Chapter G - Problems

Blinn College - Physics 2426 - Terry Honan

Problem G.1

A plane flies horizonally at a speed of $280\,m/s$ in a position where the earth's magnetic field has a magnitude 6.0×10^{-5} T and is directed at an angle of $50\,°$ below horizontal. If the wingspan of the plane is 55 m then what is the induced voltage between the tips of the wings if the plane flies to the north? Which wing tip is at higher voltage, the right or the left? How would this result change if the plane flew to the east instead?

Solution to G.1

Because the wings are horizontal and the velocity is horizontal, it is the vertical component of the magnetic field that matters. The answer is the same if the plane flies to the north or east.

$$\mathcal{E} = B_{\text{vertical}}\,\ell\,v = \left(6.0\times 10^{-5}\times \sin 50°\right)\times 55\times 280 = 0.708\text{ V}$$

To get the direction use the right hand rule to find the direction of the force on a positive test charge moving in the direction of the velocity. PUt your thumb in the direction of the plane's velocity (forward relative to the plane, of course) and put your fingers downward. Positive charges are pushed to the left, so the left wing is positively charged and thus the higher voltage.

Problem G.2

A 1.6 m long conducting rod is oriented in the east-west direction while it is slid to the north at $3\,m/s$ along parallel conducting rails separated by 1.6 m. A stationary $5\,\Omega$ resistance sits between the rails and there is a downward uniform magnetic field of 0.30 T.
(a) What is the current through the resistor?
(b) What is the rate that heat is lost in the resistor? (This is the power dissipated in the resistor.)
(c) What force is needed to move the rod at that constant speed?
(d) At what rate is work being done to slide the rod at that constant speed

Solution to G.2

(a) It is an example of motional EMF. That combined with Ohm's law gives the current.

$$\mathcal{E} = B\,\ell\,v = 0.30\times 1.6\times 3 = 1.44\,V \implies I = \frac{\mathcal{E}}{R} = \frac{1.44}{5} = 0.288\text{ A}$$

(b) The power dissipated in the resistor is

$$\mathcal{P} = V\,I = \mathcal{E}\,I = 1.44\times 0.288 = 0.415\text{ W}.$$

(c) The current creates a back magnetic force that must be cancelled by a forward applied force to keep the rod moving at a constant speed.

$$F = I\,\ell\,B = 0.288\times 1.6\times 0.30 = 0.1382 = 0.138\,N$$

(d) The rate of energy dissipated is the power dissipated

$$\mathcal{P} = F\,v = 0.1382\times 3 = 0.415\,W\,.$$

Problem G.3

A circular conducting disk with a 30 cm radius sits in a uniform magnetic field of 40 mT. If it is rotated with an angular frequency of 15 rad/s then what is the magnitude of the voltage difference between the center and the rim of the disk?

Solution to G.3

The voltage across a rod of length ℓ rotating in a field B is

$$\mathcal{E} = \frac{1}{2} B \ell^2 \omega.$$

The disk is identical to this so

$$\mathcal{E} = \frac{1}{2} B R^2 \omega = \frac{1}{2} 0.040 \times 0.30^2 \, 15 = 0.027 \text{ V}.$$

Problem G.4

A 30 turn flat 12 cm × 9 cm rectangular coil is perpendicular to a uniform magnetic field that varies from 60 mT to 25 mT in 0.3 s. What is the magnitude of the average induced EMF in the coil?

Solution to G.4

This is a simple Faraday's law problem. The field is uniform and perpendicular to the surface, so $\Phi = B A$. The flux changes because the field changes: $\Delta \Phi = A \times \Delta B$

$$|\overline{\mathcal{E}}| = N \frac{|\Delta \Phi|}{\Delta t} = N \frac{A \times |\Delta B|}{\Delta t} = 30 \frac{(0.12 \times 0.09) \times 0.035}{0.3} = 0.0378 \text{ V}$$

Problem G.5

(a) A wire with a diameter d is formed into a single circular loop of radius R. The metal in the wire has a resistivity ρ. If this loop sits in a perpendicular field that varies with time by $B(t)$ then what is the current in the loop as a function of time?

(b) A copper wire with a 1 mm diameter is formed into a single circular conducting loop with a 30 cm radius. If this sits perpendicular to a magnetic field that varies at the rate of $dB/dt = 20\, T/s$, then what is the induced current in the loop?

