Chapter E - Problems

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Problem E.1

A wire with diameter d feeds a current to a capacitor. The charge on the capacitor varies with time as $Q(t) = Q_0 \sin \omega t$. What are the current and current density in the wire as functions of time?

Solution to E.1

The current is the the time derivative of the charge.

$$I = \frac{dQ}{dt} = Q_0\, \omega \cos \omega\, t$$

The current density is just the current per area.

$$J = \frac{I}{A} = \frac{Q_0\, \omega \cos \omega\, t}{\pi\, d^2 / 4}$$

Problem E.2

The current as a function of time varies as $I(t) = 15 \sin(120\, \pi\, t)$ in SI units. What is the total charge that flows between 0 and $\frac{1}{120}$ s. (This is half a cycle of a standard 60 Hz AC frequency.)

Solution to E.2

$$Q = \int I(t)\, dt = \int_0^{1/120} 15 \sin(120\, \pi\, t)\, dt = \frac{-15}{120\, \pi} \cos(120\, \pi\, t) \Big|_0^{1/120} = \frac{1}{4\, \pi}\,\text{C} = 0.0796\,\text{C}$$

Problem E.3

The diameter of a standard 12 gauge wire is 2.05 mm. Suppose in standard houshold wiring a 15 A current flows through a 20 m length of copper wire.

(a) What is the resistance in the wire and what is the voltage drop across the wire?

(b) For each copper atom there is one free electron available for conduction. The density (mass/volume) of copper is $8920\,\text{kg} / m^3$ and its atomic mass is 63.54 u. Use the conversion $6.02 \times 10^{26}\,\text{u} = 1\,\text{kg}$ to find the mass of each atom. From that find the number of atoms per volume; this for copper is the same as the number of charge carriers per volume. Calculate the drift velocity of the free electrons and find the total time it takes one electron to migrate the full length of the wire.

Solution to E.3

(a) The resistivity of copper is $1.7 \times 10^{-8}\ \Omega \cdot \text{m}$. The cross-sectional area of the wire is

$$A = \pi\, (0.00205/2)^2 = 3.3006 \times 10^{-6}\, m^2$$

$$R = \frac{\rho\, \ell}{A} = \frac{1.7 \times 10^{-8} \times 20}{A} = 0.10301 = 0.103\,\Omega$$

The voltage drop across a wire is given by Ohm's law.

$$V = I\,R = 15\times 0.10301 = 1.55\text{ V}$$

(b) $I = n\,|q|\,v_d\,A$ q is the charge of the charge carriers, for a metal $q = -e$. A is the cross-sectional area, found above. v_d is the drift velocity and n is the number of charge carriers per volume.

The mass of each atom is:

$$m_{\text{atom}} = 63.54\text{ u}\times\frac{1\text{ kg}}{6.02\times 10^{26}\text{ u}} = 1.0555\times 10^{-25}\text{ kg}$$

Since there is one electron per atom of copper that contributes to conduction, n is the same as the number of atoms per volume. Since the density is the mass per volume we can write: $n = \rho/m_{\text{atom}}$.

$$n = \frac{\text{free electrons}}{\text{volume}} = \frac{\text{free electrons}}{\text{atom}}\times\frac{\text{atoms}}{\text{volume}} = 1\times\frac{\rho}{\text{m}_{\text{atom}}} = \frac{8920\text{ kg/m}^3}{1.0555\times 10^{-25}} = \frac{8.4511\times 10^{28}}{m^3}$$

$$I = n\,|q|\,v_d\,A \implies v_d = \frac{I}{n\,e\,A} = \frac{15}{n\,1.60\times 10^{-19}\,A} = 3.3609\times 10^{-4} = 3.36\times 10^{-4}\ \frac{\text{m}}{\text{s}}$$

The time to travel the length of the wire is given by $\ell = v_d\,t$

$$t = \frac{\ell}{v_d} = \frac{20}{v_d} = 59\,500\text{ s} = 16.5\text{ hr}$$

Note that the electrical signal travels *much* faster than that; that speed is on the order of the speed of light.

