# Chapter I - Problems

## Blinn College - Physics 2426 - Terry Honan

### Problem I.1

The voltage from an AC source varies with time as

$$V(t) = V_{\text{max}} \cos(\omega\, t + \phi)$$

where $V_{\text{max}} = 20\ \text{V}$, $\omega = 80\ s^{-1}$ and $\phi = 0.6$ .

(a) What is the root-mean-squared voltage, $V_{\text{rms}}$?

(b) What is the frequency, $f$?

(c) What is the time closest to $t = 0$ when the voltage takes on its maximum value.

**Solution to I.1**

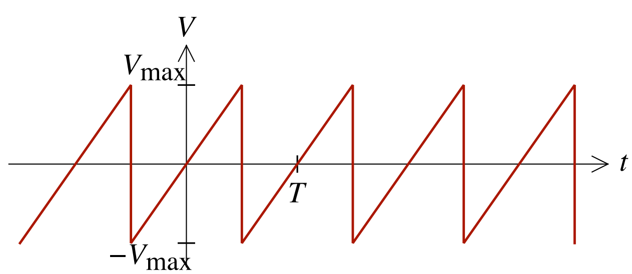
(a) $V_{\text{rms}} = \frac{1}{\sqrt{2}} V_{\text{max}} = \frac{20}{\sqrt{2}} = 14.1\ \text{V}$

(b) $f = \frac{\omega}{2\pi} = \frac{80}{2\pi} = 12.7\ \text{Hz}$

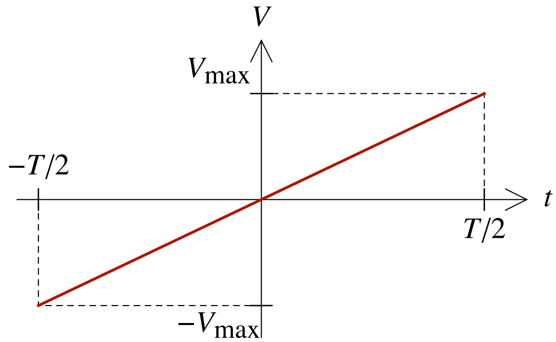
(c) $t_0 = -\frac{\phi}{\omega} = -\frac{0.6}{80} = -0,0075\ \text{s}$

### Problem I.2

For a sawtooth pattern voltage show that $V_{\text{rms}} = \frac{1}{\sqrt{3}} V_{\text{max}}$.



**Solution to I.2**




To find the average integrate over one period of the function. One period of a sawtooth voltage may be written as $V(t) = \frac{V_{max}}{T/2} t$.

$$V_{rms} = \sqrt{\overline{V^2}} = \sqrt{\frac{1}{T} \int_{-T/2}^{T/2} V^2 \, dt} = \sqrt{\frac{1}{T} \left(\frac{2 V_{max}}{T}\right)^2 \int_{-T/2}^{T/2} t^2 \, dt} = \sqrt{\frac{1}{T} \left(\frac{2 V_{max}}{T}\right)^2 \frac{T^3}{12}}$$

$$\Longrightarrow \quad V_{rms} = \frac{V_{max}}{\sqrt{3}}$$

## Problem I.3

An AC voltage source has an rms voltage of 20 V and a frequency of 40 Hz. Suppose the manufacturer's specs say that this source cannot safely put out a larger instantaneous current than 80 mA.
(a) What resistance values can be connected across this source? (Just the resistor is connected across the source.)
(b) What capacitance values can be connected across this source?
(c) What inductance values can be connected across this source?

**Solution to I.3**

The peak current $I_{max}$ can be no larger 80 mA, thus the largest value of $I_{rms}$ can be

$$I_{rms} = \frac{1}{\sqrt{2}} I_{max} = 0.05657 = 0.0566 \text{ A}.$$

This with $V_{rms}$ gives the smallest safe impedance.

$$Z_{min} = \frac{V_{rms}}{I_{rms}} = \frac{20}{0.05657} = 353.55 = 353 \, \Omega$$

The impedance for just a resistor is $R$, for just a capacitor is $X_C$ and for an inductor is $X_L$.

