Chapter J - Problems

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Problem J.1

Verify that $u(x, t) = f(x - v\,t) + g(x + v\,t)$ is a solution of the one dimensional wave equation:

$$\frac{\partial^2}{\partial x^2} u = \frac{1}{v^2} \frac{\partial^2}{\partial t^2} u \,.$$

Solution to J.1

$$\frac{\partial}{\partial x} f(x - v\,\mathrm{t}) = f'(x - v\,t)$$

$$\frac{\partial^2}{\partial x^2} f(x - v\,t) = f''(x - v\,t)$$

$$\frac{\partial}{\partial t} f(x - v\,t) = f'\left(x - v\,t\right)(-v)$$

$$\frac{\partial^2}{\partial t^2} f(x - v\,t) = f''(x - v\,t)\,(-v)^2$$

This verifies that $f(x - v\,t)$ is a solution. If we replace v with $-v$ it will still be a solution so that implies that $g(x + v\,t)$ is also a solution. Since the derivative of the sum is the sum of the derivatives it follows that the sum of the two solutions is also a solution. It is also true that this is the most general solution but that is not proven here.

Problem J.2

If the peak electric field for a plane electromagnetic wave is 1800 V/m then what is the peak magnetic field?

Solution to J.2

$$E_{\mathrm{max}} = 1800 \implies B_{\mathrm{max}} = \frac{E_{\mathrm{max}}}{c} = \frac{1800}{3\times 10^8} = 6\,\mu\mathrm{T}$$

Problem J.3

The electric field for a plane wave is given by

$$E(x, t) = \left(100\,\frac{\mathrm{V}}{\mathrm{m}}\right) \sin\!\left[\left(10^7\,\mathrm{m}^{-1}\right) x - \omega\,t\right].$$

(a) What is the magnetic field amplitude?
(b) What is the wavelength?
(c) What value must ω have?
(d) What is the frequency?

Solution to J.3

(a) From the expression given we get the amplitude is $E_{\max} = 100\ \frac{\text{V}}{\text{m}}$. This gives

$$B_{\max} = \frac{E_{\max}}{c} = \frac{100}{3\times10^8} = 333\ \text{nT}.$$

(b) The wave number k gives the wavelength.

$$k = \ 10^7 \implies \lambda = \frac{2\pi}{k} = 628\ \text{nm}$$

(c) $c = \frac{\omega}{k} \implies \omega = c\,k = 3\times10^8\times10^7 = 3\times10^{15}\ s^{-1}$

(d) $f = \ \frac{c}{\lambda} = \frac{3\times10^8}{2\pi\times10^{-7}} = 4.77\times10^{14}\ \text{Hz}$

Problem J.4

At what distance from a 50 W isotropic source (a point source) is the peak electric field 15 V/m?

Solution to J.4

$$I = \ \frac{E_{\max}^2}{2\,\mu_0\,c} = \frac{15^2}{2\times4\,\pi\times10^{-7}\times3\times10^8} = 0.29842$$

$$I = \frac{\text{Power}}{\text{Area}} = \frac{\text{Power}}{4\,\pi\,r^2} \implies 0.29842 = \frac{50}{4\,\pi\,r^2} \implies r = \ 3.65\,m$$

Problem J.5

A helium-neon laser produces a polarized beam of 632.8 nm light at a power of 5 mW.
(a) What is the peak electric field at a point where the beam has a cross-sectional area of 4 mm^2?
(b) What is the total electromagnetic energy in a 1m length of the beam?

