Chapter F - Problems

Blinn College - Physics 2426 - Terry Honan

Problem F.1

An electron moves in a region where the magnetic field points to the north. What is the direction of the force on an electron that moves
(a) to the east,
(b) to the south,
(c) upward and
(d) to the northwest.

Solution to F.1

The force on a charge Q moving with a velocity $\vec{v}$ in a magnetic field $\vec{B}$ is

$$\vec{F} = Q\,\vec{v}\times\vec{B}.$$

Use the right hand rule to get the direction of the cross product. Put your thumb in the direction of the first entry, the velocity, and your fingers in the direction of the second entry, the field. Your palm points in the direction of the cross product $\vec{v}\times\vec{B}$. The direction of the force will be in this direction when the charge is positive and opposite when the charge is negative. In this case, since the electron is negatively charged, the field is opposite the direction of the cross product.

(a) downward (b) no force (c) east (d) upward

Problem F.2

An electron with a speed of $5\times10^{6}\ m/s$ experiences an acceleration of magnitude $2\times10^{18}\ \mathrm{m/s^2}$ in a magnetic field of strength 2.6 T. What is the angle between the velocity and magnetic field?

Solution to F.2

$$\vec{F} = Q\,\vec{v}\times\vec{B} = m\,\vec{a} \implies |Q|\,v\,B\sin\theta = m\,a$$
$$\implies \sin\theta = \frac{m\,a}{e\,v\,B} = \frac{9.11\times10^{-31}\times2\times10^{18}}{1.6\times10^{-19}\times5\times10^{6}\times2.6} = 0.71172$$
$$\implies \theta = 61.2\,° \text{ or } 118.8\,°$$

Problem F.3

An electron moves in the negative x direction at $5\times10^{6}\ m/s$ in a magnetic field of $\langle -4, -5, 3\rangle$ mT. What is the force on the electron? What is magnitude of this force?

Solution to F.3

$$\begin{aligned}\vec{F} &= Q\,\vec{v}\times\vec{B} = -e\,\vec{v}\times\vec{B}\\ &= -1.6\times10^{-19}\left(-5\times10^{6}\,\hat{x}\right)\times\left(-4\,\hat{x}-5\,\hat{y}+3\,\hat{z}\right)10^{-3}\\ &= 8\times10^{-16}\left(-4\,\vec{0}-5\,\hat{z}+3\left(-\hat{y}\right)\right)=\langle 0-2.4,\,4\rangle\times10^{-15}\ \mathrm{N}\end{aligned}$$

$$F=\sqrt{F_x^2+F_y^2+F_z^2}=\sqrt{0+2.4^2+4^2}\times10^{-15}=4.66\times10^{-15}\ \mathrm{N}$$

Problem F.4

The earth's magnetic field at some postition has a component to the north of B. A horizonntal wire with a mass per length given by μ runs in the east-west direction.
(a) In which direction (east or west) should the current be in so that the magnetic force is upward?
(b) What must the current through the wire be so that the magnetic force cancels the weight of the wire?
(c) If the strength of the earth's field at some position is 5×10^{-5} T then find the value of the current in part (b) for a copper wire with a 2 mm diameter. The density of copper is $\rho = 8900\ \mathrm{kg}/m^3$.

Solution to F.4

(a) $\vec{F}=I\,\vec{\ell}\times\vec{B}$ If your fingers point north and your palm points up then the thumb is to the east. That is the direction of the current.

(b) $\vec{F}_{\text{mag}}=I\,\vec{\ell}\times\vec{B}$ aacting upward will cancel $\vec{F}_{\text{grav}}=m\,\vec{g}$ which acts downward. Equating the magnitudes gives:

$$I\,\ell\,B=m\,g$$

The mass of a segment of length ℓ is $m=\mu\,\ell$. This gives a current of

$$I=\frac{\mu\,g}{B}$$

(c) The density ρ and cross-sectional area of the wire A gives us μ, the mass per length. The mass is the volume of the wire $A\,\ell$ multiplied by the density $m=\rho\,A\,\ell$. Divide by ℓ to get μ.

$$\mu=\rho\,A=\rho\,\pi\,r^2=8900\times\pi\,0.001^2=0.02796$$

This gives a huge current.

$$I=\frac{\mu\,g}{B}=\frac{0.02796\times9.80}{5\times10^{-5}}=5480\ \mathrm{A}$$

Problem F.5

A bar magnet is rotated in a magnetic field of strength 0.030 T until the maximum torque is found. If the value of the maximum torque is $8\times10^{-4}\ N\cdot m$ then what is the magnetic moment of the dipole?