Problem E.4

Suppose a wire with restance R is stretched by 30% keeping the volume of the wire fixed. What is the new resistance of the wire?

Solution to E.4

If a wire is stretched its volume stays constant so that $A\,\ell = A'\,\ell'$. Thus $\ell' = 1.30\,\ell$ and

$$\frac{A}{A'} = \frac{\ell'}{\ell} = 1.30 \implies \frac{A'}{A} = \frac{1}{1.30}$$

Since $R = \rho\,\ell/A$ we get the ratio:

$$\frac{R'}{R} = \frac{\ell'/\ell}{A'/A} = \frac{1.30}{1/1.30} = 1.30^2 = 1.69 \implies R' = 1.69\,R$$

Problem E.5

When the temperature of a wire is decreased by 10 C° its resistance decreases by 6 %. What is its temperature coefficient?

Solution to E.5

$$\Delta R = -0.06\,R_0 \implies \frac{\Delta R}{R_0} = \alpha\,\Delta T \implies -0.06 = \alpha\,(-10) \implies \alpha = 0.006/\text{C}°$$

Problem E.6

The precise definition of the temperature coefficient is $\alpha = \dfrac{1}{\rho}\dfrac{d\rho}{d\text{T}}$.

Show that if α is constant we get: $\rho = \rho_0\,e^{\alpha\,(\text{T}-\text{T}_0)}$.

Use the approximation that for small x ($x \ll 1$), $e^x \cong 1 + x$, to show that for small $\Delta T = T - T_0$:

$$\rho = \rho_0\,e^{(\alpha\,\Delta\text{T})} \cong \rho_0\,[1 + \alpha\,(T - T_0)].$$

Solution to E.6

$$\alpha = \frac{1}{\rho}\frac{d\rho}{d\mathrm{T}} = \frac{d\ln\rho}{d\mathrm{T}} \implies d\ln\rho = \alpha\, dT$$

Integrating this and using $\rho = \rho_0$ at $\mathrm{T} = T_0$ gives

$$\int_{\rho_0}^{\rho} d\ln\rho = \int_{T_0}^{T} \alpha\, dT \implies \ln\rho - \ln\rho_0 = \alpha\,(\mathrm{T} - T_0)\,.$$

Exponentiating this gives:

$$\rho = \rho_0\, e^{\alpha\,(\mathrm{T}-\mathrm{T}_0)}.$$

If $\Delta T = T - T_0$ is small then $x = \alpha\,(T - T_0)$ is small and we can sue the expansion.

$$\rho = \rho_0\, e^{\alpha\,(T-T_0)} \cong \rho_0\,[1 + \alpha\,(T - T_0)]$$

Problem E.7

A hair dryer desiigned for a standard US outlet has a power rating of 1500 W.
(a) How much current does it draw?
(b) What is its resistance?

Solution to E.7

For a standard outlet we have $V_{\mathrm{rms}} = 120$ V. This is called the root-mean-squared voltage. We will discuss this in detail in the AC circuit chapter. For now we just need to know that for a purely resistive AC circuit we can treat it like DC if we use the rms voltage and current and the average power.

$$V = 120\text{ V} \;\text{ and }\; \mathcal{P} = 1500\text{ W}$$

(a) $\mathcal{P} = V\,I \implies I = \frac{\mathcal{P}}{V} = \frac{1500}{120} = 12.5\text{ A}$

(b) $\mathcal{P} = \frac{V^2}{R} \implies R = \frac{V^2}{\mathcal{P}} = \frac{120^2}{1500} = 9.6\,\Omega$

Problem E.8

Suppose a purely resistive Ohmic device designed for a standard outlet ($V = 120$ V) is operated at too low a voltage, 105 V. By what percent would the power of the device be decreased?

