Chapter K - Problems

Blinn College - Physics 2426 - Terry Honan

Problem K.1

A He-Ne (helium-neon) laser has a wavelength of 632.8 nm. If this is shot at an incident angle of 55° into a glass block with index $n = 1.52$ then
(a) what is the frequency of this light,
(b) what is the sseed of the light in the glass,
(c) what is the wavelength of the light in the glass and
(d) what is the angle the beam makes from the normal to the surface when inside the glass?

Solution to K.1

(a) The wavelength is 632.8 nm in a vacuum. The frequency is

$$f = \frac{c}{\lambda} = \frac{3.00\times10^{8}}{632.8\times10^{-9}} = 4.74\times10^{14}\ \text{Hz}$$

(b) When light moves from one medium to another its speed changes

$$v = \frac{c}{n} = \frac{3.00\times10^{8}}{1.52} = 1.97\times10^{8}\ \frac{\text{m}}{\text{s}}$$

(c) Since the frequency of light does not change and its speed does it follows that the wavelength must descrease by the same factor as the speed.

$$\lambda = \frac{\lambda_0}{n} = \frac{632.8}{1.52} = 416\ \text{nm}$$

(d) Take the index of air to be $n_1 = 1$. Snell's law gives the refracted angle.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \implies 1 \sin 55\,° = 1.52 \sin\theta_2 \implies \theta_2 = 32.6\,°$$

Problem K.2

When under water a swimmer sees the sun make a 50° angle from the surface. What is the true angle of the sun above the water's surface?

Solution to K.2

The light moves from the air to the water so $n_1 = 1$ and $n_2 = 1.333$. Since angles are always measured from the normal to the surface we have $\theta_2 = 90\,° - 50\,° = 40\,°$.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \implies 1 \sin\theta_1 = 1.333 \sin 40\,° \implies \theta_2 = 59.0\,° \implies 90\,° - 59.0\,° = 31.0\,°$$

Problem K.3

What is the critical angle for total internal reflection for a fiber optic tube with index 1.40? If the tube is submerged in water then what is the critical angle?

Solution to K.3

$$\text{tube in air} \implies \sin\theta_{\text{crit}} = \frac{n_2}{n_1} = \frac{1}{1.40} \implies \theta_{\text{crit}} = 45.6\,°$$

$$\text{tube in water} \implies \sin\theta_{\text{crit}} = \frac{n_2}{n_1} = \frac{1.33}{1.40} \implies \theta_{\text{crit}} = 71.8\,°$$

Problem K.4

Consider a clear thick flat horizontal surface with a small imperfection 1.10 cm below the surface.When a penny is place directly over the imperfection it cannot be seen at all but when a dime is placed over the imperfection it can be seen by viewing from an angle. What does this imply about the index of refraction of the plastic? A dime has a radius of 0.875 cm and a penny has a 0.950 cm.

Solution to K.4

The incident angle from the imperfection to the edge of the coin satisfies: $\tan\theta_1 = r/h$, where h is the depth.

$$\theta_{1,\text{dime}} = \tan^{-1}\frac{0.875}{1.10} = 38.501\,° \quad \text{and} \quad \theta_{1,\text{penny}} = \tan^{-1}\frac{0.950}{1.10} = 40.815\,°$$

It follows that the critical angle is between the incident angles for the dime and penny. This gives inequalities for the index.

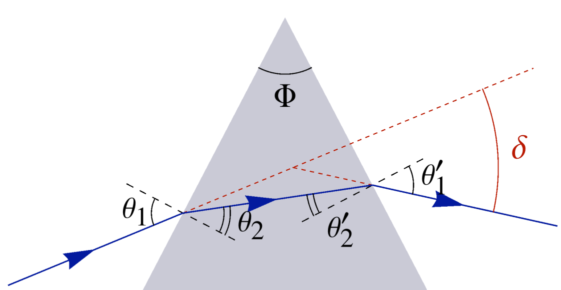
$$\theta_{1,\text{dime}} \le \theta_{\text{crit}} \le \theta_{1,\text{penny}} \implies \sin\theta_{1,\text{dime}} \le \sin\theta_{\text{crit}} = \frac{1}{n} \le \sin\theta_{1,\text{penny}}$$

$$\implies \frac{1}{\sin\theta_{1,\text{penny}}} \le n \le \frac{1}{\sin\theta_{1,\text{dime}}} \implies 1.53 \le n \le 1.61$$