(a) $R = Z \geq Z_{min} = 353 \, \Omega$

(b) $Z = X_c = \frac{1}{2 \pi f C} \geq Z_{min} \quad \Longrightarrow \quad C \leq \frac{1}{2 \pi f Z_{min}} = \frac{1}{2 \pi 40 \times 353.55} = 11.3 \, \mu\text{F}$

(c) $Z = X_L = 2 \pi f L \geq Z_{min} \quad \Longrightarrow \quad L \geq \frac{Z_{min}}{2 \pi f} = \frac{353.55}{2 \pi 40} = 1.41 \text{ H}$

## Problem I.4

What is the maximum charge on the plates of a capacitor with capacitance $C$ when connected across an AC source with frequency $f$ and rms voltage $V_{\text{rms}}$?

**Solution to I.4**

$$Q_{\max} = C\, V_{\max} = \; C\, V_{\text{rms}} \sqrt{2}$$

## Problem I.5

A voltage source of produces an output which varies as a function of time as

$$V(t) = 50 \text{ V } \cos\left[\left(300\, s^{-1}\right)\, t\right].$$

(a) What are the root-mean-squared voltage and frequency of the source?

(b) Suppose this voltage is connected across a series combination of a 100 $\mu$F capacitor and a 40 $\Omega$ resistor. What is the rms current through the circuit and what is the phase angle?

(c) Write the current as a function of time.

**Solution to I.5**

(a) From the function of time we can read off that.

$$V_{\max} = 50 \text{ V and } \omega = 300\, s^{-1}.$$

We can then find $V_{\text{rms}}$ and $f$ from this.

$$V_{\text{rms}} = \frac{1}{\sqrt{2}}\, V_{\max} = 35.344 = 35.4 \text{ V} \;\; \text{and} \;\; f = \frac{\omega}{2\,\pi} = 47.7 \text{ Hz}$$

(b) We have $R = 40\ \Omega$ and $C = 100 \times 10^{-6}$ F. Since there is no inductor we have $X_L = 0$. The capacitive reactance is

$$X_C = \frac{1}{\omega\, C} = 33.333\ .$$

We can then find the impedance and rms current, and then the phase angle.

$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2} = 52.068 \implies I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = 0.67902 = 0.679 \text{ A}$$

$$\phi = \tan^{-1} \frac{X_L - X_C}{R} = -39.8 = -0.695 \text{ rad}$$

(c) To get the current as a function of time we must first find $I_{\max}$. Starting from $V_{\max}$ we can use $I_{\max} = V_{\max} / Z$ but since we have already found $I_{\text{rms}}$ it is easier to use

$$I_{\max} = \sqrt{2}\ I_{\text{rms}} = 0.960 \text{ A}.$$

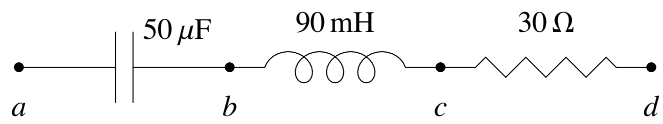
We saw in the notes that

$$I(t) = I_{\max} \cos(\omega\, t) \implies V(t) = V_{\max} \cos(\omega\, t + \phi).$$

When the voltage is varies as $\cos \omega\, t$ then the current must shift in the reverse direction.

$$V(t) = V_{\max} \cos(\omega\, t) \implies I(t) = I_{\max} \cos(\omega\, t - \phi) = 0.960 \text{ A } \cos\left[\left(300\, s^{-1} + 0.695\right)\right]$$

## Problem I.6




A 50 $\mu$C capacitor, a 90 mH inductor and a 30 $\Omega$ resistor are connected in series across a standard outlet in the order listed, $C$ then $L$ then $R$. An AC multimeter measures the rms voltage and rms current.

(a) When the multimeter is set as an ammeter and put into the circuit, what does it read?
(b) What does the multimeter set as a voltmeter read when connected between $a$ and $b$, between $b$ and $c$, between $c$ and $d$, between $a$ and $c$, and between $b$ and $d$?
(c) By what phase angle is the voltage ahead of the current?
(d) What is the average power dissipated in the capacitor, in the inductor and in the resistor?

**Solution to I.6**

For a standard outlet $V_{\text{rms}} = 120$ V and $f = 60$ Hz. The components have values: $C = 50 \times 10^{-6}$ F, $L = 0.090$ H and $R = 30\ \Omega$.

