vec{E} \cdot d\vec{A} = \frac{1}{\epsilon_0} Q_{\text{enc}}$, $\oint \vec{B} \cdot d\vec{A} = 0$
 $\oint \vec{B} \cdot d\vec{r} = \mu_0 I_{\text{enc}} + \mu_0 \epsilon_0 \frac{d\Phi_e}{dt}$, $\oint \vec{E} \cdot d\vec{r} = -\frac{d\Phi_m}{dt}$

■ **Inductance** Mutual: $\mathcal{E}_2 = -M \frac{dI_1}{dt}$, Self: $\mathcal{E} = -L \frac{dI}{dt}$
 Long Solenoid: $L = \mu_0 n^2 A \ell = \mu_0 \frac{N^2}{\ell} A$
 Energy in Inductor: $U = \frac{1}{2} L I^2$

■ **Energy Density** $u = \frac{1}{2\mu_0} B^2$ (in magnetic field)
 $u = u_e + u_m = \frac{\epsilon_0}{2} E^2 + \frac{1}{2\mu_0} B^2$ (in electromagnetic field)

■ **RC Circuits** $\tau = RC = \text{time const.}$, $\mathcal{E} = \text{EMF}$
 Discharging: $Q(t) = Q_0 e^{-t/\tau}$ Charging: $Q(t) = C \mathcal{E} (1 - e^{-t/\tau})$

■ **RL Circuits** $\tau = \frac{L}{R} = \text{time const.}$
 Current Growth: $I(t) = \frac{\mathcal{E}}{R} (1 - e^{-t/\tau})$, Current Decay: $I(t) = I_0 e^{-t/\tau}$

■ **LC Circuits** $\omega_0 = \frac{1}{\sqrt{LC}}$, $Q(t) = Q_{\text{max}} \cos(\omega_0 t + \phi)$

■ **LCR Circuits** $Q(t) = Q_0 e^{-\gamma t} \cos(\omega t + \phi)$, $\gamma = \frac{R}{2L}$, $\omega = \sqrt{\omega_0^2 - \gamma^2}$

■ **General AC Circuits** $V_{\text{rms}} = \frac{1}{\sqrt{2}} V_{\text{max}}$, $I_{\text{rms}} = \frac{1}{\sqrt{2}} I_{\text{max}}$
 $\omega = 2\pi f$, $I(t) = I_{\text{max}} \cos \omega t$, $V(t) = V_{\text{max}} \cos(\omega t + \phi)$
 $Z = \frac{V_{\text{max}}}{I_{\text{max}}} = \frac{V_{\text{rms}}}{I_{\text{rms}}}$ (Impedance), $\mathcal{P}_{\text{ave}} = V_{\text{rms}} I_{\text{rms}} \cos \phi$ (Ave. Power)

| | Z | ϕ |
|--------|----------------------------|------------------------------|
| Just R | R | 0 |
| Just C | $X_C = \frac{1}{\omega C}$ | $-90^\circ = -\frac{\pi}{2}$ |
| Just L | $X_L = \omega L$ | $90^\circ = \frac{\pi}{2}$ |

■ **Series RCL** $Z = \sqrt{R^2 + (X_L - X_C)^2}$, $\tan \phi = \frac{X_L - X_C}{R}$, $\mathcal{P}_{\text{ave}} = I_{\text{rms}}^2 R$
 Resonance: $Z = Z_{\text{min}} = R \Leftrightarrow X_L = X_C \Leftrightarrow \omega = \frac{1}{\sqrt{LC}}$

■ **Transformer** $\frac{V_2}{V_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2}$

■ **Electromagnetic Radiation in Vacuum**

$\vec{E} = \hat{y} E_{\text{max}} \cos(kx - \omega t)$, $\vec{B} = \hat{z} B_{\text{max}} \cos(kx - \omega t)$
 $\omega = 2\pi f = \frac{2\pi}{T}$, $k = \frac{2\pi}{\lambda}$, $f\lambda = \frac{\omega}{k} = c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$, $c = \frac{E_{\text{max}}}{B_{\text{max}}} = \frac{E}{B}$

Intensity = $I = \frac{\text{Power}}{\text{Area}} = \frac{U}{A \Delta t}$, $I = \frac{E_{\text{max}}^2}{2\mu_0 c} = c u_{\text{ave}} = S_{\text{ave}}$, $\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$

■ **Radiation Pressure and Momentum**

Momentum carried by radiation: $p = \frac{U}{c}$, Pressure = $\frac{\text{Force}}{\text{Area}}$

| | Momentum to Surface | Pressure on Surface |
|----------------------------------|--------------------------------|--------------------------------|
| Perfect Absorber | $p = \frac{U}{c}$ | $P = \frac{I}{c}$ |
| Perfect Reflector | $p = 2 \frac{U}{c}$ | $P = 2 \frac{I}{c}$ |
| $\kappa = \text{fraction refl.}$ | $p = (1 + \kappa) \frac{U}{c}$ | $P = (1 + \kappa) \frac{I}{c}$ |

■ **In a Medium** $f\lambda = v = c/n$

At interface: $n_1 \lambda_1 = n_2 \lambda_2$
 Snell's Law: $n_1 \sin \theta_1 = n_2 \sin \theta_2$

Total Internal Refl.: $\theta_1 > \theta_c$ where $\sin \theta_c = \frac{n_2}{n_1}$

■ **Geometric Optics** $\frac{1}{f} = \frac{1}{s} + \frac{1}{s'}$, $m = \frac{h'}{h} = -\frac{s'}{s}$

Sph. Mirrors: $f = \frac{R}{2}$, $R > 0$ (concave), $R < 0$ (convex), $R \rightarrow \infty$ (flat)

Thin Lenses: $f > 0$ (converging), $f < 0$ (diverging)

Spherical Interface: $\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R}$, $m = \frac{h'}{h} = -\frac{n_1 s'}{n_2 s}$

Flat Interface: $\frac{s'}{s} = -\frac{n_2}{n_1}$, $m = \frac{h'}{h} = 1$

Lensmaker Formula: $\frac{1}{f} = (n - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$

■ **Interference and Diffraction**

$\tan \theta = \frac{y}{L}$, Small θ or $y \ll L \Rightarrow \sin \theta = \frac{y}{L}$

Double Slit: Intensity: $I = I_{\text{max}} \cos^2 \left(\frac{\pi d \sin \theta}{\lambda} \right)$

Const. Int.: $d \sin \theta = m \lambda$, Dest. Int.: $d \sin \theta = \left(m + \frac{1}{2} \right) \lambda$

Diffraction Grating: Const. Int: $d \sin \theta = m \lambda$, Dest. Int. elsewhere

Single Slit: Destr. Int.: $a \sin \theta = m \lambda$, $m \neq 0$

| Thin Films | Constructive Interference | Destructive Interference |
|------------|---|---|
| $n < n'$ | $2t = m \frac{\lambda}{n}$ | $2t = \left(m + \frac{1}{2} \right) \frac{\lambda}{n}$ |
| $n > n'$ | $2t = \left(m + \frac{1}{2} \right) \frac{\lambda}{n}$ | $2t = m \frac{\lambda}{n}$ |

Phase Shift on Reflection: $n < n' \Rightarrow 180^\circ$ shift, $n > n' \Rightarrow$ no shift

■ **Polarization** $I = \frac{1}{2} I_0$, $I = I_0 \cos^2 \theta$, $\tan \theta_P = \frac{n_2}{n_1}$ (Pol. \perp)