Solution to F.5

The angle of maximum torque is $\theta = 90\,°$.

$$\vec{\tau} = \vec{\mu} \times \vec{B} \implies \tau = \mu B \sin\theta \implies \tau_{\max} = \mu B$$

$$\implies \mu = \frac{\tau_{\max}}{B} = \frac{8\times10^{-4}}{.030} = 0.0267\ \text{A}\cdot\text{m}^2$$

Problem F.6

An electron is shot with a horizontal initial velocity in an upward uniform magnetic field of 1.5 mT. It moves in a circle in the field.
(a) Does it move clockwise or counterclockwise?
(b) How long does each orbit take?
(c) If the radius of the circle is 1.3 cm then what is the speed of the electron?

Solution to F.6

(a) At some instant call the direction of the velocity vector forward. Put your thumb forward and your fingers upward, in the direction of the field. Your palm points to the right,so the force on the negatively charged electron is to the left. Since the force is directed toward the center of the circle the motion is counterclockwise.

(b) Applying the second law, using the expression $a_c = v^2/r$ for the centripetal acceleration gives.

$$F_{\text{net}} = m\,a \implies |Q|\,v\,B = m\frac{v^2}{r} \implies r = \frac{m\,v}{|Q|\,B} = \frac{m\,v}{e\,B}$$

The angular frequency is

$$\omega = \frac{v}{r} = \frac{e\,B}{m}.$$

The period is related to the angular frequency

$$T = \frac{2\pi}{\omega} = \frac{2\pi\,m}{e\,B} = \frac{2\pi\,9.11\times10^{-31}}{1.60\times10^{-19}\times0.0015} = 2.38\times10^{-8}\ \text{s}$$

(c) Using the formula for r we can find v.

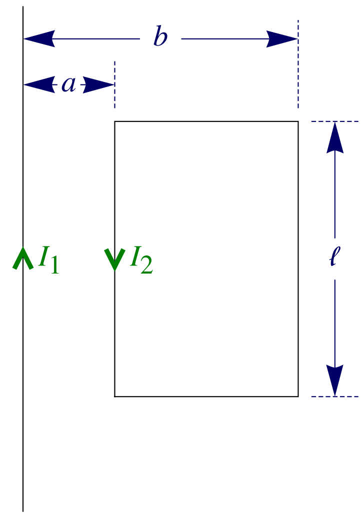
$$r = \frac{m\,v}{e\,B} \implies v = \frac{r\,e\,B}{m} = \frac{0.013\times1.60\times10^{-19}\times0.0015}{9.11\times10^{-31}} = 3.42\times10^{6}\ \frac{\text{m}}{\text{s}}$$

Problem F.7

Consider a long wire with a current I_1 and a loop with a current I_2 as shown. What is the force on the loop due to the long wire? If the values of the currents and distances are

$$I_1 = 20\ \text{A},\ I_2 = 8\ \text{A},\ a = 2\ \text{cm},\ b = 5\ \text{cm and}\ \ell = 2\ \text{m}$$

then what is the value of the net force?



Solution to F.7

We must first find the force on a segment of length ℓ due to a long wire a distance r away where I_1 and I_2 are the parallel currents in the long wire and segment and loop, respectively.

$B_1 = \frac{\mu_0 I_1}{2\pi r}$ is the field due to the long wire at the segment. The direction is into the page by the right hand rule.

$F_{21} = I_2 \ell B_1 = \frac{\mu_0}{2\pi} I_1 I_2 \frac{\ell}{r}$ is the force on the segment due to the long wire. The direction of the force on a segment is found by putting your thumb in the direction of the current and fingers in the direction of the field. This gives a force away from the long wire at $r = a$ and toward at $r = b$. Note that the perpendicular segments cancel each other and are neglected.