Solution to G.5

(a) The length of the wire is the circumference $\ell = 2 \pi R$ and its cross-sectional area is $A = \pi (d/2)^2$. The resistance of the wire is

$$R = \frac{\rho \ell}{A} = \frac{\rho \, 2 \pi R}{\pi (d/2)^2} = \frac{8 \rho R}{d^2}$$

The changing magnetic field produces a changing flux.

$$\Phi = B A = B \pi R^2 \implies \frac{d\Phi}{dt} = \pi R^2 \frac{dB}{dt}$$

Faraday's law relates this to the induced EMF and Ohm's law gives the induced current.

$$|I|\,R = |\mathcal{E}| = \left|\frac{d\Phi}{dt}\right| \implies |I|\,\frac{8\,\rho\,R}{d^2} = \left|\pi\,R^2\,\frac{dB}{dt}\right| = \pi\,R^2\left|\frac{dB}{dt}\right| \implies |I| = \frac{\pi\,R\,d^2}{8\,\rho}\left|\frac{dB}{dt}\right|$$

(b) Plugging numbers into the expression from part (a) gives our result. The resistivity of copper is $1.7\times10^{-8}\,\Omega\cdot\text{m}$.

$$|I| = \frac{\pi\,R\,d^2}{8\,\rho}\left|\frac{dB}{dt}\right| = \frac{\pi\,0.30\times0.001^2}{8\times1.7\times10^{-8}}\,20 = 138.6\text{ A}$$

Problem G.6

A flat circular coil with a 30 cm radius and 200 turns sits in a uniform and constant (time independent) magneitc field of 40 mT.
(a) If the coil is initially in the plane of the field and then over a time of 3 s is rotated 90° until it is perpendicular to the field. What is the average induced EMF magnitude over this time?
(b) Suppose this coil is rotated about an axis parallel to the plane of the coil and perpendicular to the field. If it is rotated at a frequency of 15 Hz (15 times each second) then what is the peak EMF induced in the coil.

Solution to G.6

This is a Faraday's law problem. The field is uniform and the surface is flat, so $\Phi = B\,A\cos\theta$. The flux changes because the field changes in this case because the angle changes. The angle is between the normal to the surface and the field. Initially, the field is parallel to the plane, so it is perpendicular to the normal $\theta_i = 90\,°$. This give $\Phi_i = 0$. When the coil is perpendicular to the field it's normal is parallel, so $\theta_f = 0$ and $\Phi_f = B\,A$. Thus $\Delta\Phi = B\,A - 0 = B\,A$.

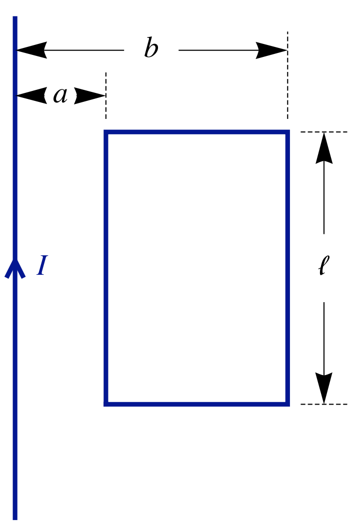
$$|\overline{\mathcal{E}}| = N\,\frac{|\Delta\,\Phi|}{\Delta\,t} = N\,\frac{|B\,A|}{\Delta\,t} = 200\,\frac{0.040\times\left(\pi\,0.30^2\right)}{3} = 0.754\text{ V}$$

(b) This is just an AC generator. The angular frequency ω (in rad/s) is related to the frequency by $\omega = 2\,\pi\,f$, since there are $2\,\pi$ radians in each full rotation. The peak EMF is $\mathcal{E}_{\max}$.

$$\mathcal{E}_{\max} = N\,\text{B}\,A\,\omega = \;N\,B\,A\,2\,\pi\,f = 200\times0.040\times\pi\,0.30^2\times2\,\pi\,15 = 213\text{ V}$$

Problem G.7

Consider a long wire with a current I next to a single rectangular conducting loop as shown.



(a) What is the magnetic flux through the loop due to the current in the long wire?

(b) Given $I(t)$ obtain an expression for the induced EMF in the loop due to the current.

(c) If the current is increasing, is this a clockwise or counterclockwire EMF?

(d) Suppose the current varies as $I(t) = I_0 \cos(\omega t + \phi)$. If the loop has a resistance R then what is the induced current as a function of time?

Solution to G.7

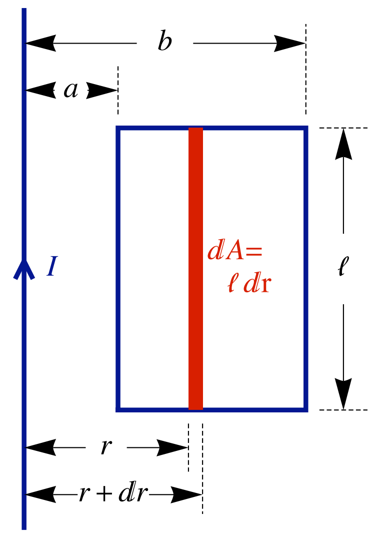
(a) The magnetic field as a function of r, the distance from the wire, is given by

$$B = \frac{\mu_0 I}{2\pi r}.$$

It direction, by the right hand rule, is *into the page*. (What we need is the direction of the field where the loop is.) The flux is the integral of the field over the surface. Here, it is perpendicular to the surface.

$$\Phi = \int \vec{B} \cdot d\vec{A} = \int B\, dA$$

Since the field varies with r we should choose narrow strips at a distance r from the long wire and of width dr. This gives $dA = \ell\, dr$ as shown in red in the diagram below.



