Chapter A - Problems

Blinn College - Physics 2426 - Terry Honan

Problem A.1

(a) The size of an atomic nucleus is on the order of 10^{-15} m. What is the magnitude of the electric force between two protons separated by 10^{-15} m?

(b) What is the magnitude of the gravitational force between two protons separated by 10^{-15} m?

(c) What is the ratio of the electric to gravitational force magnitudes? Show that this number is independent of the distance between the two.

(d) Two identical particles have the same charge Q and same mass m. If the electric repulsion exactly cancels the gravitational attraction, what must be the value of $|Q|/m$?

Solution to A.1

(a) $F_{\text{elec}} = k_e \dfrac{|Q_1|\cdot|Q_2|}{r^2} = k_e \left(\dfrac{e}{r}\right)^2 = \; 9.0\times10^9 \times \left(\dfrac{1.60\times10^{-19}}{10^{-15}}\right)^2 = 230.\,\text{N}$

(b) $F_{\text{grav}} = G\,\dfrac{m_1\cdot m_2}{r^2} = G\left(\dfrac{m_{\text{proton}}}{r}\right)^2 = \; 6.67\times10^{-11} \times \left(\dfrac{1.67\times10^{-27}}{10^{-15}}\right)^2 = 1.86\times10^{-34}\,\text{N}$

(c) $\dfrac{F_{\text{elec}}}{F_{\text{grav}}} \; = \; 1.24\times10^{36}$

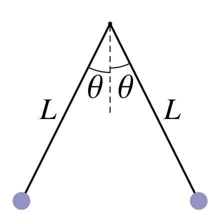
To show it is independent of distance r use the above expressions and notice the cancellation of r:

$$\frac{F_{\text{elec}}}{F_{\text{grav}}} = \frac{k_e|Q_1|\cdot|Q_2|/r^2}{G\,m_1\cdot m_2/r^2} = \frac{k_e|Q_1|\cdot|Q_2|}{G\,m_1\cdot m_2}.$$

(c) $F_{\text{elec}} = F_{\text{grav}} \implies k_e\,\dfrac{Q^2}{r^2} = G\,\dfrac{m^2}{r^2} \implies \dfrac{|Q|}{m} = \sqrt{\dfrac{G}{k_e}} = \sqrt{\dfrac{6.67\times10^{-11}}{9.0\times10^9}} = \; 8.61\times10^{-11}\,\text{C/kg}$

Problem A.2

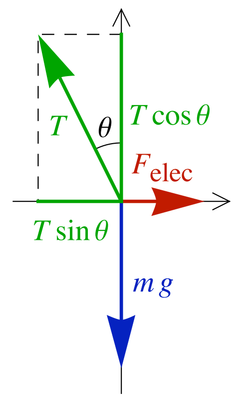
Two identical small light spheres of mass m hang from strings of length L from the same pivot point so that the strings are vertical and the spheres touch. Both are then given the same positive charge causing the spheres to separate as shown. If each string makes an angle of θ from vertical then what is the charge Q?



Solution to A.2

The electric force between the two is $F_{\text{elec}} = k_e \frac{Q^2}{r^2}$ and the distance between the two spheres is $r = 2\,L\sin\theta$. Combining gives:

$$F_{\text{elec}} = k_e \left(\frac{Q}{2\,L\sin\theta}\right)^2.$$



Above is the free-body diagram for the sphere on the right. Since there is no acceleration the forces in each direction cancel. This gives:

$$T\sin\theta = F_{\text{elec}} \;\text{ and }\; T\cos\theta = m\,g$$

Dividing gives

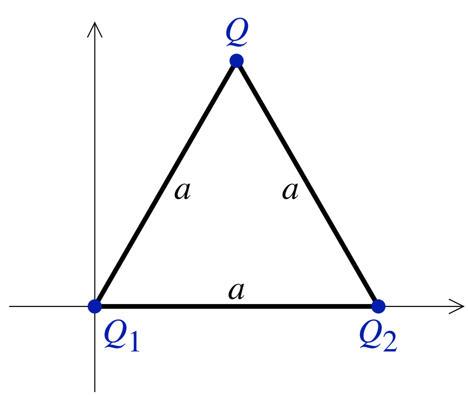
$$\tan\theta = \frac{F_{\text{elec}}}{m\,g}$$

We can now solve for the charge Q.