Problem K.5

The indices of refraction of red and violet light in a glass prism are 1.51 and 1.53, respectively. Suppose the prism has an apex angle of 55° and the light is incident at 40°. What is the total angle of deflection for red, for violet and what is the total angle of dispersion?

Solution to K.5



For a given θ_1 and n we can find the other angles. To get θ_2 use Snell's law at the first interface.

$$\sin\theta_1 = n\sin\theta_2$$

We can relate θ_2' to θ_2. Consider the triangle formed by the apex and the internal portion of the ray. The three internal angles of this add to 180°.

$$\Phi + (90\,° - \theta_2) + \left(90\,° - \theta_2'\right) = 180\,° \implies \theta_2 + \theta_2' = \Phi.$$

At the second interface we can then find θ_1' using Snell's law.

$$\sin\theta_1' = n\sin\theta_2'$$

The total angle of dispersion is δ in the diagram. It is the total amount the ray bends due to the prism. At the first interface it is deflected by $\theta_1 - \theta_2$ at the second it is deflected by $\theta_1' - \theta_2'$. The total amount of deflection is the sum of the two.

$$\delta = \theta_1 - \theta_2 + \theta_1' - \theta_2' \implies \delta = \theta_1 + \theta_1' - \Phi$$

Applying these expressions using the numbers given gives the following.

$$\Phi = 55\,° \text{ and } \theta_1 = 40\,°$$

	For red $n = 1.51$	For violet $n = 1.53$
$\sin\theta_1 = n\sin\theta_2$	$\theta_2 = 25.194\,°$	$\theta_2 = 24.842\,°$
$\theta_2 + \theta_2' = \Phi$	$\theta_2' = 29.806\,°$	$\theta_2' = 30.158\,°$
$\sin\theta_1' = n\sin\theta_2'$	$\theta_1' = 48.693\,°$	$\theta_1' = 50.232\,°$
$\delta = \theta_1 + \theta_1' - \Phi$	$\delta = 33.639$	$\delta = 35.232\,°$

The total angle of dispersion is the difference of these two deflection angles

$$\delta_{\text{violet}} - \delta_{\text{red}} = 1.59\,°.$$

Note that this could also be found by taking the difference between the values of θ_1'.

Problem K.6

The minimum angle of deviation when a light beam is shot through a prism with a 45° apex angle is 31°. What is the index of refraction for the prism?

Solution to K.6

Refer to the previous problem. The angle of deflection δ has its minimum value δ_{min} when the incoming and outgoing rays are symmetrical.

$$\theta_1 = \theta_1' \text{ and } \delta = \theta_1 + \theta_1' - \Phi \implies \theta_1 = \frac{\delta_{\text{min}} + \Phi}{2} = \frac{31\,° + 45\,°}{2} = 38\,°$$

$$\theta_2 = \theta_2' \text{ and } \theta_2 + \theta_2' = \Phi \implies \theta_2 = \frac{\Phi}{2} = \frac{45\,°}{2} = 22.5\,°$$

Snell's law gives the index.

$$\sin\theta_1 = n\sin\theta_2 \implies n = \frac{\sin\theta_1}{\sin\theta_2} = \frac{\sin 38\,°}{\sin 22.5\,°} = 1.61$$

Problem K.7

Consider a convex mirror with a 40 cm radius.
(a) If an object is 30 cm from the mirror then where is the image and what is its magnification?
(b) Repeat (a) if the object is 60 cm from the mirror.