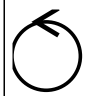
To integrate over the entire surface we choose the limits of integration to be $a \le r \le b$. This gives

$$\Phi = \int B\, dA = \int_a^b \frac{\mu_0 I}{2\pi r} \ell\, dr = \frac{\mu_0 I \ell}{2\pi} \int_a^b \frac{dr}{r} = \frac{\mu_0 I \ell}{2\pi} \ln \frac{b}{a}.$$

(b) If the current changes then the flux changes and, by Faraday's law, there is an induced EMF in the loop.

$$\mathcal{E} = -\frac{d\Phi}{dt} = -\frac{\mu_0 \ell}{2\pi} \ln \frac{b}{a} \times \frac{dI}{dt}.$$

(c) The right hand rule gives direction of the field. To the right of the long wire it is into the page. The increasing current gives an increasing flux and then $d\Phi/dt$ is into the page as well. The induced flux is opposite, out of the page, and the induced current is then counterclockwise.

Φ	$\frac{d\Phi}{dt}$	Φ_{ind}	$\mathcal{E}$ or I
$\times$	$\times$	$\bullet$	

(d) With $I(t) = I_0 \cos(\omega t + \phi)$ we can put this into the result of (b) and get

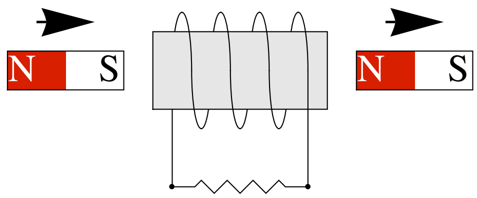
$$I(t) = I_0 \cos(\omega t + \phi) \implies \frac{d}{dt} I(t) = -\omega I_0 \sin(\omega t + \phi)$$
$$\implies \mathcal{E} = -\frac{\mu_0 \ell}{2\pi} \ln \frac{b}{a} \times \frac{dI}{dt} = \omega I_0 \frac{\mu_0 \ell}{2\pi} \ln\left(\frac{b}{a}\right) \sin(\omega t + \phi)$$

Ohm's law gives $\mathcal{E} = I R$. The induced current is $\mathcal{E}/R$.

$$I = \omega I_0 \frac{\mu_0 \ell}{2\pi R} \ln\left(\frac{b}{a}\right) \sin(\omega t + \phi)$$

Problem G.8

A magnet is moved through a solenoid as shown. What is the direction of the induced current through the resistor as the magnet enters and then leaves.



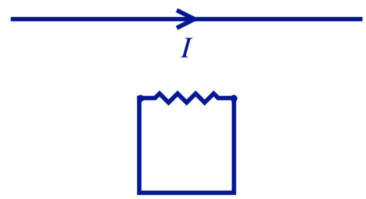
Solution to G.8

(a) As the magnet enters the direction (or sense) of the flux is to the left, since the field is toward a South pole. As it approaches more of the lines from the magnet pass through the coil, so the sense of $d\Phi/dt$ is the same as the flux. The induced flux, by Lens's Law is always opposite to $d\Phi/dt$, so it is to the right. The direction of the induced EMF and current is found by the right hand rule. The way the coil is wrapped implies the current through the resistor is tot he left.
As the magnet leaves the field is away from the North pole and is still to the left. Now the flux is decreasing so $d\Phi/dt$ is opposite and to the right. The induced flux is opposite that and that implies a current to the right.

	Φ	$\frac{d\Phi}{dt}$	Φ_{ind}	$\mathcal{E}$ or I	Current through R
magnet enters (at left)	$\longleftarrow$	$\longleftarrow$	$\longrightarrow$		to the left
magnet leaves (at right)	$\longleftarrow$	$\longrightarrow$	$\longleftarrow$		to the right

Problem G.9

A square loop with a resistor is moved next to a wire with a constant current I. What is the direction of the current through the resistor
(a) when the loop is moved to the top of the page (toward the wire),
(b) when moved to the left,
(c) when moved to the bottom of the page and
(d) when moved to the right?