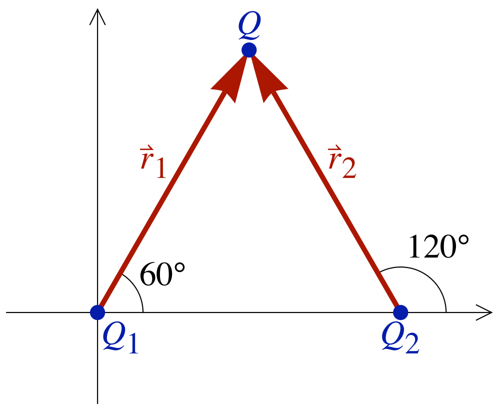
$$k_e\left(\frac{Q}{2\,L\sin\theta}\right)^2 = m\,g\tan\theta \implies Q = 2\,L\sin\theta\sqrt{\frac{m\,g}{k_e}\tan\theta}$$

Problem A.3

Three charges $Q = 7.0\,\mu\text{C}$, $Q_1 = 2.0\,\mu\text{C}$ and $Q_2 = -4.0\,\mu\text{C}$ are arranged around the corners of and equilateral triangle as shown. Each side of the triangle is $a = 0.5$ m. What is the net force of the charge Q (at the top)?



Solution to A.3



To calculate the force on a point charge Q due to a distribution of charges Q_i use: $\vec{F} = k_e Q \sum Q_i \frac{\hat{r}_i}{r_i^2}$ where $\frac{\hat{r}_i}{r_i^2} = \frac{\vec{r}_i}{r_i^3}$ and where $\vec{r}_i$ is the vector from Q_i to Q.

Note that a unit vector is some direction labelled by θ (measured counterclockwise from the positive x axis) is given by: $\hat{r} = \cos\theta\ \hat{x} + \sin\theta\ \hat{y} = \langle \cos\theta, \sin\theta \rangle$

$$r_1 = r_2 = a = 0.50\,m. \text{ Note that: } 10^9 \times 10^{-6} \times 10^{-6} = 10^{-3}.$$

$$\vec{F} = k_e Q \left(Q_1 \frac{\hat{r}_1}{r_1^2} + Q_2 \frac{\hat{r}_2}{r_2^2} \right) = k_e \frac{Q}{a^2} (Q_1 \hat{r}_1 + Q_2 \hat{r}_2)$$
$$= \frac{9.0\times10^{-3}}{0.5^2} \times 7.0\ [\,2.0 \langle \cos 60°, \sin 60° \rangle - 4.0 \langle \cos 120°, \sin 120° \rangle\,]$$
$$= \langle .756, -.436 \rangle N = (.756\,\hat{x} - .436\,\hat{y})\,N.$$

Problem A.4

A small sphere with a 2 gram mass sits in an upward electric field of 60 $\frac{\text{N}}{\text{C}}$ in the Earth's gravitational field. For the electric force to cancel the weight of the sphere what charge must be given to the sphere? How many electrons must be removed from a neutral sphere to give this charge?

Solution to A.4

$$\vec{0} = \vec{F}_{\text{net}} = \vec{F}_{\text{elec}} + \vec{F}_{\text{grav}} = Q\vec{E} + m\vec{g} \implies F_{\text{elec}} = F_{\text{grav}} \implies |Q|\,E = m\,g \implies |Q| = \frac{m\,g}{E}$$

To balance the weight, $\vec{F}_{\text{elec}}$ must be upward. Since the field is upward the charge is positive.

$$Q = |Q| = \frac{.002 \times 9.80}{60} = 3.267 \times 10^{-4}\ \text{C}.$$

The number of electrons is given by $n = Q/e$:

$$n = \frac{3.267\times10^{-4}}{1.60\times10^{-19}} = 2.04 \times 10^{15}.$$

Problem A.5

A 5 μC charge sits at the origin and a -8 μC charge sits at (2 m, -3 m).

