Chapter D - Problems

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Problem D.1

Two electrically neutral conductors sit near each other. If a 12 V car battery is connected across the conductors then a total charge of 30 pC flows putting charges of ± 30 pC on the conductors.

(a) If two car batteries are used to create a 24 V potential difference between the conductors, then what is the total charge that flows?

(b) What is the capacitance of this configuration?

Solution to D.1

(a) $Q = C\,V \implies \frac{Q_2}{Q_1} = \frac{V_2}{V_1} \implies \frac{Q_2}{30} = \frac{24}{12} \implies Q_2 = 60\text{ pC}$

(b) $Q = C\,V \implies C = \frac{Q}{V} = \frac{30\text{ pC}}{12\text{ V}} = 2.5\text{ pF}$

Problem D.2

An empty parallel plate capacitor with a plate separation of 2 mm is connected across a 12 V battery.

(a) What is the electric field between the plates?

(b) What is the surface charge density on each plate?

(c) If the charge on the each plate is 300 pC then what is its capacitance and what is the surface area of the plates in cm^2?

Solution to D.2

$V = 12\,V,\ d = 2\times 10^{-3}\text{ m}$

(a) $E = \frac{V}{d} = \frac{12}{2\times 10^{-3}} = 6000\ \frac{\text{V}}{\text{m}}$

(b) $\sigma = \varepsilon_0\,E = 8.85\times 10^{-12}\times 6000 = 5.31\times 10^{-8}\ \frac{\text{C}}{m^2}$

(c) $C = \frac{Q}{V} = \frac{300\times 10^{-12}}{12} = 25\text{ pF}$

We can find the surface area either by using $C = C_0 = \varepsilon_0\,A/d$ or by using $Q = \sigma\,A$. These formulas give the same answer because we are now just stepping through the derivation of the formula for C_0. The second form is easier to use here.

$$Q = \sigma\,A \implies A = \frac{Q}{\sigma} = \frac{300\times 10^{-12}}{5.31\times 10^{-8}} = 56.5\times 10^{-4}\,m^2 = 56.5\text{ cm}^2$$

Problem D.3

A 30 m long coaxial cable has an inner conductor with a 2.5 mm diameter and an outer conductor with an inside diameter of 7 mm and an outer diameter of 8 mm. Suppose it is connected across a 1.5 V battery with the positive terminal connected to the inner conductor. What is the charge on each conductor? Assume the insulator between the conductors behaves as a vacuum.

Solution to D.3

(a) We must first find the capacitance. For a cylindrical capacitor

$$C = C_0 = \frac{2\pi \varepsilon_0 \ell}{\ln(b/a)} = \frac{2\pi\, 8.85\times10^{-12}\times 30}{\ln(7/2.5)} = 1.6202\times10^{-9} = 1.62\ \text{nF}\,.$$

Note that a and b are radii in the expression and diameters are given, but the ratio of the radii b/a is the same as the ratio of diameters. Also the ratio of two distances is the same in mm as m.

(b) The magnitude of the charge that flows is $Q = C\,V$.

$$Q = C\,V = 1.6202\times10^{-9}\times 1.5 = 2.43\ \text{nC}$$

Since the inside conductor is attached to the positive terminal it gains a positive charge and the outer conductor then gains a negative charge.

$$\text{Inner Conductor: } Q = +2.43\ \text{nC} \quad \text{and} \quad \text{Outer Conductor: } Q = -2.43\ \text{nC}$$

Problem D.4

A conducting sphere with a 12 cm radius sits inside a hollow conducting sphere with an inside radius of 15 cm and an outside radius of 20 cm, where all spherical surfaces are concentric. Suppose this is connected across a potential difference of 200 V. Assume the region between the conductors is a vacuum.
(a) What is the magnitude of the charge that flows?
(b) How much energy is stored in this configuration?

Solution to D.4

(a) We first need to find the capacitance. All of the capacitance is in the gap between the two conductors and thus the outside radius of the outside conductor is irrelevant. For an empty spherical capacitor

$$C = C_0 = \frac{1}{k_e\left(\frac{1}{a} - \frac{1}{b}\right)} = \frac{1}{9.0\times10^9\left(\frac{1}{0.12} - \frac{1}{0.15}\right)} = 6.66667\times10^{-11}\ \text{F}$$

The charge is given by $Q = C\,V$ with C from above and $V = 200$ V.

$$Q = C\,V = 13.3\ \text{nC}$$

(b) the energy is given by $U = \frac{1}{2}C\,V^2$ using ghe same values of C and V.

$$U = \frac{1}{2}C\,V^2 = 1.33\ \mu\text{J}$$

Problem D.5

A uniform electric field with a 300 V/m magnitude is in a room with dimensions 4 m × 6 m × 3 m.
(a) What is the electric energy density in this field?
(b) What is the total electric energy in the room?