Solution to E.8

$$\mathcal{P} = \frac{V^2}{R} \implies \frac{\mathcal{P}'}{\mathcal{P}} = \left(\frac{V'}{V}\right)^2 = \left(\frac{105}{120}\right)^2 = 0.766$$

The power decreases by 23.4 %.

Problem E.9

kW·hr is the unit of energy used in electric bills. What is 1 kW·hr in J? If the electric utility charges \$0 .09 per kW·hr, then what is the cost of running a 100 W bulb constantly for a 31 day month?

Solution to E.9

$$\text{kW}\cdot\text{h} = 1000\,W \times 3600\,s = 3.6 \times 10^6\,J$$

Do the calculation of cost using just kW·h instead of J. For a 100 W bulb we have Power = 0.10 kW.

$$\text{Energy} = \text{Power} \times \text{time} = 0.10\,\text{kW} \times 31\text{ days} \times \frac{24\text{ hr}}{\text{day}} = 74.4\,\text{kW}\cdot\text{hr}$$

$$\text{Cost} = \text{Energy} \times \text{Rate} = 74.4\,\text{kW}\cdot\text{hr} \times \frac{\$0.09}{\text{kW}\cdot\text{hr}} = \$6.70$$

Problem E.10

A student tries to determine the internal resistance of a battery. A voltmeter connected across the battery without a load reads the EMF $\mathcal{E}$. When the battery is connected across a known load resistance R_0, the voltmeter reads across the battery reads V_0. What is the internal resistance r?

Solution to E.10

$$V_t = \mathcal{E} - I\,r$$

The voltmeter across the battery reads it's terminal voltage $V_t = V_0$. This is across the resistance R_0 giving the current.

$$V_0 = I\,R_0 \implies I = \frac{V_0}{R_0}$$

$$V_t = \mathcal{E} - I\,r \implies V_0 = \mathcal{E} - \frac{V_0}{R_0}\,r \implies r = R_0\left(\frac{\mathcal{E}}{V_0} - 1\right)$$

Problem E.11

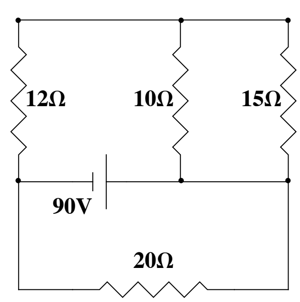
An unknown resistance R is connected across a fixed voltage source. When a 200 Ω resistor is placed in parallel with it the current delivered by the source quadruples. What is the resistance?

Solution to E.11

Quadrupling the current at a fixed voltage implies 1/4 the resistance.

$$R_{\text{eq}} = \frac{R}{4} = \left(\frac{1}{R} + \frac{1}{200}\right)^{-1} \implies \frac{4}{R} = \frac{1}{R} + \frac{1}{200} \implies R = 600\,\Omega$$

Problem E.12



Complete the table below with the voltage across, the current through and the total power dissipated in each resistor.

	12 Ω	10 Ω	15 Ω	20 Ω
V				
I				

$\mathcal{P}$				

Also, give the total current and power delivered by the battery. How are these values related to those in the table?

Solution to E.12

Once one element in a column is known the other two can easily be found using $V = I\,R$ and $\mathcal{P} = V\,I$.

The $20\,\Omega$ resistor is connected across the 90 V source.

$$V_{20} = 90 \implies I_{20} = \frac{V_{20}}{R_{20}} = \frac{90}{20} = 4.5 \implies \mathcal{P}_{20} = V_{20}\,I_{20} = 90 \times 4.5 = 405$$

The equivalent resistance of the other three resistors is: $12 + \left(\frac{1}{10} + \frac{1}{15}\right)^{-1} = 18$. All of the current through this equivalent flows through the $12\,\Omega$ resistor.

$$I_{12} = I_{18} = \frac{90}{18} = 5 \implies V_{12} = I_{12}\,R_{12} = 5 \times 12 = 60 \implies \mathcal{P}_{12} = V_{12}\,I_{12} = 60 \times 5 = 300$$