Combining these results we get a force away from the long wire, since the field at a is stronger, and a value of

$$F_{21} = \frac{\mu_0}{2\pi} I_1 I_2 \left(\frac{\ell}{a} - \frac{\ell}{b}\right).$$

$$I_1 = 20\text{ A}, I_2 = 8\text{ A}, a = 0.02\text{ m}, b = 0.05\text{ m and } \ell = 2\text{ m}$$
$$\Longrightarrow \quad F_{21} = 0.00192\text{ N}$$

Problem F.8

A Helmholtz coil is an apparatus connsisting of two identical flat circular coils of radius R, each with N turns. Take the coils to both be parallel to the xy plane and centered at the z axis at $z = -z_0$ and $z = +z_0$. If both coils have counterclockwise currents of I, then what is the field as a function of position along the z axis?

Solution to F.8

In the notes we derived an expression for the magnetic field a perpendicular distance z_0 above the center of a single circular conducting loop of radius R in the xy-plane. The current I is counterclockwise when viewed from above, where above means the positive z direction. The derived express was

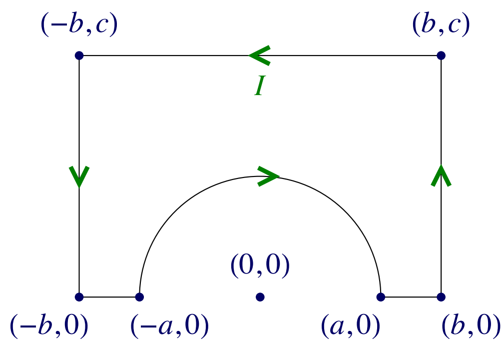
$$\vec{B} = \hat{z}\,\frac{\mu_0\,I}{2}\,\frac{R^2}{\left(R^2+z_0^2\right)^{3/2}}.$$

Here we want the field at the origin due to N turns centered at $-z_0$ and N turns centered at z_0. The above expression gives the field for each loop at $-z_0$. It also gives the field for each loop at z_0. There are then a total of $2\,N$ loops. This gives the total field of

$$\vec{B} = \hat{z}\,\mu_0\,N\,I\,\frac{R^2}{\left(R^2+z_0^2\right)^{3/2}}.$$

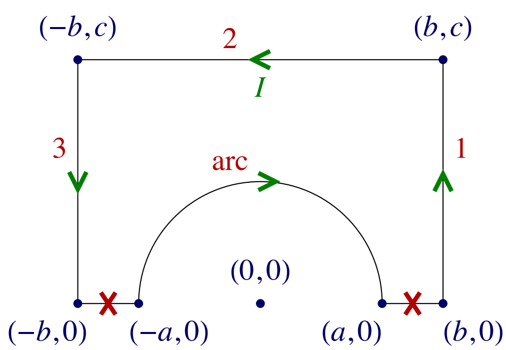
Problem F.9

What is the magnetic field at the origin, (0,0)? GIve both its magnitude and direction.



Solution to F.9

Here we have 5 segments and an arc. Two of the segments, shown as crossed out, do not contribute; in the Biot-Savart law any segment directed at or away from the point P doesn't contribute. The other three segments are labelled 1, 2 and 3.



The field due to the arc is directed into the page. This is found with the right hand rule; wrap your fingers around in the direction of the arc and then the thumb points into the page.
The field due to each of the three segments is directed out of the page. Using the right hand rule again, put your thumb in the direction of the current and your finngers wrap around in the direction of the field; at the point $P = (0, 0)$ this is out of the page.
The field due to the arc will dominate the segments. It is closer and the current has the same magnitude. The direction of the net field will then be into the page. When adding vectors in opposite directions we subtract their magnitudes so we can write the total field as:

$$B_{\text{tot}} = B_{\text{arc}} - B_1 - B_2 - B_3.$$

The angle for the arc is $\theta = 180\,° = \pi$ and the radius is $R = a$ so we get:

$$B_{\text{arc}} = \frac{\mu_0\,I}{4\,\pi\,R}\,\theta = \frac{\mu_0\,I}{4\,a}$$

For a segment we have

$$B_{\text{seg}} = \frac{\mu_0 I}{4\pi a}(\sin\phi_2 - \sin\phi_1),$$

where a is the perpendicular distance from the point P to the line containing the segment. Take the positive x direction to be the direction of the current and $x = 0$ the point on the line closest to P. The segment is from x_1 to x_2. ϕ is the angle associcated with x. The sign of ϕ follows from the sign of x.