Solution to K.7

For all cases of spherical mirrors or lenses we have

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} \quad \text{and} \quad m = \frac{h'}{h} = -\frac{s'}{s}.$$

For a convex mirror $R < 0$, so $f = R/2 = -40/2 = -20$ cm.

(a) $s = 30$ cm

$$s' = \left(\frac{1}{f} - \frac{1}{s}\right)^{-1} = \left(\frac{1}{-20} - \frac{1}{30}\right)^{-1} = -12 \text{ cm}$$

$$m = -\frac{s'}{s} = -\frac{-12}{30} = 0.4$$

(b) $s = 60$ cm

$$s' = \left(\frac{1}{f} - \frac{1}{s}\right)^{-1} = \left(\frac{1}{-20} - \frac{1}{60}\right)^{-1} = -15 \text{ cm}$$

$$m = -\frac{s'}{s} = -\frac{-15}{60} = 0.25$$

Problem K.8

A concave mirror has a 60 cm radius.
(a) If a 5 cm high object is placed 90 cm from the mirror then where is the image and what is its height?
(b) Repeat (a) if the same object is placed 20 cm from the mirror.
(c) Trace the rays for both cases.

Solution to K.8

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} \quad \text{and} \quad m = \frac{h'}{h} = -\frac{s'}{s}.$$

For a concave mirror $f = \dfrac{R}{2} = \dfrac{60}{2} = 30$ cm.

(a) $s = 90$ cm

$$s' = \left(\frac{1}{f} - \frac{1}{s}\right)^{-1} = \left(\frac{1}{30} - \frac{1}{90}\right)^{-1} = 45 \text{ cm}$$

$$\frac{h'}{h} = -\frac{s'}{s} \implies \frac{h'}{5} = -\frac{45}{90} = -0.5 \implies h' = -2.5\,\text{cm}$$

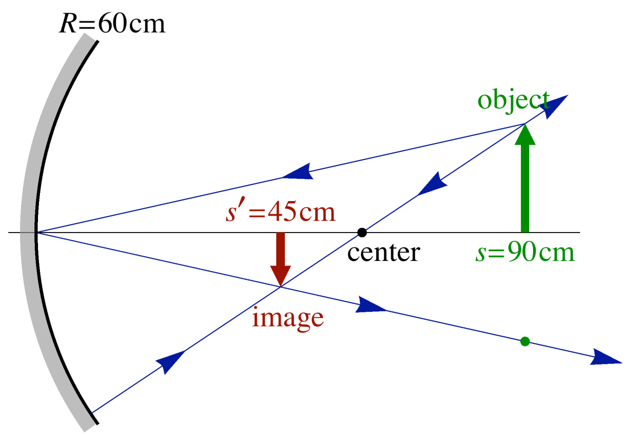
(b) $s = 20\,\text{cm}$

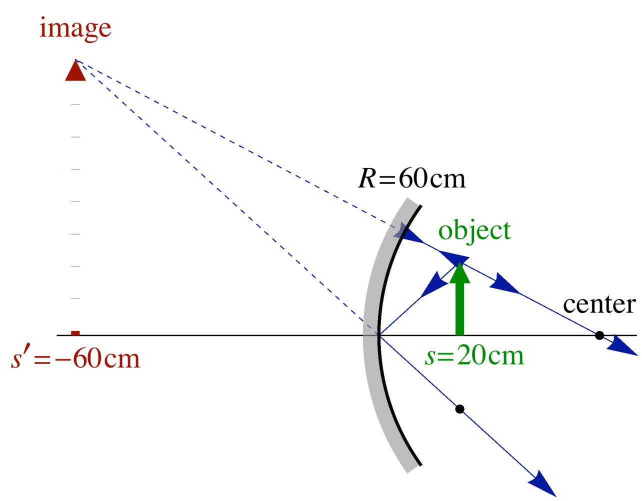
$$s' = \left(\frac{1}{f} - \frac{1}{s}\right)^{-1} = \left(\frac{1}{30} - \frac{1}{20}\right)^{-1} = -60\,\text{cm}$$

$$\frac{h'}{h} = -\frac{s'}{s} \implies \frac{h'}{5} = -\frac{-60}{20} = 3 \implies h' = 15\,\text{cm}$$

$$\frac{h'}{h} = -\frac{s'}{s} = -\frac{-60}{20} = 3$$

(c) To ray trace mark the center, a distance R from the mirror, and mark the point at the position of the object below the axis by the object's height.