(a) What is the force on the -8 μC charge?

(b) What is the electric field at (0, -2 m)?

Solution to A.5

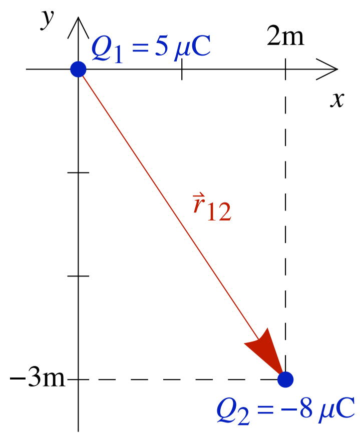
(a) To calculate the force on a point charge Q_2 due to a charge Q_1 use:

$$\vec{F}_{12} = k_e\, Q_1\, Q_2\, \frac{\hat{r}_{12}}{r_{12}^2} \quad \text{where} \quad \frac{\hat{r}_{12}}{r_{12}^2} = \frac{\vec{r}_{12}}{r_{12}^3}$$

and where $\vec{r}_{12}$ is the vector from Q_1 to Q_2.

Take $Q_1 = 5\,\mu\text{C}$ and $Q_2 = -8\,\mu\text{C}$. Here we have $\vec{r}_{12} = \langle 2, -3\rangle = 2\,\hat{x} - 3\,\hat{z}$. Note that: $10^9 \times 10^{-6} \times 10^{-6} = 10^{-3}$.

$$\vec{F}_{12} = k_e\, Q_1\, Q_2\, \frac{\vec{r}_{12}}{r_{12}^3} = 9\times 10^{-3} \times 5 \times (-8)\, \frac{\langle 2,-3\rangle}{\left(2^2+3^2\right)^{3/2}} = \langle -0.0154\,,\; 0.0230\,\rangle\,\text{N}$$



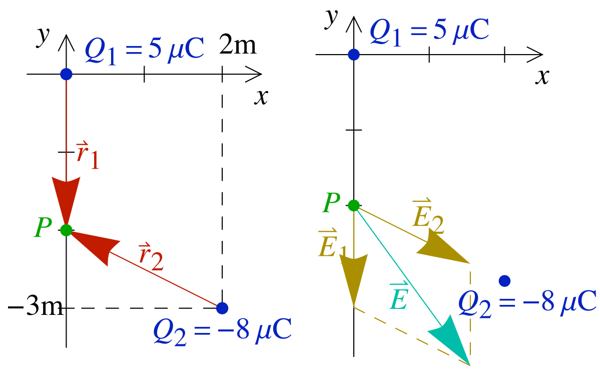
(b) To calculate the electric field at a point P due to a distribution of charges Q_i use:

$$\vec{E} \;=\; k_e \sum Q_i\, \frac{\hat{r}_i}{r_i^2} \quad \text{where} \quad \frac{\hat{r}_i}{r_i^2} = \frac{\vec{r}_i}{r_i^3} \quad \text{and where } \vec{r}_i \text{ is the vector from } Q_i \text{ to } P.$$

Since P is $(0, -2\,\text{m})$ we have: $\vec{r}_1 = -2\,\hat{y} = \langle 0, -2\rangle$ and $\vec{r}_2 = -2\,\hat{x} + \hat{y} = \langle -2, 1\rangle$. Note that: $10^9 \times 10^{-6} = 10^3$. Since the vector $\vec{r}_1$ is along an axis, the unit vector in its direction is simple $\hat{r}_1 = \frac{\vec{r}_1}{r_1} = -\,\hat{y} = \langle 0, -1\rangle$.

