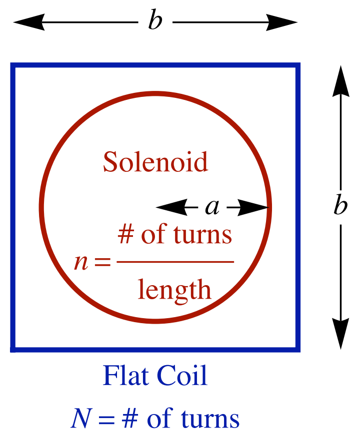
Chapter H - Problems

Blinn College - Physics 2426 - Terry Honan

Problem H.1

Refer to **Problem G.9** where a $b \times b$ square flat coil with N turns sits outside of a long solenoid with n turns per length and a circular cross-section radius a. What is the mutual inductance between the solenoid and the flat coil?



Solution to H.1

In **Problem G.9** we had the induced EMF in the flat coil as

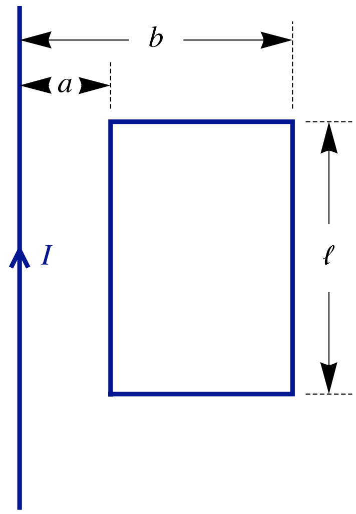
$$\mathcal{E}_2 = -N \mu_0 n \pi a^2 \frac{dI_1}{dt}$$

where I am now referring to the EMF in the coil as $\mathcal{E}_2$. Using the definition of mutual inductance

$$\mathcal{E}_2 = -M \frac{dI_1}{dt} \implies M = N \mu_0 n \pi a^2.$$

Problem H.2

Refer to **Problem G.7** where we discussed a long wire next to a rectangular conduction loop. What is the mutual inductance between the long wire and the loop?



Solution to H.2

We saw in Problem G.7 that

$$\mathcal{E} = -\frac{d\Phi}{dt} = -\frac{\mu_0 \ell}{2\pi} \ln \frac{b}{a} \times \frac{dI}{dt}.$$

Take the current of the long wire to be I_1 and the induced EMF in the loop to be $\mathcal{E}_2$ then use the definition of mutual inductance:

$$\mathcal{E}_2 = -M \frac{dI_1}{dt} \text{ and } \mathcal{E}_2 = -\frac{\mu_0 \ell}{2\pi} \ln \frac{b}{a} \times \frac{dI_1}{dt} \implies M = \frac{\mu_0 \ell}{2\pi} \ln \frac{b}{a}.$$

Problem H.3

What is the average induced EMF when the current through a coil with a 50 mH inductance varies from 5 A to 3 A in 0.04 s?

Solution to H.3

$$|\overline{\mathcal{E}}| = L \left| \frac{\Delta I}{\Delta t} \right| = 0.050 \times \frac{2}{0.04} = 2.5 \text{ V}$$

Problem H.4

The current through a 15 mH inductor varies as

$$I(t) = I_{max} \sin \omega t \text{ where } I_{max} = 6 \text{ A and } \omega = 200\, s^{-1}.$$

What is the EMF across the inductor as a function of time?

Solution to H.4

$$\mathcal{E} = -L\,\frac{dI}{dt} = \; -L\,\frac{d}{dt}\,I_{\max}\,\sin\omega\,t = \; -L\,\omega\,I_{\max}\,\cos\omega\,t$$
$$= \; -0.015\times 200\times 6\,\cos\omega\,t = \; -(18\;V)\,\cos\left[\left(200\;\text{s}^{-1}\right)\,t\right]$$

Problem H.5

A solenoid has 250 turns, a length of 40 cm and a diameter of 3 cm. The current through the solenoid varies as

$$I(t) = 15\,t^3 - 40\,t + 30 \quad \text{(in SI units)}.$$

(a) What is the inductance of the solenoid?
(b) What is the voltage drop across the solenoid at $t = 2$ s?
(c) What is the energy stored in the solenoid at $t = 2$ s?