Problem K.9

When an object is 30 cm away from a spherical mirror an image is created that is inverted and four times larger than the object. What is the radius of the mirror? Is it concave or convex? Trace the rays for this arrangement.

Solution to K.9

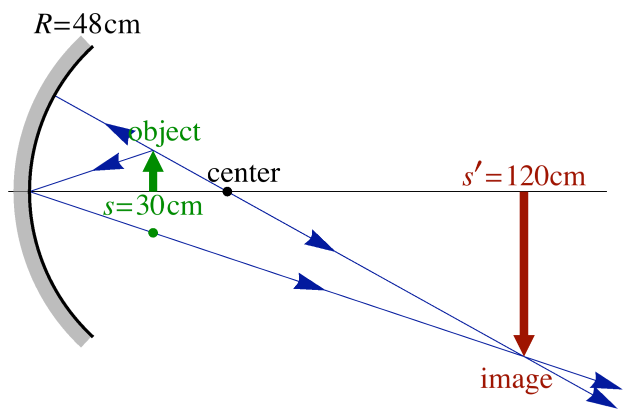
$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} \quad \text{and} \quad m = \frac{h'}{h} = -\frac{s'}{s}.$$

The image distance is $s = 30$. The image is inverted so h' is negative.

$$-4 = \frac{h'}{h} = -\frac{s'}{s} \implies s' = 4\,s = 120$$

$$\frac{R}{2} = f = \left(\frac{1}{s} + \frac{1}{s'}\right)^{-1} = \left(\frac{1}{30} + \frac{1}{120}\right)^{-1} = 24 \implies R = 48\text{ cm}$$

Since f and R are positive it is a concave mirror.



Problem K.10

A concave make-up mirror has a radius R. To create an upright image that is four times larger than the object, where must the object be placed?

Solution to K.10

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} \quad \text{and} \quad m = \frac{h'}{h} = -\frac{s'}{s}.$$

The focal length is half the radius, $f = R/2$. The image is upright so its magnification is positive.

$$+4 = m = \frac{h'}{h} = -\frac{s'}{s} \implies s' = -4\,s$$

$$\frac{2}{R} = \frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{s} + \frac{1}{-4\,s} \implies s = \frac{R}{2}\left(1 - \frac{1}{4}\right) = \frac{3}{8}\,R$$

Problem K.11

Arctic explorers see a frozen animal that is 50 cm below the surface of a glacier. How far below the surface does the animal appear when viewed from directly above? The index of refraction of ice is 1.31 .

Solution to K.11

$$\frac{s'}{s} = -\frac{n_2}{n_1} \implies \frac{s'}{50} = -\frac{1}{1.31} \implies s' = -38.2\text{ cm}$$

The negative sign implies it is a virtual image. In this case it means it is below the surface by 38.2 cm.

Problem K.12

A glass ($n = 1.50$) sphere with a 15 cm radius has a small imperfection 5 cm above the center. When viewed from directly above where is the image? Is it above or below the surface

Solution to K.12

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R}$$

The ray is passing from inside the glass $n_1 = 1.50$ to air $n_2 = 1$. The radius is taken to be positive when the center is on the side where the light ends up. In this case the center is where the light originates, so $R = -15\text{ cm}$. The object is $s = 15 - 5 = 10\text{ cm}$ from the surface.

$$\frac{1.5}{10} + \frac{1}{s'} = \frac{1 - 1.5}{-15} \implies s' = -8.57\text{ cm}$$

It appears 8.57 cm below the surface.