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = k_e\, Q_1\, \frac{\hat{r}_1}{r_1^2} + k_e\, Q_2\, \frac{\vec{r}_2}{r_2^3} = \; 9\times 10^3 \times 5\, \frac{\langle 0,-1\rangle}{2^2} \;+\; 9\times 10^3 \times (-8)\, \frac{\langle -2,1\rangle}{\left(2^2+1^2\right)^{3/2}}$$

$$= \; -\,11.25\times 10^3\,\langle 0,\,1\rangle + \left\langle 12.88\times 10^3,\, -6.440\times 10^3\right\rangle = \langle 12.9,\, -\,17.7\rangle \times 10^3\, \frac{N}{C}$$



Problem A.6

A charge $3\,Q$ is at $x = 0$ and a charge of $-Q$ is at $x = d$. Where, other than infinity, is the electric field zero?

Solution to A.6

$\vec{E} = \vec{E}_1 + \vec{E}_2$ For the electric field to be zero, $\vec{E}_1$, the field due to Q_1 and $\vec{E}_2$, the field due to Q_2, must be equal in magnitude and opposite in direction. This could only occur somewhere on the x axis. We will first find where the magnitudes are equal.

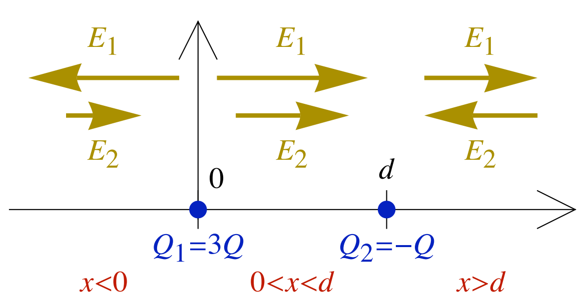
$$E_1 = E_2 \implies k_e \frac{|Q_1|}{r_1^2} = k_e \frac{|Q_2|}{r_2^2} \implies k_e \frac{3\,Q}{x^2} = k_e \frac{Q}{(x-d)^2} \implies \left(\frac{x-d}{x}\right)^2 = \frac{1}{3} \implies \frac{x-d}{x} = \pm\frac{1}{\sqrt{3}}$$

This gives two solutions.

$$\frac{x-d}{x} = \frac{1}{\sqrt{3}} \implies x - d = \frac{1}{\sqrt{3}}\,x \implies x = \frac{d}{1-\frac{1}{\sqrt{3}}} = 2.366\ d$$

$$\frac{x-d}{x} = -\frac{1}{\sqrt{3}} \implies x - d = -\frac{1}{\sqrt{3}}\,x \implies x = \frac{d}{1+\frac{1}{\sqrt{3}}} = 0.634\ d$$

Of these two solutions one is between 0 and d, $0 < x < d$ and the other is beyond d, $x > d$. We now consider the directions of the fields $\vec{E}_1$ and $\vec{E}_2$. Since fields point away from positive charges and toward we can see the directions are as shown.



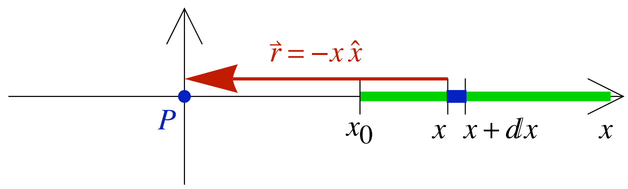
Between the two charges the fields are in the same direction and thus cannot cancel. For $x > d$ they can cancel. The solution then is:

$$x = 2.366\ d\,.$$

Problem A.7

What is the electric field at the origin due to a line of charge from x_0 to ∞ along the positive x-axis with a uniform linear charge density (charge/length) of λ.

Solution to A.7



Since the charge is along the x axis it is natural to choose x as the variable of integration. Take its limits of integration to be: $x_0 \leq x < \infty$. The vector $\vec{r}$ is from x along the x axis to the origin.

$$\vec{r} = -x\,\hat{x} \implies \hat{r} = -\,\hat{x} \text{ and } r = x$$

dq is the charge between x and $x + dx$. Since the charge per length is $\lambda_0$the charge is a region of width dx is $dq = \lambda_0\, dx$.

$$\vec{E} = k_e \int \frac{\hat{r}}{r^2}\, dq = \; k_e \int_{x_0}^{\infty} \frac{-\hat{x}}{x^2}\, \lambda_0\, dx \implies \vec{E} = -\hat{x}\,\lambda_0\, k_e \int_{x_0}^{\infty} \frac{dx}{x^2}$$

Students will typically be asked to leave their answer in the form of a *well defined* definite integral. If so, the form above is then sufficient. Here, the integral is simple and the result is: $\vec{E} = -\hat{x}\, k_e\, \frac{\lambda_0}{x_0}$. Thus the magnitude is $k_e\, \frac{\lambda_0}{x_0}$ and it is in the negative x direction.