Solution to H.5

(a) This is a long solenoid. The inductance of a long solenoid is:

$$L = \mu_0\,\frac{N^2}{\ell}\,A = 4\,\pi\times 10^{-7}\,\frac{250^2}{0.40}\,\left(\pi\,0.015^2\right) = 1.3879\times 10^{-4} = 0.139\;\text{mH}$$

(b) The voltage drop is $V = L\,dI/dt$. We need to differentiate the current.

$$\frac{dI(t)}{dt} = 45\,t^2 - 40 \;\Longrightarrow\; \frac{dI}{dt}\,(2) = 45\times 2^2 - 40 = 140$$
$$\Longrightarrow\; V = L\,\frac{dI}{dt} = L\times 14 = 0.0194\;V$$

(c) The energy stored in an inductor is $U = \frac{1}{2}\,L\,I^2$.

$$I(2) = 15\times 2^3 - 40\times 2 + 30 = 70 \;\Longrightarrow\; U = \frac{1}{2}\,L\,I^2 = \frac{1}{2}\,L\times 70^2 = 0.340\;\text{J}$$

Problem H.6

A typical electric field at the earth's surface is 100 V/m and a typical magnetic field value is 5×10^{-5} T. What are the electric and magnetic energy density, u_e and u_m.

Solution to H.6

$$u_e = \frac{1}{2}\,\epsilon_0\,E^2 = \; \frac{1}{2}\,8.85\times 10^{-12}\times 100^2 = 4.43\times 10^{-8}\;\frac{\text{J}}{m^3}$$

$$u_m = \frac{B^2}{2\,\mu_0} = \; \frac{\left(0.5\times 10^{-4}\right)^2}{2\times 4\,\pi\times 10^{-7}} = 9.95\times 10^{-4}\;\frac{\text{J}}{m^3}$$

Problem H.7

A 2 μF capacitor is given an initial charge of 6 μC and is then connected across a 15 kΩ resistor to discharge it.
(a) What is the charge on the capacitor 40 ms after the resistor is connected?
(b) What is the current through the resistor 40 ms after it is connected?
(c) What is the maximum magnitude current through the resistor?

Solution to H.7

The time constant is

$$\tau = R\,C = 15\ \mathrm{k\Omega} \times 2\ \mu\mathrm{F} = 30\ \mathrm{ms}$$

(a) For a discharging capacitor:

$$Q\,(t) = Q_0\, e^{-t/\tau} = (6\ \mu\mathrm{C})\, e^{-40/30} = 1.58\ \mu\mathrm{C}$$

(b) To get the current differentiate the charge.

$$I(\mathrm{t}) = \frac{dQ}{dt} = -\frac{Q_0}{\tau}\, e^{-t/\tau} = -\frac{6\ \mu\mathrm{C}}{30\ \mathrm{ms}}\, e^{-40/30} = -5.27 \times 10^{-5}\ \mathrm{A}$$

(c) The maximum current is at $t = 0$.

$$I_{\mathrm{max}} = \frac{Q_0}{\tau} = \frac{6\ \mu\mathrm{C}}{30\ \mathrm{ms}} = 2 \times 10^{-4}\ \mathrm{A}$$

Problem H.8

A 2 MΩ resistor and a 3 μC capacitor are connected across a 12 V battery. What is the timeconstant of the circuit. Express the charge and currents as functions of time.

Solution to H.8

$$R = 2 \times 10^{6}\ \Omega\,,\quad C = 3 \times 10^{-6}\ F \text{ and } \mathcal{E} = 12\ \mathrm{V}$$

$$\tau = R\,C = 6\,s$$

$$Q(t) = \mathcal{E}\,\mathrm{C}\left(1 - e^{-\mathrm{t}/\tau}\right) = 36\ \mu\mathrm{C}\left(1 - e^{-t/(6\ \mathrm{s})}\right)$$

$$\mathrm{I}(t) = \frac{d}{dt}\,Q(t) = \frac{\mathcal{E}\,C}{R\,C}\, e^{-\mathrm{t}/\tau} = \frac{\mathcal{E}}{\mathrm{R}}\, e^{-\mathrm{t}/\tau} = 6\ \mu\mathrm{A}\, e^{-t/(6\ \mathrm{s})}$$

Problem H.9

If a charging capacitor reaches 60% of its maximum charge after *0.90* s then what is the time constant of the circuit?

Solution to H.9

$$Q(t) = Q_{max}\left(1 - e^{-t/\tau}\right) \implies 0.6 = \frac{Q}{Q_{max}} = \left(1 - e^{-t/\tau}\right) \implies 0.4 = e^{-t/\tau} \implies -\tau \ln 0.4 = t = 0.9\,s \implies \tau = 0.982\text{ s}$$

Problem H.10

A 2.5 H inductor is connected in a loop circuit with a DC voltage source and a resistor. If the current increases to 90% of its final value in 3 s then what is the resistance in the circuit?