Solution to J.5

(a) $I = \frac{\mathcal{P}}{A} = \frac{\text{Power}}{\text{Area}} = \frac{5\times10^{-3}}{4\times10^{-6}} = 1250$

$$1250 = I = \frac{E_{\max}^2}{2\,\mu_0\,c} = \frac{E_{\max}^2}{2\times4\,\pi\times10^{-7}\times3\times10^8} \implies E_{\max} = 971\ \frac{\text{V}}{\text{m}}$$

(b) To find the energy U in a length L of a beam, relate U to $\overline{u}$, the energy density and relate $\overline{u}$ to I.

$$I = \frac{\mathcal{P}}{A} \text{ and } I = \overline{u}\,c \implies \overline{u} = \frac{\mathcal{P}}{A\,c}$$

$$U = \bar{u}\,\text{Volume} = \bar{u}\,A\,L = \frac{\mathcal{P}}{A\,c}\,A\,L = \frac{\mathcal{P}}{c}\,L = \frac{5\times10^{-3}}{3\times10^{8}}\times1 = 1.67\times10^{-11}\text{ J}$$

Problem J.6

A 5 MWatt radio station emits radio waves isotropically.
(a) What is the intensity of radio waves 5 m from the source?
(b) What is the force on a 2 cm × 2 cm square sheet that is a perfect conductor oriented normally to the radiation?

Solution to J.6

(a) Isotropic means the same in all directions. This means here that the energy is being spread out spherically and thus the relevant area is the surface area of a sphere.

$$I = \frac{\mathcal{P}}{A} = \frac{\mathcal{P}}{4\,\pi\,r^2} = \frac{5\times10^{6}}{4\,\pi\,5^2} = 15\,916 = 15\,900\ \frac{\text{W}}{m^2}$$

(b) A perfect conductor is a perfect reflector of radio waves. The force is the pressure times the area, $F = P\,A$. The pressure on a perfect reflector is given by

$$P = 2\,\frac{I}{c} \implies F_{\text{sheet}} = P\,A_{\text{sheet}} = 2\,\frac{I}{c}\,A_{\text{sheet}} = 2\,\frac{15\,916}{3\times10^{8}}\,(0.02)^2 = 4.24\times10^{-8}\text{ N}$$

Problem J.7

A 100 mW laser reflects normally off a perfect mirror. What is the force on the mirror?

Solution to J.7

Use Force = Pressure × Area and Power = Intensity × Area.

$$F = P\,A = 2\,\frac{I}{c}\times A = \ 2\,\frac{\mathcal{P}}{\text{c}} = 2\,\frac{0.100}{3\times10^{8}} = 6.67\times10^{-10}\text{ N}$$

Problem J.8

An electromagnetic plane wave of intensity $750\ \text{W}/\text{m}^2$ is normally incident on a 0.5 m × 1 m rectangular surface that reflects half the energy.
(a) What is the total energy absorbed by the surface in 1 min?
(b) What is the total momentum given to the surface in 1 min?

Solution to J.8

The total energy hitting the surface is

$$U = \ I\times\text{Area}\times\text{t} = \ 750\times(0.5\times1)\times60 = 22.5\text{ kJ}.$$

(a) If half the energy is reflected then half is absorbed. The energy absorbed is:

$$U_{\text{absorbed}} = \frac{U}{2} = 11.3\,\text{kJ}\,.$$

(b) The fraction of energy reflected is $\kappa = \dfrac{1}{2}$ and the momentum given to a surface is

$$\text{p} = (1 + \kappa)\,\frac{\text{U}}{\text{c}} = 1.5\,\frac{22.5\times 10^{3}}{3\times 10^{8}} = 1.125\times 10^{-4}\,\text{kg}\,\frac{\text{m}}{\text{s}}$$

Problem J.9

The AM radio band is from 540 kHz to 1600 kHz and the FM band is between 88 MHz and 108 MHz. What are the range of wavelengths in each band?

Solution to J.9

$$\lambda = \frac{c}{f} \text{ where } c = 3\times 10^{8}\,\frac{\text{m}}{\text{s}}.$$

AM band: $f = 540\,\text{kHz} \implies \lambda = 556\,\text{m}$ and $f = 1600\,\text{kHz} \implies \lambda = 188\,\text{m}$

FM band: $f = 88\,\text{MHz} \implies \lambda = 3.41\,\text{m}$ and $f = 108\,\text{MHz} \implies \lambda = 2.78\,\text{m}$