The voltages across the other two parallel resistors are equal

$$V_{10} = V_{15} = 90 - V_{12} = 90 - 60 = 30 \implies I_{10} = \frac{V_{10}}{R_{10}} = \frac{30}{10} = 3 \text{ and } I_{15} = \frac{V_{15}}{R_{15}} = \frac{30}{15} = 2$$

$$\implies \mathcal{P}_{10} = V_{10}\,I_{10} = 30 \times 3 = 90 \text{ and } \mathcal{P}_{15} = V_{15}\,I_{15} = 30 \times 2 = 60$$

	12 Ω	**10 Ω**	**15 Ω**	**20 Ω**
V	60 V	30 V	30 V	90 V
I	5 A	3 A	2 A	4.5 A
$\mathcal{P}$	300 W	90 W	60 W	405 W

The overall equivalent is: $R_{\text{eq}} = \left(\frac{1}{18} + \frac{1}{20}\right)^{-1} = \frac{180}{19} = 9.4737$ The total current delivered by the battery is $I_{\text{tot}} = \frac{V_{\text{tot}}}{R_{\text{eq}}} = \frac{90}{180/19} = 9.5\,A$ and the total power delivered by the battery is

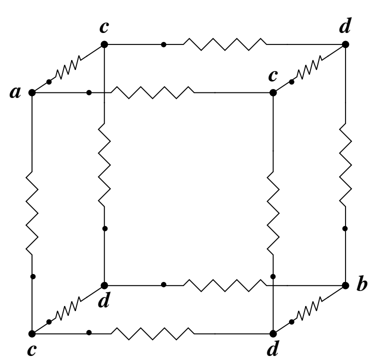
$$\mathcal{P}_{\text{tot}} = V_{\text{tot}}\,I_{\text{tot}} = 90 \times 9.5 = 855\,W.$$

Note that the total current is the sum of the currents through the two branches $9.5 = 5 + 4.5$ and the total power is the sum of all powers $855 = 300 + 90 + 60 + 405$.

Problem E.13

Suppose a resistance R is put on each edge of a cube. What is the equivalent resistance between opposite corners of the cube?

Solution to E.13

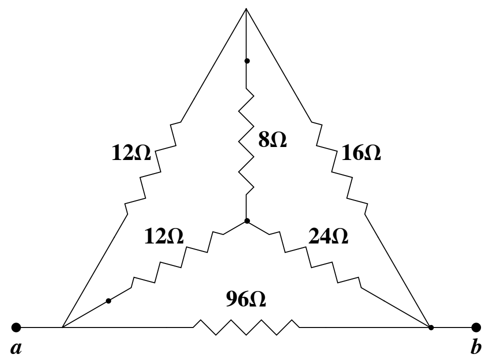


Using symmetry we can identify corners as having the same voltage. Nodes labelled by c are one resistor away from node a and must have the same voltage. Similarly nodes labelled by d should have the same voltage. We may then, since they are at the same voltage, short out the c nodes and short out the d nodes. There are three resistors between a and c; these are in parallel and thus have an equivalent of $R/3$. There are six resistors between c and d, which thus have an equivalent of $R/6$. The threee between d and b give $R/3$. The resulting three resistors are in series giving an overall equivalent of:

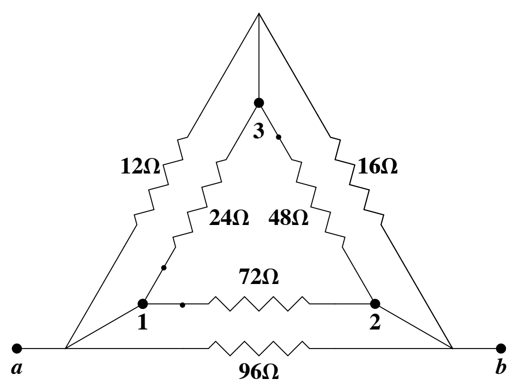
$$R_{eq} = \frac{R}{3} + \frac{R}{6} + \frac{R}{3} = \frac{5}{6} R$$

Problem E.14

What is the equivalent resistance between a and b?



