Chapter G

Faraday's Law and Electrodynamics

Blinn College - Physics 2426 - Terry Honan

Faraday's law describes how a generator works. A changing magnetic flux through a conducting loop induces an EMF (Electromotive Force) around the loop. Faraday's law will become our fourth of Maxwell's equations.

G.1 - Faraday's Law

We can demonstrate magnetic induction by moving a magnet toward a solenoid connected to a galvanometer, where a deflection of the galvanometer's needle indicates a current. Moving the magnet toward the solenoid induces an EMF in the solenoid, which creates a current in the solenoid-galvanometer circuit. Pulling the magnet away from the solenoid induces a current in the opposite direction. If the magnet is at rest in the solenoid no current is induced. Moreover, increasing the number of magnets increases these effects. It is clear that the induced EMF depends on the change in the flux. If $\overline{\mathcal{E}}$ is the average induced EMF and Φ is the magnetic flux through each loop of the solenoid then we get the proportionality

$$\overline{\mathcal{E}} \propto \Delta\Phi.$$

If the speed of the magnet is increased the effect is enhanced and slowing it diminishes it. This suggests an inverse proportionality with the time.

$$\overline{\mathcal{E}} \propto \frac{1}{\Delta t}.$$

Since in a coil all the loops are connected in series it follows that $\overline{\mathcal{E}}$ is proportional to the number of loops N.

Combining these proportionalities gives

$$\overline{\mathcal{E}} \propto N \frac{\Delta\Phi}{\Delta t}$$

and it turns out that the proportionality becomes an equality when we consider the magnitude of the average induced EMF $|\overline{\mathcal{E}}|$ and the absolute value of the change in flux with time

$$|\overline{\mathcal{E}}| = N \left|\frac{\Delta\Phi}{\Delta t}\right|.$$

If we consider the polarity (sign) of the induced EMF then this adds a sign, which is known as Lenz's law.

$$\overline{\mathcal{E}} = -N \frac{\Delta\Phi}{\Delta t}.$$

We will discuss Lenz's law in detail later.

Taking the limit as Δt goes to zero these expressions become

$$|\mathcal{E}| = N \left|\frac{d\Phi}{dt}\right| \text{ and } \mathcal{E} = -N \frac{d\Phi}{dt}.$$

This is analogous to the definitions of average and instantaneous velocity in kinematics where $\overline{v} = \frac{\Delta x}{\Delta t}$ and $v = \frac{dx}{dt}$.

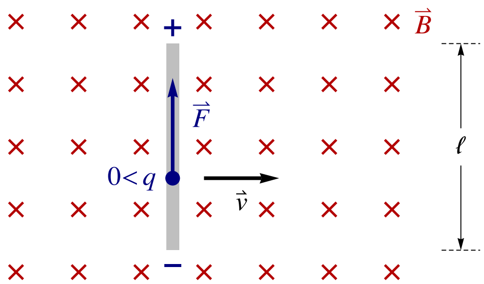
G.2 - Motional EMF

If a conductor moves in a magnetic field there is a magnetic force on the charge carriers. This magnetic force does work on the charge carriers. The EMF is the work per charge.

$$\mathcal{E} = \frac{W}{q}.$$

Translation of a Conducting Rod in a Uniform Field

Consider a conducting rod of length ℓ translating in a uniform magnetic field. Take the rod, its velocity and the field to be mutually perpendicular.