Problem A.8

Consider a ring of radius $R = 0.10$ m uniformly charged with $Q = 75\ \mu$C. The ring sits in the xy-plane with its center at the origin.

(a) What is the magnitude of the electric field at $z = 1$ cm, 5 cm and 30 cm?

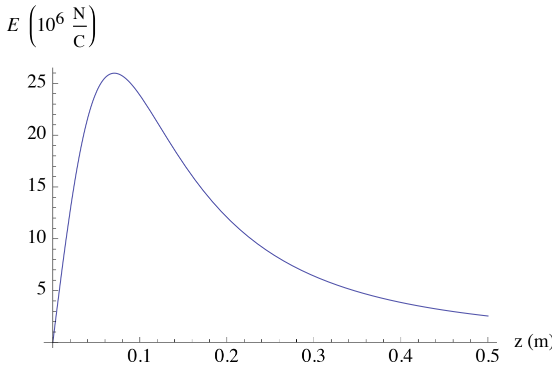
(b) Where is the field its maximum and what is that maxum value?

Solution to A.6

For a charged ring in the xy-plane with the origin at its center the magnitude of the electric field at z_0 along the z-axis is given by:

$$E = k_e\, Q\, \frac{z_0}{\left(R^2 + z_0^2\right)^{3/2}}$$

(See the notes for this derivation.)



(a) Using the value of $k_e = 9.0 \times 10^9 \ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}$ we get:

$$z_0 = 0.01\,m \implies E = 6.65\times 10^6 \ \frac{\text{N}}{\text{C}}$$
$$z_0 = 0.05\text{ m} \implies E = 24.1\times 10^6 \ \frac{\text{N}}{\text{C}}$$
$$z_0 = 0.30\text{ m} \implies E = 6.40\times 10^6 \ \frac{\text{N}}{\text{C}}$$

(b) To maximize we use the usual procedure of calculus: Set the derivative to zero and solve for the optimized position.

$$0 = \frac{dE}{dz_0} = -3\,k_e\,Q\,\frac{z_0^2}{\left(R^2+z_0^2\right)^{5/2}} + k_e\,Q\,\frac{1}{\left(R^2+z_0^2\right)^{3/2}} \implies 3\,z_0^2 = R^2 + z_0^2 \implies z_0 = R\big/\sqrt{2}$$

The value of E is

$$E = \left(k_e\,Q\,\frac{z_0}{\left(R^2+z_0^2\right)^{3/2}}\right)_{z_0=R/\sqrt{2}} = \frac{k_e\,Q}{3\sqrt{3}\,R^2}$$

Inserting the values gives:

$$z_0 = 0.0707\text{ m} \ \text{ and } \ E = 2.60\times 10^7 \ \frac{\text{N}}{\text{C}}$$

Problem A.9

Consider a uniformly charged cylindrical shell (a hollow thin-walled tube) of radius R, length L and with a total charge Q.

(a) What is the electric field at the geometrical center of the tube?

(b) What is the magnitude of the electric field along the central axis at one end of the tube? Leavc your answer in the form of a well-defined definite integral. Do not integrate.

Solution to A.9

(a) At the geometrical center $E = 0$, by symmetry.

(b) Take the central axis of the cylinder to be the z-axis and take the cylinder to be between $z = 0$ and $z = L$. We want to find the field at the origin. Our integration variable is then z and its limits are $0 < z < L$. We now have a ring of charge $dq = \frac{Q}{L} dz$ between z and $z + dz$.