Solution to E.14



There are no resistors in this network that are either in series or parallel, so we must use the node reduction formula. Applying this to the three internal resistors we get:

$$R_{\shortparallel} = \left(\frac{1}{12} + \frac{1}{24} + \frac{1}{8}\right)^{-1} = 4$$

$$R'_{12} = \frac{R_1 R_2}{R_{\shortparallel}} = \frac{12\times 24}{4} = 72$$

$$R'_{13} = \frac{R_1 R_3}{R_{\shortparallel}} = \frac{12\times 8}{4} = 24$$

$$R'_{23} = \frac{R_2 R_3}{R_{\shortparallel}} = \frac{24\times 8}{4} = 48$$

Now identify that the $24\,\Omega$ and $12\,\Omega$ resistors are in parallel as are the $48\,\Omega$ and $16\,\Omega$ resistors. The resulting two resistors are in series giving:

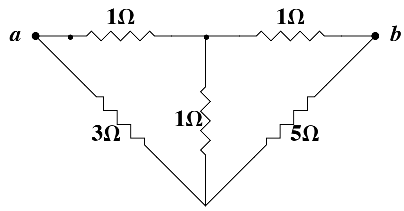
$$\left(\frac{1}{24} + \frac{1}{12}\right)^{-1} + \left(\frac{1}{48} + \frac{1}{16}\right)^{-1} = 8 + 12 = 20\,.$$

The $20\,\Omega$, $72\,\Omega$ and $96\,\Omega$ resistors are then in parallel:

$$R_{eq} = \left(\frac{1}{20} + \frac{1}{72} + \frac{1}{96}\right)^{-1} = 13.46\,\Omega\,.$$

Problem E.15

What is the equivalent resistance between nodes *a* and *b*?

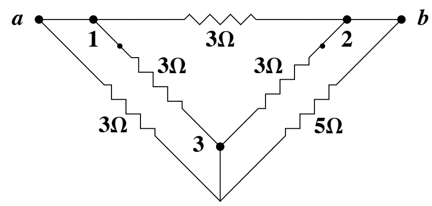


Solution to E.15

This is an example where no two resistors are in series or parallel so we must use the node-reduction method.

The three $1\,\Omega$ resistors are the easiest to eliminate (instead of the $1\,\Omega$, $3\,\Omega$ and $5\,\Omega$) because the new resistors are equal.

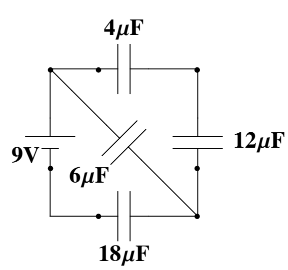
$$R_{\parallel} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1} = \frac{1}{3} \implies R'_{ij} = \frac{R_i R_j}{R_{\parallel}} = \frac{1 \times 1}{1/3} = 3$$



Combine the pair of $3\,\Omega$ resistors in the lower left in parallel and also combine the $3\,\Omega$ and $5\,\Omega$ resistors in the lower right in parallel. The resulting two resistors are then in series and this is then in parallel with the $3\,\Omega$ at the top.

$$\left(\frac{1}{3} + \frac{1}{3}\right)^{-1} + \left(\frac{1}{3} + \frac{1}{5}\right)^{-1} = \frac{27}{8} \implies R_{eq} = \left(\frac{1}{3} + \frac{1}{27/8}\right)^{-1} = \frac{27}{17}\,\Omega\,.$$

Problem E.16



(a) What is the equivalent capacitance across the 9 V source?

(b) Complete the table below with the voltage across each capacitor and the charge on each.