Problem K.13

A goldfish is inside a spherical bowl with a 12 cm radius. Assume the glass of the bowl is thin so that the glass layer can be neglected. If a person views the bowl with his face at the same level as the center of the bowl and the goldfish is on the same radial line as the face. Where does the goldfish see the face if the face is 20 cm from the edge of the bowl? Where if the face is 5 cm from the bowl.

Solution to K.13

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R}$$

The ray is passing from inside the air $n_1 = 1$ to water $n_2 = 1.33$. The radius is taken to be positive because the center is inside the bowl $R = +12\text{ cm}$.

$$\frac{1}{s} + \frac{1.33}{s'} = \frac{1.33 - 1}{+12}$$

$$s = 20\text{ cm} \implies s' = -59.1\text{ cm}$$

$$s = 5\text{ cm} \implies s' = -7.71\text{ cm}$$

The negative q means the face appears outside of the bowl.

Problem K.14

What is the focal length of a biconvex lens with surfaces of 10 cm and 15 cm radii, if the lens is made of a plastic material with $n = 1.60$? Would this change is the lens is reversed?

Solution to K.14

Use the lensmaker formula $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$. The radius is taken to be positive if its center is on the side where the light emerges from the lens. Since the lens is biconvex the $R_1 > 0$ and $R_2 < 0$.

$$\frac{1}{f} = (1.60 - 1)\left(\frac{1}{10} - \frac{1}{-15}\right) \implies f = 10\,\text{cm}$$

Reversing the lens makes $R_1 = 15\,\text{cm}$ and $R_2 = -10\,\text{cm}$ which gives the same value for f. This is a general feature of lenses.

Problem K.15

When an object is placed 10 cm from a thin lens a virtual image is seen 15 cm from the lens. What is the focal length of the lens? What type of lense is it, converging or diverging?

Solution to K.15

Here we have $s = 10\,\text{cm}$ and since it is a virtual image $s' = -15\,\text{cm}$.

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} \implies f = \left(\frac{1}{s} + \frac{1}{s'}\right)^{-1} = \left(\frac{1}{10} + \frac{1}{-15}\right)^{-1} = 30\,\text{cm}$$

Problem K.16

A slide projector uses a converging lens to project a 35 mm wide slide to fill a 70 cm wide screen. If the distance from the slide to the screen is to be 6.3 m then where must the slide be placed and what focal length must the lens have?

Solution to K.16

$h = 3.5\,\text{cm}$ and $h' = -70\,\text{cm}$. Note that the image height is negative because it is inverted. Another way of seeing this is the image must be real because it is projected and a real image must have a positive q; this requires a negative h'.

$$-\frac{s'}{s} = \frac{h'}{h} = \frac{-70}{3.5} = -20 \implies s' = 20\,s$$

The slide (object) to screen (image) distance is $s + s'$,

$$s + s' = 630\,\text{cm} \implies s + 20\,s = 630\,\text{cm} \implies s = \frac{630}{21} = 30\,\text{cm}.$$

$$s' = 630 - s = 600 \implies f = \left(\frac{1}{s} + \frac{1}{s'}\right)^{-1} = \left(\frac{1}{30} + \frac{1}{600}\right)^{-1} = 28.6\,\text{cm}.$$

Problem K.17

An object is 20 cm from a diverging lens with $f = -32\,\text{cm}$. Where is the image and what is its magnification? Trace the rays for this arrangement.

Solution to K.17

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} \quad \text{and} \quad m = \frac{h'}{h} = -\frac{s'}{s}.$$

$$f = -32\text{ cm and } s = 20\text{ cm} \implies s' = \left(\frac{1}{f} - \frac{1}{s}\right)^{-1} = \left(\frac{1}{-32} - \frac{1}{20}\right)^{-1} = -12.3\text{ cm}$$

$$m = -\frac{s'}{s} = -\frac{-12.3}{20} = 0.615$$