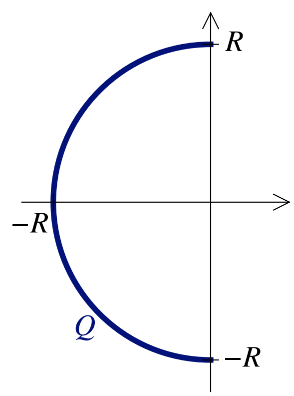
$E = k_e Q \frac{z_0}{(R^2 + z_0^2)^{3/2}}$ is the magnitude of the electric field due to a ring of radius R a distance z_0 from the center along the central axis.

This gives the infinesimal field due to a ring of dQ as: $dE = k_e \frac{z}{(R^2 + z^2)^{3/2}} dQ$. We need to integrate this to get the answer.

$$E = \int dE = k_e \frac{Q}{L} \int_0^L \frac{z}{(R^2 + z^2)^{3/2}} dz$$

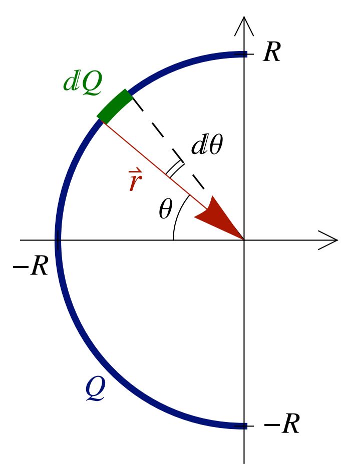
The direction of the field points away from the cylinder.

Problem A.10



What is the field at the origin due to a uniformly charged semicircular charge distribution of radius R with charge Q as shown?

Solution to A.10



Take θ to be the angle above the negative x-axis as shown. The limits of integration become: $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$.

$dQ = \frac{Q}{\pi} d\theta$ is the infinesimal charge between θ and $\theta + d\theta$. (This is the total charge divided by the total angle multiplied by the infinitesimal angle.)

The unit vector from $d Q$ to the origin is: $\hat{r} = \cos\theta\,\hat{x} - \sin\theta\,\hat{y} = \langle\cos\theta, -\sin\theta\rangle$

$$\vec{E} = k_e \int \frac{\hat{r}}{r^2}\, dq = k_e \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\langle\cos\theta, -\sin\theta\rangle}{R^2}\,\frac{Q}{\pi}\,d\theta$$

Because of symmetry we know that the y-component of the field is 0.

$$\vec{E} = \hat{x}\,k_e\,\frac{Q}{\pi R^2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\theta \quad d\theta = \hat{x}\,k_e\,\frac{2Q}{\pi R^2}$$

Problem A.11

An electron accelerates from rest to $2\times10^6\ \frac{\mathrm{m}}{\mathrm{s}}$ in an electric field of magnitude 800 $\frac{\mathrm{N}}{\mathrm{C}}$.

(a) What is the magnitude of the accelerarion of an electron in this electric field?

(b) How long does it take for an electron to accelerate to that speed?

(c) How far does the electron move in this time?

(d) What is the kinetic energy of the electron after this acceleration?

(e) What is the work done by the electric field?

Solution to A.11

(a) $a = \frac{F}{m} = \frac{eE}{m} = \frac{1.60\times10^{-19}\times800}{9.11\times10^{-31}} = 1.41\times10^{14}\ \mathrm{m}/\mathrm{s}^2$

(b) $v = v_0 + a\,t = 0 + a\,t \implies t = \frac{v}{a} = \frac{2.0\times10^{6}}{1.41\times10^{14}} = 1.42\times10^{-8}\ \mathrm{s}$

(c) $\Delta x = \frac{1}{2}a\,t^2 = \frac{1}{2}\,1.41\times10^{14}\left(1.42\times10^{-8}\right)^2 = 0.0142\ \mathrm{m}$

(d) $K = \frac{1}{2}m\,v^2 = \frac{1}{2}\,9.11\times10^{-31}\left(2.0\times10^{6}\right)^2 = 1.822\times10^{-18}\ \mathrm{J}$

(e) $W = F\,\Delta x = e\,E\,\Delta x = 1.60\times10^{-19}\times800\times0.0142 = 1.822\times10^{-18}\ \mathrm{J}$