	18 μF	6 μF	4 μF	12 μF
V				
Q				

Solution to E.16

(a) The 4 μF and 12 μF capacitors are in series, so their equivalent is: $\left(\frac{1}{4}+\frac{1}{12}\right)^{-1}=3\ \mu\text{F}$.

This is in parallel with the 6 μF capacitor, giving $3+6=9\ \mu$F.

This, in turn, is in series with the 18 μF giving $C_{eq}=\left(\frac{1}{9}+\frac{1}{18}\right)^{-1}=6\ \mu\text{F}$.

(b) The charge on the 18 μF is the same as on the equivalent. This is the charge delivered by the battery. $Q_{18}=Q_{eq}=C_{eq}\,V_{tot}=6\times 9=54\ \mu\text{C}$.

The voltage across it is $V_{18}=\frac{Q_{18}}{C_{18}}=\frac{54}{18}=3\ V$.

The voltage across the 6 μF capacitor and the equivalent 3 μF capacitor is: $V_3=V_6=V_{tot}-V_{18}=9-3=6\ V$.

The charge on the 6 μF capacitor is: $Q_6=C_6\,V_6=6\times 6=36\ \mu\text{C}$

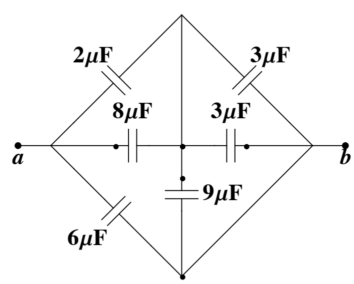
The charges on the 4 μF and 12 μF capacitors are equal to that on their equivalent: $Q_4=Q_{12}=Q_3=C_{eq}\,V_3=3\times 6=18\ \mu\text{C}$.

The voltages across the 4 μF and 12 μF capacitors can now be found: $V_4=\frac{Q_4}{C_4}=\frac{18}{4}=4.5$ V and $V_{12}=\frac{Q_{12}}{C_{12}}=\frac{18}{12}=1.5$ V

	18 μF	**6 μF**	**4 μF**	**12 μF**
V	3 V	6 V	4.5 V	1.5 V
Q	54 μC	36 μC	18 μC	18 μC

Problem E.17

What is the equivalent capacitance between a and b?

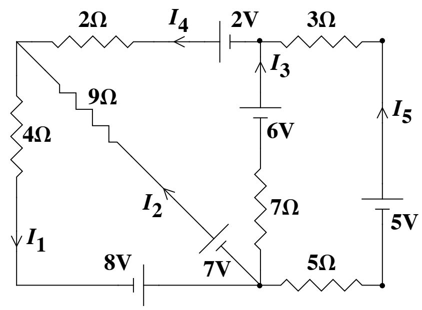


Solution to E.17

A node is a point of constant voltage in a circuit. The two wires without components are nodes. To simplify this circuit shrink these wires to a point. It follows, then, that the 2 μF and 8 μF capacitors are in parallel (giving 2 + 8 = 10), as are the 3 μF, 3 μF and 9 μF (giving 3 + 3 + 9 = 15). The two equivalents are then in series $\left(\frac{1}{10}+\frac{1}{15}\right)^{-1}=6$ and this is in parallel with the 6 μF capacitor. The overall equivalent is then: $C_{eq}=6+6=12\ \mu\text{F}$.

Problem E.18

Give a set of 5 linear equations that can be solved for the currents. You need not solve the equations.



Solution to E.18

The junction rule gives:

$$I_1 = I_2 + I_3 + I_5$$
$$I_3 + I_5 = I_4$$
$$I_2 + I_4 = I_1$$

Recall that any one of the junction rule equations is the sum of the others, so to get an independent set eliminate one of these three equations.

The loop rule gives:

$$7 - 9\,I_2 - 4\,I_1 + 8 = 0$$
$$2 - 2\,I_4 + 9\,I_2 - 7 - 7\,I_3 + 6 = 0$$
$$-6 + 7\,I_3 - 5\,I_5 + 5 - 3\,I_5 = 0$$