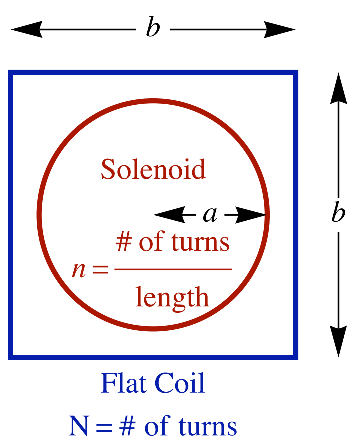


Solution to G.9

The direction of the field due to the long wire, at the loop, is into the page. When moving to the left or the right the magnitude of the field doesn't change, so $d\Phi/dt = 0$. Whan moving toward the wire the flux increases and when moving away it decreases.

	Φ	$\frac{d\Phi}{dt}$	Φ_{ind}	$\mathcal{E}$ or I	Current through R
(a) to top	×	×	•	↺	to the left
(b) to left	×	0	0	0	0
(c) to bottom	×	•	×	↻	to the right
(d) to right	×	0	0	0	0

Problem G.10



A long solenoid with n turns/length and with a circular cross-section of radius a sits with its axis perpendicular to the page. Sitting outside of the solenoid is a square flat coil (entirely in the plane) with N turns, with sides of length b $(b > 2\,a)$ and with a total resistance R. A current of I_1 flows through the solenoid.

(a) What is the magnetic flux through the flat coil due to the current in the solenoid?

(b) If the current through the solenoid varies at a rate of $\frac{dI_1}{dt}$ then what is I_2, the induced current through the coil?

(c) If the current I_1 is clockwise and decreasing then what is the direction of I_2?

Solution to G.10

(a) The magnetic field inside the solenoid is $B = \mu_0\, n\, I_1$. The flux is

$$\Phi = \int \vec{B} \cdot d\vec{A} = \vec{B} \cdot \vec{A} = B\,A = \mu_0\, n\, I_1\, \pi\, a^2$$

Note that the area used the area of the solenoid because the field due to the solenoid is zero outside the solenoid.

(b) A changing current causes a changing flux and by Faraday's law induces an EMF. Ohm's law gives the current.

$$I_2\, R = \mathcal{E} = -N \frac{d\Phi}{dt} = -N\, \mu_0\, n\, \pi\, a^2 \frac{dI_1}{dt} \implies I_2 = -\frac{\mu_0\, n\, N\, \pi\, a^2}{R} \frac{dI_1}{dt}$$

(c) The magnetic field due to a clockwise I_1 is, by the right hand rule, into the page. The right hand rule gives the direction of the field above the wire as into the page. Since the current is decreasing the flux is decreasing so $d\Phi/dt$ is out of the page. The induced flux is opposite that - into. This gives a clockwise current.

Φ	$\frac{d\Phi}{dt}$	Φ_{ind}	$\mathcal{E}$ or I
×	•	×	↻

Problem G.11

A solenoid with a circular cross-section with a radius R and has n turns per length. It sits with a vertical axis; take the positive z direction to be upward. A varying current of $I(t)$ flows through the solenoid, where a positive I represents a counterclockwise current.
(a) What is the magnetic field as a function of time and r, the perpendicular distance from the central axis of the solenoid?
(b) What is the electric field as a function of time and r?

Solution to G.11

(a) The magnetic field is in the z direction by the right hand rule. It is given by the usual expression inside and is zero outside.

$$\vec{B} = \mu_0\, n\, I(t)\, \hat{z} \text{ for } r < R \text{ and } \vec{B} = \vec{0} \text{ for } r > R$$

(b) Use the $\mathcal{E} = \oint \vec{E} \cdot d\vec{r}$ form of Faraday's law to find the electric field. Note that this is analogous to using Ampere's law to find the Magnetic field for a uniform current in a wire of radius R.

$$\oint \vec{E} \cdot d\vec{r} = -\frac{d}{dt}\,\Phi_m$$

Choose a contour of integration to be a circle of radius r, as was done in the Ampere's law problem. The left hand side takes the form $\oint \vec{E} \cdot d\vec{r} = E\,2\,\pi\,r$, where a positive E corresponds to a counterclockwise field. Write the magnetic flux inside the contour as $\Phi_{m,\text{inside}}$.

$$\Phi_{m,\text{inside}} = B\,A_{\text{inside}} = \mu_0\,n\,I\,A_{\text{inside}}$$

where since the magnetic field stops at $r = R$ we have

$$A_{\text{inside}} = \pi\,r^2 \text{ for } r < R \;\text{ and }\; A_{\text{inside}} = \pi\,R^2 \text{ for } r > R.$$

We can now put this all together.

$$\oint \vec{E} \cdot d\vec{r} = -\frac{d}{dt}\,\Phi_m \implies E\,2\,\pi\,r = -\mu_0\,n\,A_{\text{inside}}\,\frac{dI}{dt}$$

$$E = -\frac{\mu_0\,n\,r}{2}\,\frac{dI}{dt} \text{ for } r < R \;\text{ and }\; E = -\frac{\mu_0\,n\,R^2}{2\,r}\,\frac{dI}{dt} \text{ for } r > R$$