If q is some charge carrier then the force on it is

$$\vec{F} = q\,\vec{v}\times\vec{B}. \implies F = q\,v\,B$$

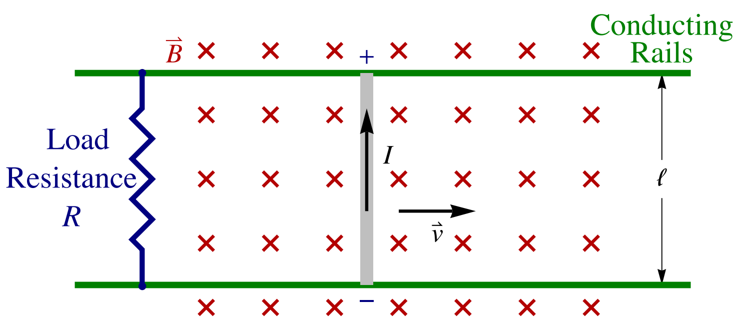
The work is

$$W = \int \vec{F}\cdot d\vec{r} = F\,\ell = q\,v\,B\,\ell.$$

Using $\mathcal{E} = W/q$ gives the induced EMF across the rod

$$\mathcal{E} = v\,B\,\ell$$

How can one measure this EMF? If a voltmeter is connected across the ends of the rod and it is moved with it then the same EMF is induced in the leads to the meter and it reads zero. To avoid this, imagine the rod moving with its ends sliding along a conducting rail. If the voltmeter is connected between the rails then it would read this voltage.



A DC Generator and Conservation of Energy

In fact, what we have here is a simple DC generator. Instead of connecting a voltmeter between the rails we could connect anything and it would be given a steady DC voltage. If a DC motor is connected then this motor could do work. We must address the question of conservation of energy. Where does this energy come from?

To see this place a load resistor R across the conducting rails. Ohm's law gives the current through the load

$$I = \frac{\mathcal{E}}{R}.$$

The rate of power dissipation in the load, which is the power output of the generator, is

$$\mathcal{P}_{\text{out}} = I\,\mathcal{E} = I\,v\,B\,\ell,$$

where we are making the idealizing assumption that all the resistance in the circuit is in the load.

This is a complete circuit, so all the current through the load passes through the rod. A current through a conductor in a magnetic field creates a magnetic force, $\vec{F}_{\text{mag}}$.

$$\vec{F}_{\text{mag}} = I\,\vec{\ell}\times\vec{B} \implies F_{\text{mag}} = I\,\ell\,B$$

To keep the rod moving at a constant speed there must be zero net force, so there must be some external applied force $\vec{F}_{\text{app}}$. Making another idealizing assumption of no friction we get

$$\vec{F}_{\text{app}} = -\vec{F}_{\text{mag}} \implies F_{\text{app}} = F_{\text{mag}} = I\,\ell\,B$$

The applied force does work and this is the source of the energy. The rate that it does work is the power $\mathcal{P}_{\text{in}}$. Power is related to force by

$$\mathcal{P} = \frac{d\,\text{Work}}{d\,t} = \frac{\vec{F}\cdot\boldsymbol{d}\vec{r}}{d\,t} = \vec{F}\cdot\vec{v}$$

so we get in this case

$$\mathcal{P}_{\text{in}} = F_{\text{app}}\,v = I\,\ell\,B\,v.$$

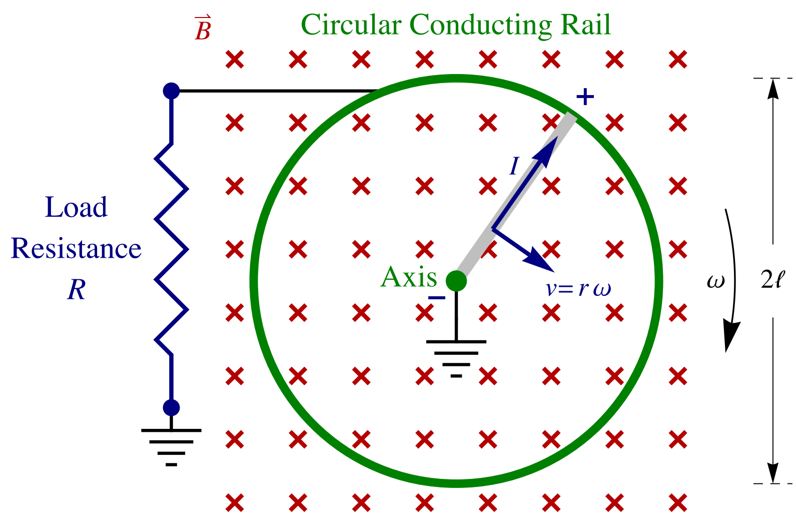
Comparing this with the power output gives

$$\mathcal{P}_{\text{in}} = \mathcal{P}_{\text{out}}.$$

If we relax our idealizing assumptions and include mechanical friction and resistance in the circuit elsewhere than the load then we get