Before doing other calculations we must first find the reactances and impedance.

$$X_C = \frac{1}{2\,\pi\, f\, C} = 53.052 = 53.0\ \Omega,$$
$$X_L = 2\,\pi\, f\, L = 33.929 = 33.9\ \Omega \text{ and}$$
$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2} = 35.576 = 35.6\ \Omega$$

(a) The rms current is found by $V_{\text{rms}} = I_{\text{rms}}\, Z$.

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = 3.3730 = 3.37 \text{ A}$$

(b) To get the voltage between two points use $V_{\text{rms}} = I_{\text{rms}}\, Z'$ where $Z'$ is the impedance between the two positions. All voltages given here are rms values.

The impedance between $a$ and $b$ is the capacitive reactance.

$$Z_{ab} = X_C \implies V_{ab} = I_{\text{rms}}\, X_C = 179 \text{ V}$$

The impedance between $b$ and $c$ is the inductive reactance.

$$Z_{bc} = X_L \implies V_{bc} = I_{\text{rms}}\, X_L = 114 \text{ V}$$

The impedance between $c$ and $d$ is the resistance.

$$Z_{cd} = R \implies V_{cd} = I_{\text{rms}}\, R = 101 \text{ V}$$

Between $a$ and $c$ we have $X_C$ and $X_L$ but no resistance.

$$Z_{ac} = \sqrt{(X_L - X_C)^2} = 19.122 \implies V_{ac} = I_{\text{rms}}\, Z_{ac} = 64.5\text{ V}$$

Between $b$ and $d$ we have $X_L$ and $R$ but no capacitor, so $X_C = 0$.

$$Z_{bd} = \sqrt{R^2 + {X_L}^2} = 45.2901 \implies V_{bd} = I_{\text{rms}}\, Z_{bd} = 153\text{ V}$$

(c) The phase angle is found by $\tan\phi = \frac{X_L - X_C}{R}$.

$$\phi = \tan^{-1}\frac{X_L - X_C}{R} = -32.5\,°$$

(d) The average power dissipated in an AC circuit is $\overline{\mathcal{P}} = V_{\text{rms}}\, I_{\text{rms}} \cos\phi$. For a series RCL circuit this reduces to $\overline{\mathcal{P}} = I_{\text{rms}}^2\, R$. All the power dissipated in an AC circuit is lost in the resistor.

$$\overline{\mathcal{P}}_R = I_{\text{rms}}^2\, R = 341\text{ W} \quad\text{and}\quad \overline{\mathcal{P}}_C = 0 = \overline{\mathcal{P}}_L$$

## Problem I.7

The resonance circuit of a radio tuner is uses a 1.4 $\mu$H inductor and a variable capacitor to tune a 99.7 MHz radio station. What value of the capacitance is needed.

**Solution to I.7**

$$f = \frac{1}{2\pi\sqrt{LC}} \implies 99.7\times10^6 = \frac{1}{2\pi\sqrt{1.4\times10^{-6}\times C}} \implies C = 1.82\text{ pF}$$

## Problem I.8

A 2 Ω resistor, an 8 mH inductor and a 50 $\mu$F capacitor are connected in series.
(a) What is the resonant frequency?
(b) Suppose this is connected across a voltage source with a fixed $V_{\text{rms}} = 5$ V and a variable frequency. At what frequency is the rms current its maximum and what is the maximum value of $I_{\text{rms}}$?

**Solution to I.8**

(a) The resonance frequency is

$$f_{\text{res}} = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.008\times50\times10^{-6}}} = 252\text{ Hz}$$

(b) The impedance is its smallest at resonance, $I_{\text{rms}} = V_{\text{rms}}/Z$, so the maximum current is at the resonance frequency.

$$Z = Z_{\text{min}} = R \implies \max(I_{\text{rms}}) = \frac{V_{\text{rms}}}{Z_{\text{min}}} = \frac{V_{\text{rms}}}{R} = \frac{5}{2} = 2.5\text{ A}$$

## Problem I.9

An ideal transformer is used to raise the voltage from standard household voltage to an rms voltage of 240 V. If the primary coil has 300 turns carrying an rms current of 5 A, then how many turns are in the secondary coil and what is the current through the secondary coil? What is the power delivered to the transformer?

**Solution to I.9**

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = \frac{I_1}{I_1} \implies \frac{240}{120} = \frac{N_2}{300} = \frac{5}{I_2} \implies N_2 = 600 \text{ and } I_2 = 2.5\text{ A}$$

$$\mathcal{P} = V_1 I_1 = 120 \times 5 = 600\text{ W}$$