For segment 1: $a \to b$, $\phi_1 = 0$ and $\sin\phi_2 = \frac{c}{\sqrt{b^2+c^2}}$.

$$B_1 = \frac{\mu_0 I}{4\pi b}(\sin\phi_2 - \sin\phi_1) = \frac{\mu_0 I}{4\pi b}\left(\frac{c}{\sqrt{b^2+c^2}} - 0\right) = \frac{\mu_0 I}{4\pi\sqrt{b^2+c^2}}\,\frac{c}{b}$$

For segment 2: $a \to c$, $\sin\phi_1 = \frac{-b}{\sqrt{b^2+c^2}}$ and $\sin\phi_2 = \frac{b}{\sqrt{b^2+c^2}}$.

$$B_2 = \frac{\mu_0 I}{4\pi c}(\sin\phi_2 - \sin\phi_1) = \frac{\mu_0 I}{4\pi c}\left(\frac{b}{\sqrt{b^2+c^2}} - \frac{-b}{\sqrt{b^2+c^2}}\right) = \frac{\mu_0 I}{4\pi\sqrt{b^2+c^2}}\,2\,\frac{b}{c}$$

For segment 3: $a \to b$, $\sin\phi_1 = \frac{-c}{\sqrt{b^2+c^2}}$ and $\phi_2 = 0$.

$$B_3 = \frac{\mu_0 I}{4\pi b}(\sin\phi_2 - \sin\phi_1) = \frac{\mu_0 I}{4\pi b}\left(0 - \frac{-c}{\sqrt{b^2+c^2}}\right) = \frac{\mu_0 I}{4\pi\sqrt{b^2+c^2}}\,\frac{c}{b}$$

Putting this all together we get

$$B_{\text{tot}} = B_{\text{arc}} - B_1 - B_2 - B_3 = \frac{\mu_0 I}{4a} - \frac{\mu_0 I}{2\pi\sqrt{b^2+c^2}}\left(\frac{b}{c} + \frac{c}{b}\right)$$

The direction is *into the page*.

Problem F.10

Consider flat $\ell \times \ell$ square coil with N turns that sits in the xy plane and with a clockwise (when viewed from above) current I? (Take the positive z direction to be up.)
(a) What is the magnetic field at the center?
(b) If this sits in an external magnetic field of magnitude B in the y direction the what is the torque on it?

Solution to F.10

(a) The field for N turns is simple N times the field for a single loop. For each square loop there are four segments contributing equal fields.

$$B = N\,B_{\text{loop}} \text{ and } B_{\text{loop}} = 4\,B_{\text{seg}} \implies B = 4\,N\,B_{\text{seg}}$$

Here we have enough information to find the direction of the field also. A clockwise current in the xy plane gives a field in the negative z direction.

$$\vec{B} = -B\,\hat{z}$$

We now need to concentrate on the field due to just one segment.

$$B_{\text{seg}} = \frac{\mu_0 I}{4\pi a} (\sin\phi_2 - \sin\phi_1),$$

For an $\ell \times \ell$ square the distance from the center to the center of one side is

$$a = -\ell/2$$

The angle varies from $-45°$ to $45°$.

$$\phi_1 = -45° \text{ and } \phi_2 = 45° \implies \sin\phi_2 - \sin\phi_1 = \frac{1}{\sqrt{2}} - \frac{-1}{\sqrt{2}} = \sqrt{2}$$

It follows that the field becomes

$$B_{\text{seg}} = \frac{\mu_0 I}{2\pi\ell}\sqrt{2} \implies \vec{B} = -\hat{z}\, 4\, N\, B_{\text{seg}} = -\hat{e}_z \frac{\mu_0 N I}{\pi\ell} 2\sqrt{2}$$

(b) The magnetic moment of an N turn coil is $\vec{\mu} = N I \vec{A}$. Here $\vec{A} = A\,\hat{n}$ where A is the area of the loop and $\hat{n}$ is the unit normal to the surface. To get the direction use the right hand rule. Wrap your fingers in the direction of the current and your thumb gives the direction of $\hat{n}$ and thus of $\vec{\mu}$. Here we get a moment in the negative z direction.