$$\mathcal{P}_{\text{in}} > \mathcal{P}_{\text{out}}.$$

Rotation of a Conducting Rod in a Uniform Field - The DC Generator



The above describes a crude DC generator, but it is not a practical device because it requires an infinite track and an infinitely long region of uniform field. To make a more practical generator consider a rod rotating in a uniform field. Take the rod to be of length ℓ rotating with an angular velocity ω in a uniform field B. The velocity varies with r, the distance from the center by

$$v = r\,\omega.$$

The force on a charge carrier becomes

$$\vec{F} = q\,\vec{v}\times\vec{B}. \implies F = q\,v\,B = q\,r\,\omega\,B$$

and the work is

$$W = \int \vec{F}\cdot d\vec{r} = \int_0^{\ell} F\,d\,r = q\,\omega\,B\int_0^{\ell} r\,d\,r = \frac{1}{2}\,q\,\omega\,B\,\ell^2.$$

$\mathcal{E} = W/q$ gives the EMF

$$\mathcal{E} = \frac{1}{2}\,\omega\,B\,\ell^2.$$

This is a DC generator. If a bicycle wheel with N spokes rotates similarly in a magnetic field the same formula applies since voltage sources are connected in parallel. A rotating disk is also a DC generator where ℓ becomes the disk radius; this may be viewed as the limit of an infinite number of spokes.

G.3 - Faraday's Law Examples

Usually when we use Faraday's law we will only consider the magnitude of the induced EMF or current. We will see how to find the polarity in the Lenz's law discussion that follows. We can neglect polarity information by inserting absolute values into Faraday's law.

$$|\overline{\mathcal{E}}| = N\left|\frac{\Delta\Phi}{\Delta t}\right|. \text{ and } |\mathcal{E}| = N\left|\frac{d\Phi}{dt}\right|$$

If one is given some problem involving induced currents then this is related to the induced EMF by Ohm's law

$$\mathcal{E} = I\,R$$

The Translating Rod

There is an equivalence between motional EMF and Faraday's law. For the case of the translating rod that we analyzed above using motional EMF, we ought to be able to derive the same results using Faraday's law directly. If a load resistor is placed in the circuit then the complete circuit consists of the rod, rails and load. As the rod moves the area inside the circuit loop is increasing and so is the flux. In the time dt the rod moves by $v\,dt$ and the area increases by

$$dA = \ell\,v\,dt$$

The flux, since the field is uniform and perpendicular to the surface, is $\Phi = B\,A$ and since the field is constant we get $\frac{d\Phi}{dt} = B\,\frac{dA}{dt}$. The induced EMF becomes

$$|\mathcal{E}| = \left|\frac{d\Phi}{dt}\right| = B\,\frac{dA}{dt} = B\,\ell\,v$$

which is the same as the expression derived using motional EMF considerations.

The Rotating Rod

We can similarly derive the expression for a rotating rod. This is the same as the translating rod except the expression for the dA becomes

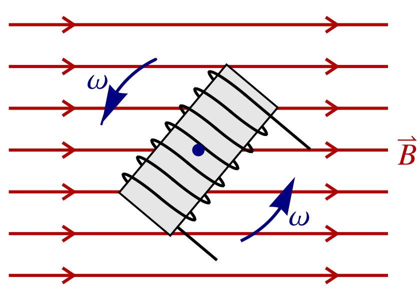
$$dA = \frac{1}{2}\,\ell^2\,\omega\,dt.$$

To get this we have used the area formula for an arc $A = \frac{1}{2}\,R^2\,\theta$ and that the angle subtended by the rod in time $d\theta$ is given by $d\theta = \omega\,dt$. The induced EMF can be found

$$|\mathcal{E}| = \left|\frac{d\Phi}{dt}\right| = B\,\frac{dA}{dt} = \frac{1}{2}\,B\,\ell^2\,\omega$$

which is the same as the previously derived expression.