Solution to D.5

(a) The energy density is u.

$$u = \frac{1}{2}\,\varepsilon_0\,E^2 = \frac{1}{2}\,8.85\times 10^{-12}\,300^2 = 3.98\times 10^{-7}\,\frac{\mathrm{J}}{m^3}$$

(b) Since energy density is energy/volume, the total energy is:

$$\text{Energy} = U = u \times \text{volume} = 3.98\times 10^{-7}\times(4\times 6\times 3) = 28.7\,\mu\mathrm{J}.$$

Problem D.6

The water molecule H_2O had a dipole moment of 6.3×10^{-30} C·m. What is the maximum torque on a water molecule in an electric field of magnitude 5000 V/m? How much work is required to rotate this from an aligned position to an anti-aligned (opposite the field) position in the same field?

Solution to D.6

The torque vector for a dipole in an electric field is $\vec{\tau} = \vec{p}\times\vec{E}$, so the magnitude of the torque is just $\tau = p\,E\sin\theta$. The maximum torque is when $\sin\theta = 1$ ($\theta = 90\,°$), so

$$\tau_{\text{max}} = p\,E = 6.3\times 10^{-30}\,5000 = 3.15\times 10^{-26}\,\mathrm{N\,m}.$$

The work is the change in the potential energy $W = \Delta\mathrm{U}$ and potential energy is $U = -\vec{p}\cdot\vec{E} = -p\,E\cos\theta$.

$$W = \Delta\,U = -p\,E\cos 180\,° - -\,p\,E\cos 0\,° = 2\,p\,E = 2\times 6.3\times 10^{-30}\,5000$$
$$=\ 6.3\times 10^{-26}\,\mathrm{J}$$

Problem D.7

A parallel plate capacitor with a plate area of 30 cm^2 and a plate separation of 2 mm is connected across a 250 V source.
(a) What are the electric field between the plates, the charge on the plates and the total energy stored?
(b) While still connected across the 250 V source suppose the plates are dropped into distilled water. What are the electric field between the plates, the charge on the plates and the total energy stored?
(c) Suppose instead the 250 V source is disconnected before the plates are dropped into distilled water. What are the electric field between the plates, the charge on the plates and the total energy stored?

Solution to D.7

$$d =\ 0.002\,m,\quad A =\ .0030\,\mathrm{m}^2,\quad V = 250\,\mathrm{V}\ \text{ and }\ \kappa = 80\ \text{(for water)}$$

(a) To find the electric field we don't need the area A. The electric field is always related to the voltage by $\Delta\,V = -\int\vec{E}\cdot d\vec{r}$ so here we simple get $V =\ E\,d$.

$$E = \frac{V}{d} = \frac{250}{0.002} = 1.25\times 10^5\,\frac{\mathrm{V}}{\mathrm{m}}$$

To find the energy we first need to find the capacitance of the empty capacitor C_0.

$$C_0 = \frac{\varepsilon_0 A}{d} = 8.85 \times 10^{-12} \frac{0.0030}{0.002} = 1.3275 \times 10^{-11} \text{ F}$$

For part (a) the capacitor is empty so $C = C_0$. The charge on the plates is

$$Q = C V = C_0 \, 250 = 3.32 \times 10^{-9} \text{ C}.$$

The energy in the capacitor is

$$U = \frac{1}{2} C V^2 = \frac{1}{2} C_0 \, 250^2 = 4.14 \times 10^{-7} \text{ J}.$$

(b) If the capacitor is dropped in water while connected to the voltage source then the voltage will stay fixed. The electric field will keep the same value.

$$E = \frac{V}{d} = \frac{250}{0.002} = 1.25 \times 10^5 \ \frac{\text{V}}{\text{m}}$$

The capacitance increases by a factor of $\kappa = 80$.

$$C = \kappa C_0 = 80 \, C_0 = 80 \times 1.3275 \times 10^{-11} = \ 1.062 \times 10^{-9} \text{ F}$$

The charge and energy change since C has changed.

$$Q = C V = C \, 250 = 2.65 \times 10^{-7} \text{ C}.$$

$$U = \frac{1}{2} C V^2 = \frac{1}{2} C \, 250^2 = 3.32 \times 10^{-5} \text{ J}.$$

(c) When the capacitor is disconnected from the battery and dropped in water then the charge on the plates will stay constant. The capacitance is the same as in part (b) and the charge is then the sames as part (a)

$$C = 1.062 \times 10^{-9} \text{ F} \ \text{ and } \ Q = 3.32 \times 10^{-9} \text{ C}.$$

The voltage and electric field change.

$$V = \frac{Q}{C} = 3.125 \text{ V} = \ \frac{250 \text{ V}}{80} \ \Longrightarrow \ E = \frac{V}{d} = \frac{3.125}{0.002} = 1560 \ \frac{\text{V}}{\text{m}}$$

The energy also changes

$$U = \frac{1}{2} C V^2 = \frac{Q^2}{2 C} = 5.19 \times 10^{-9} \text{ J}.$$