Solution to H.10

For such a circuit the current is

$$I(t) = I_{max}\left(1 - e^{-t/\tau}\right)$$

$$0.90 = \frac{I}{I_{max}} = 1 - e^{-3/\tau} \implies e^{-3/\tau} = 0.10 \implies \tau = -\frac{3}{\ln(0.1)} = 1.3029$$

$$\tau = \frac{L}{R} = \frac{2.5}{R} \implies R = 1.92\,\Omega$$

Problem H.11

An inductance L and a capacitance C both have the same time constant when combined with the same resistance.
(a) What is the resistance and what is the value of the time constant?
(b) If $L = 12$ mH and $C = 30\ \mu$F then what is R and what is the value of the time constant?

Solution to H.11

(a) The time constants are $\tau = R\,C$ and $\tau = L/C$.

$$\frac{L}{R} = \tau = R\,C \implies R = \sqrt{\frac{L}{C}}$$

$$\tau = R\,C = C\sqrt{\frac{L}{C}} = \sqrt{L\,C}$$

(b) Using the values $L = 12$ mH and $C = 30\ \mu$F gives

$$R = \sqrt{\frac{L}{C}} = \sqrt{\frac{12\times10^{-3}}{30\times10^{-6}}} = 20\,\Omega \text{ and } \tau = \sqrt{L\,C} = 0.6\text{ ms.}$$

Problem H.12

A 1 μF capacitor is fully charged across a 40 V battery. It is then disconnected from the battery and connected actoss a 10 mH inductor inductor. What is the maximum current through the inductor during the LC oscillations?

Solution to H.12

The initial charge on the capacitor is the maximum charge of the oscillation. It is

$$Q_{\max} = C\,V = \;\; 10^{-6} \times 40 = 4 \times 10^{-5}\ \text{C}.$$

The angular frequency of the oscillation is

$$\omega = \frac{1}{\sqrt{L\,C}} = \frac{1}{\sqrt{10^{-6} \times 0.010}} = 10^{4}\ \frac{\text{rad}}{\text{s}}.$$

For an LC circuit the charge varies as: $Q(t) = Q_{\max} \cos(\omega\,t + \phi)$. The current is the derivative of the charge. $I(t) = dQ/dt = -\omega\,Q_{\max} \sin(\omega\,t + \phi)$. Since sine varies between +1 and -1 we get

$$I_{\max} = \omega\,Q_{\max} = \;\; 0.4\,A$$

Problem H.13

A 840 nF capacitor with an initial 105 μC is connected across a 3.30 mH inductor at $t = 0$.
(a) What is the energy in the capacitor at $t = 2$ ms?
(b) What is the energy in the inductor at $t = 2$ ms?
(c) What is the total energy?

Solution to H.13

For the charge as a function of time we want our sinusoidal function to be cosine since the charge has its maximum value at $t = 0$.

$$Q\,(t) = \;\; Q_{\max} \cos \omega\,t \implies I(t) = \frac{d}{dt} Q\,(t) = -\omega\,Q_{\max} \sin \omega\,t.$$

$$\text{where}\;\; \omega = \frac{1}{\sqrt{L\,C}} = \frac{1}{\sqrt{3.3 \times 10^{-3} \times 840 \times 10^{-9}}} = 18\,993 \;\;\text{and}\;\; Q_{\max} = Q_0 = \;\; 105 \times 10^{-6}$$

At $t = 2$ ms ,

$$Q(2\text{ ms}) = Q_{\max} \cos\left(\omega\;\; 2 \times 10^{-3}\right) = 1.00683 \times 10^{-4}$$

$$I(2\text{ ms}) = -\omega\,Q_{\max} \sin\left(\omega\;\; 2 \times 10^{-3}\right) = -0.56597$$

(a) $U_C = \dfrac{Q^2}{2\,C} = \dfrac{\left(1.00683 \times 10^{-4}\right)^2}{2 \times 840 \times 10^{-9}} = 6.03\text{ m J}$

(b) $U_L = \dfrac{1}{2}\,L\,I^2 = \dfrac{1}{2}\;3.3 \times 10^{-3} \times 0.56597^2 = 0.529\text{ m J}$

(c) $U_{\text{total}} = \dfrac{Q_{\max}^2}{2\,C} = \dfrac{\left(105 \times 10^{-6}\right)^2}{2 \times 840 \times 10^{-9}} = 6.56\text{ m J}$

Note that $U_{\text{total}} = U_C + U_L$.