The AC Generator



Consider a solenoid or coil with its central axis rotating with angular velocity ω in a uniform magnetic field of magnitude B. Take the central axis to be perpendicular to the rotational axis and the rotational axis to be perpendicular to the field. The coil has N turns and each loop has a cross-sectional area A. Take the angle between the axis of the coil and the field to be θ. This varies by

$$\theta = \omega\,t$$

The flux through a loop is

$$\Phi = \vec{B} \cdot \vec{A} = B\,A \cos\theta = B\,A \cos\omega\,t.$$

By Faraday's law $\mathcal{E} = -N\,\frac{d\Phi}{dt}$ we get the induced EMF as a function of time to be

$$\mathcal{E}(t) = N\,B\,A\,\omega \sin\omega\,t.$$

This is an AC voltage. We will discuss AC in detail in the AC circuit chapter. We will state for future reference that the peak EMF is

$$\mathcal{E}_{\max} = N\,B\,A\,\omega$$

and the frequency of AC is the same as the rotational frequency.

G.4 - Lenz's Law

The minus sign in Faraday's law is known as Lenz's law. It gives the polarity of the induced EMF. If the circuit is completed with a load resistance the induced EMF causes an induced current. The induced current induces a magnetic field $\vec{B}_{\text{ind}}$ which in turn induces a flux Φ_{ind} through the loop.

The induced flux tends to cancel the change in the flux.

This cancellation is exact for a superconductor.

The magnetic flux through a superconducting loop is a constant.

Using Lenz's Law

A simple procedure for applying Lenz's law follows. Make a table consisting of Φ, $\frac{d\Phi}{dt}$, Φ_{ind} and $\mathcal{E}$. Flux is a scalar but it has a sign; we can associate the signs of our flux variables with the directions of the normals to the loop.

The direction associated with Φ is the direction of the field through the loop.

The direction associated with $d\Phi/dt$ is the same as Φ when the flux is increasing, and opposite to Φ when the flux is decreasing.

By Lenz's law, the direction associated with Φ_{ind} is always opposite to that of $\frac{d\Phi}{dt}$.

Get the direction of $\mathcal{E}$ (and I) by the right-hand rule.

G.5 - Maxwell's Equations

EMF and Induced Electric Fields

The induced EMF can be related to an induced electric field. The electric field pushes the charge carriers in the conductor around the circuit. The work done on a charge carrier of charge q is

$$W = \int \vec{F} \cdot d\vec{r} = q \int \vec{E} \cdot d\vec{r}.$$

This is the energy gain for each charge carrier. The EMF is defined as the energy gain per charge carrier $\mathcal{E} = W/q$, allowing us to write the induced EMF in terms of the induced electric field.

$$\mathcal{E} = \int \vec{E} \cdot d\vec{r}.$$

Faraday's law can now be written as

$$\oint \vec{E} \cdot d\vec{r} = -\frac{d\Phi_m}{dt}.$$

This is the fourth of Maxwell's equations. The m subscript on the flux has been included to emphasize that we are dealing with the magnetic flux. The other form of Faraday's applied to a conducting loop, where this applies whether or not there is a conductor. A changing magnetic flux generally induces an electric field, if there is a conducting wire then this electric field causes an EMF in the wire. The integrals over electric fields along a line are analogous to the integrals over magnetic fields in Ampere's law. We choose a closed contour for integration and Φ_m is the magnetic flux through any surface that has the contour as its boundary.

Note that the N has been omitted in the above expression. It is implied that for a wire we integrate along the entire wire; thus if the wire circulates N times, we integrate around the loop N times.

Summary of Electromagnetism

We have been gradually building up our list of Maxwell's equations. We started with electrostatics, where charges were not allowed to move. Gauss's law

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\varepsilon_0}$$

was introduced as being mathematically equivalent to Coulomb's law, where Coulomb's law refers to the inverse square law for electric fields. Coulomb's law is not correct beyond electrostatics but Gauss's law is. If charges are allowed to move then the inverse square law implies that the effect of moving a charge is felt instantaneously at large distances. This is known as action at a distance and is not present in electrodynamics.