$$\vec{\mu} = N I \vec{A} = -\hat{z}\, N I \ell^2$$

In a magnetic field of $\vec{B} = B\,\hat{e}_y$ this coil exeriences a torque.

$$\vec{\tau} = \vec{\mu}\times\vec{B} = -N I \ell^2 B\, \hat{z}\times\hat{y} = N I \ell^2 B\, \hat{x}$$

Problem F.11

Consideer a long hollow wire with an inside radius a and outside radius b.
(a) If the wire has a uniform current density J then what is the magnetic field strength as a function of r, the distance from the central axis?
(b) Suppose instead of being given the current density J you were given the current I. Write the field as a function of position in terms of I.

Solution to F.11

(a) For any case with cylindrical symmetry we choose the contour for Ampere's law to be a circle of radius r. Integrating around the contour gives $B\, 2\pi r$.

$$\oint \vec{B}\cdot d\vec{r} = \mu_0 I_{\text{inside}} \implies B = \frac{\mu_0 I_{\text{inside}}}{2\pi r}$$

Consider the case of $r < a$ first. All the current is between radius a and radius b so there is on current inside a contour of radius $r < a$.

$$\text{For } r < a\text{: } I_{\text{inside}} = 0 \implies B = 0$$

The case of $a < r < b$ is more difficult. The total cross-sectional area of the wire is:

$$A_{\text{total}} = \pi\left(b^2 - a^2\right).$$

This is the area of a disk of radius b with the area of a disk of radius a removed. For the case between a and b the area we need is between a and r. Call this A_{inside}

$$A_{\text{inside}} = \pi\left(r^2 - a^2\right).$$

Generally, for a uniform current density we can write $I = J\,A$.

$$\text{For } a < r < b\text{:}\;\; I_{\text{inside}} = J\,A_{\text{inside}} = J\,\pi\left(r^2 - a^2\right) \implies B = \frac{\mu_0\,J}{2\,r}\left(r^2 - a^2\right)$$

Outside the wire ($r > b$) the total current is inside the contour.

$$\text{For } r > b\text{:}\;\; I_{\text{inside}} = J\,A_{\text{total}} = J\,\pi\left(b^2 - a^2\right) \implies B = \frac{\mu_0\,J}{2\,r}\left(b^2 - a^2\right)$$

(b) In terms of the current I instead of J we get simple results for inside and outside the wire.

$$\text{For } r < a\text{:}\;\; I_{\text{inside}} = 0 \implies B = 0$$

$$\text{For } r > b\text{:}\;\; I_{\text{inside}} = I \implies B = \frac{\mu_0\,I}{2\,\pi\,r}$$

For the case between $a$and b the I_{inside} is the same fraction of the total current I as the fraction of the area.

$$\text{For } a < r < b\text{:}\;\; I_{\text{inside}} = I\,\frac{A_{\text{inside}}}{A_{\text{total}}} = I\,\frac{r^2 - a^2}{b^2 - a^2} \implies B = \frac{\mu_0\,I}{2\,\pi\,r}\,\frac{r^2 - a^2}{b^2 - a^2}$$

Problem F.12

A solenoid has a circular cross-section with a 3 cm radius, a length of 80 cm and 300 turns. It carries a curent of 5 A.
(a) What is the magnetic field strength inside the solenoid?
(b) What is the magnitude of the solenoid's magnetic moment?

Solution to F.12

(a) The field inside a solenoid is $B = \mu_0\,n\,I$ where n is the number of turns per length.

$$B = \mu_0\,n\,I = 4\,\pi \times 10^{-7}\,\frac{300}{0.80}\,5 = 2.36\text{ mT}$$

(b) The magnetic moment of a single loop is $\vec{\mu} = I\,\vec{A}$. Here $\vec{A} = A\,\hat{n}$ where A is the area of the loop and $\hat{n}$ is the unit normal to the surface. For a coil with N turns then $\vec{\mu} = N\,I\,\vec{A}$. For the solenoid the magnitude of the magnetic moment is

$$\mu = N\,I\,A = 300 \times 5 \times \pi\,0.03^2 = 4.24\text{ A}\cdot\text{m}^2$$