Next we considered magnetostatics, where we allow for electric currents but assume all currents are constant so that all magnetic fields are constant. Gauss's law for magnetism was introduced as a mathematical statement of the absence of isolated magnetic poles.

$$\oint \vec{B} \cdot d\vec{A} = 0$$

Just as with its electrical counterpart, the assumption of statics is not needed. Ampere's law

$$\oint \vec{B} \cdot d\vec{r} = \mu_0 I_{\text{enclosed}}$$

was introduced in the context of magnetostatics. Maxwell realized that this was not correct if the assumption of statics is relaxed and showed that this deficiency can be remedied by the introduction of an extra term to Ampere's law. This will be discussed in the next section.

Gauss's law involves only electric fields. Gauss's law for magnetism and Ampere's law involve only magnetic fields. Faraday's law

$$\oint \vec{E} \cdot d\vec{r} = -\frac{d\Phi_m}{dt}$$

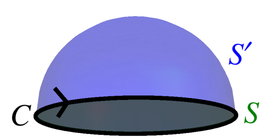
is genuinely electromagnetic, meaning that it involves both fields. It is also electrodynamic; this means that no assumptions of statics are made and, in fact, the induction effect requires dynamical (changing) fields.

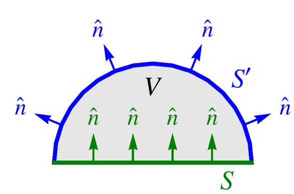
Maxwell's Displacement Current Addition to Ampere's Law

If C is any closed contour and S is some surface with C as its boundary $C = \partial S$, then Ampere's law can be written as

$$\int_C \vec{B} \cdot d\vec{r} = \mu_0 I(S).$$

where the integral is over the contour and $I(S)$ is the current through S. Different surfaces may share C as their boundary; for example, the equator may be viewed as the boundary of the northern hemisphere, of the southern hemisphere or of the disk through the center of the Earth.





Let S and S' be any two surfaces that share C as their boundary. In magnetostatics the currents through S and S' must be equal. Beyond magnetostatics, the currents can be different; this occurs when the charge enclosed between the surfaces varies. These two surfaces may be viewed as the boundary of some three dimensional volume V

$$\partial V = S' - S$$

and $I(S') - I(S)$ is the total current leaving V. It follows that if $Q(V)$ is the total charge of V then the current leaving V is the rate that $Q(V)$ is decreasing

$$I(S') - I(S) = -\frac{d}{dt} Q(V).$$

If the electric flux through some surface S is written as $\Phi(S)$ then Gauss's law states that

$$Q(V) = \varepsilon_0 \int_{\partial V} \vec{E} \cdot d\vec{A} = \varepsilon_0\, \Phi_e(\partial V) = \varepsilon_0\, \Phi_e(S') - \varepsilon_0\, \Phi_e(S).$$

Combining the above two expressions gives

$$I(S') - I(S) = -\varepsilon_0 \frac{d}{dt} \Phi_e(S') + \varepsilon_0 \frac{d}{dt} \Phi_e(S).$$

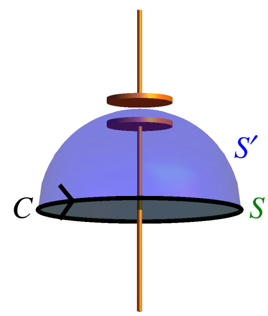
Rearranging this, we can see that although the current is different through S and S', if we add the displacement current $\varepsilon_0 \frac{d}{dt} \Phi_e$ to the current we get a result that is the same for both surfaces.

$$I(S') + \varepsilon_0 \frac{d}{dt} \Phi_e(S') = I(S) + \varepsilon_0 \frac{d}{dt} \Phi_e(S).$$

This new added term, the displacement current $\varepsilon_0 \frac{d}{dt} \Phi_e = \varepsilon_0 \frac{d}{dt} \int \vec{E} \cdot d\vec{A}$, is Maxwell's addition to Ampere's law. The modification of Ampere's law, sometimes referred to the Ampere-Maxwell law, becomes

$$\oint \vec{B} \cdot d\vec{r} = \mu_0\, I_{\text{enclosed}} + \mu_0\, \varepsilon_0 \frac{d}{dt} \Phi_e$$

Another way of viewing this addition is in terms of Electromagnetic duality, the symmetry between electric and magnetic fields. Faraday's law says that a changing magnetic flux induces an electric field so we should expect that similarly, a changing electric flux induces a magnetic field.



The diagram above shows a wire and a capacitor. The surface S is pierced by the wire and the surface S' passes through the gap between the capacitor's plates. If there is a current passing through the wire then that current passes through S but not S'. The current increases the charge on the plates of the capacitor and increases the flux through the surface S'; the increasing flux means that there is a displacement current through S'.

Maxwell's Equations

With the addition of the new term to Ampere's law, Maxwell had a set of four equations that was far more than the sum of its parts; the equations gave a complete and consistent dynamical theory of the electromagnetic field.

$$\oint \vec{E} \cdot d\vec{A} = \frac{1}{\varepsilon_0} Q_{\text{enclosed}}$$

$$\oint \vec{B} \cdot d\vec{A} = 0$$

$$\oint \vec{B} \cdot d\vec{r} = \mu_0\, I_{\text{enclosed}} + \mu_0\, \varepsilon_0 \frac{d}{dt} \Phi_e$$

$$\oint \vec{E} \cdot d\vec{r} = -\frac{d}{dt} \Phi_m$$

Newton's second law specifies the dynamics of particles and systems of particles. Maxwell's equations does the same for electric and magnetic fields; it describes their dynamics. Changing magnetic fields induce electric fields by Faraday's law and changing electric fields induce magnetic fields through the Ampere-Maxwell law.

Maxwell's equations form the basis of electrodynamics; it gives the dynamics of the electromagnetic field. With Coulomb's law, and similarly with Newton's law of gravity, we have action-at-a-distance, which is a direct interaction between distant charges (or masses.) With a field theory like electrodynamics we avoid action-at-a-distance. The basic idea is this: particles create fields, fields propagate through space at the speed of light or slower and then fields exert forces on particles. The force between particles is not direct but is mediated by a field. The propagation of the fields is determined by Maxwell's equations. Maxwell's equations also describe how to find the fields from the charges. To find the force on a charge due to a field we must use one extra thing, the Lorentz force law, which combines the electric force law with the magnetic one giving

$$\vec{F} = Q\left(\vec{E} + \vec{v}\times\vec{B}\right).$$

G.6 - Aside on Vector Calculus

S is some 2D surface in 3D. ∂S is its bounding contour.

Recall that the orientation of a surface is a consistent choice of normals, where any orientable surface has two orientations. The orientation of a contour is the direction of integration along the contour. The boundary of a surface is a closed contour. The orientation on the contour ∂S is related to the orientation of S by the right hand rule.

V is some volume, a 3D region. ∂V is its bounding surface.

The boundary of a volume is a closed surface. We choose the outward normal as the orientation of ∂V.

Operations of Vector Calculus

The Gradient Operator - $\vec{\nabla}$

The gradient operator is

$$\vec{\nabla} = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z} = \left\langle\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right\rangle.$$

As an operator it only makes sense when it acts on something else. Another example is the differential operator $\frac{d}{dx}$ that only makes sense when it acts on some function.

Gradient - $\vec{\nabla} f$

The gradient of a scalar field f is the vector field given by

$$\vec{\nabla} f = \hat{x}\frac{\partial f}{\partial x} + \hat{y}\frac{\partial f}{\partial y} + \hat{z}\frac{\partial f}{\partial z} = \left\langle\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right\rangle.$$

Divergence - $\vec{\nabla}\cdot\vec{F}$

The divergence of a vector field $\vec{F}$ is a scalar field.

$$\vec{\nabla}\cdot\vec{F} = \frac{\partial}{\partial x}F_x + \frac{\partial}{\partial y}F_y + \frac{\partial}{\partial z}F_z.$$

Curl - $\vec{\nabla}\times\vec{F}$

The curl of a vector field is a vector field.

$$\begin{aligned}\vec{\nabla}\times\vec{F} &= \hat{x}\left(\frac{\partial}{\partial y}F_z - \frac{\partial}{\partial z}F_y\right)\\ &+\hat{y}\left(\frac{\partial}{\partial z}F_x - \frac{\partial}{\partial x}F_z\right)\\ &+\hat{z}\left(\frac{\partial}{\partial x}F_y - \frac{\partial}{\partial y}F_x\right)\end{aligned}$$

Theorems of Vector Calculus

The theorems of vector calculus are all generalizations of the fundamental theorem of calculus. Generally, they relate the integral of some derivative over a region to an integral over the boundary of the region.

The Fundamental Theorem of Calculus

$$\int_a^b \frac{d\, f(x)}{d\, x}\, d\, x = f(b) - f(a)$$

This says that the integral of the derivative of any function $f(x)$ over the closed interval $[a, b]$ is equal to the function evaluated over the boundary of the interval, which are the two points a and b, with the signs giving the orientation.

Gauss's Theorem or the Divergence Theorem

$$\int_V \vec{\nabla} \cdot \vec{F} \, d\text{Vol} = \int_{\partial V} \vec{F} \cdot d\vec{A}$$

Stokes' Theorem

$$\int_S \left(\vec{\nabla} \times \vec{F}\right) \cdot d\vec{A} = \int_{\partial S} \vec{F} \cdot d\vec{r}$$

Differential Form of Maxwell's Equations

We can rewrite Maxwell's equations in terms of the divergence and curl. The first step is to write the four equations in terms of volumes V surfaces S and their boundaries. Starting with Gauss's law we can write the total charge inside a volume as an integral over the charge density (charge/volume) ρ

$$Q_{\text{enclosed}} = Q(V) = \int_V \rho \, d\text{Vol}.$$

Next, apply Gauss's theorem to the left hand side. The resulting expression is true for any volume V, so we can neglect the integrals and equate the integrands. This becomes the differential form of Gauss's law:

$$\oint \vec{E} \cdot d\vec{A} = \frac{1}{\varepsilon_0} Q_{\text{enclosed}} \implies \int_{\partial V} \vec{E} \cdot d\vec{A} = \frac{1}{\varepsilon_0} \int_V \rho \, d\text{Vol}$$

$$\implies \int_V \vec{\nabla} \cdot \vec{E} \, d\text{Vol} = \frac{1}{\varepsilon_0} \int_V \rho \, d\text{Vol} \implies \vec{\nabla} \cdot \vec{E} = \frac{1}{\varepsilon_0} \rho.$$

Similarly, we can write Gauss's law for magnetism in a differential form

$$\int_{\partial V} \vec{B} \cdot d\vec{A} = 0 \implies \vec{\nabla} \cdot \vec{B} = 0.$$

A current through a surface is related to the current density integrated over it

$$I_{\text{enclosed}} = I(S) = \int_S \vec{J} \cdot d\vec{A}.$$

Next apply Stokes' theorem to the left hand side of the expression. Since the resulting expression for any surface S we can equate the integrands.

$$\oint \vec{B} \cdot d\vec{r} = \mu_0 \, I_{\text{enclosed}} + \mu_0 \, \varepsilon_0 \frac{d}{dt} \Phi_e$$

$$\implies \int_{\partial S} \vec{B} \cdot d\vec{r} = \mu_0 \int_S \vec{J} \cdot d\vec{A} + \mu_0 \, \varepsilon_0 \frac{d}{dt} \int_S \vec{E} \cdot d\vec{A}$$

$$\implies \int_S \left(\vec{\nabla} \times \vec{B}\right) \cdot d\vec{A} = \mu_0 \int_S \vec{J} \cdot d\vec{A} + \mu_0 \, \varepsilon_0 \int_S \frac{\partial}{\partial t} \vec{E} \cdot d\vec{A}$$

$$\implies \vec{\nabla} \times \vec{B} = \mu_0 \, \vec{J} + \mu_0 \, \varepsilon_0 \frac{\partial}{\partial t} \vec{E}$$

A similar calculation gives the differential form of Faraday's law

$$\int_{\partial S} \vec{E} \cdot d\vec{r} = -\frac{d}{dt} \int_S \vec{B} \cdot d\vec{A} \implies \vec{\nabla} \times \vec{E} = -\frac{\partial}{\partial t} \vec{B}$$

Summarizing, we can write Maxwell's equations in terms of divergence and curl; this is known as the differential form.

$$\vec{\nabla} \cdot \vec{E} = \frac{1}{\varepsilon_0} \rho$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial}{\partial t} \vec{E}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial}{\partial t} \vec{B}$$