Chapter L - Problems

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Problem L.1

Young's double slit experiment is performed by shooting a He-Ne laser beam ($\lambda = 632.8$ nm) through two slits separated by 0.15 mm onto a screen 8 m away.
(a) What is the distance between bright fringes on the screen?
(b) What is the distance between dark fringes?
(c) What is the distance from the center of the central maximum to the third dark fringe?
(d) What is the smallest distance from the central maximum where the intensity is 3/4 its maximum?

Solution to L.1

$$\lambda = 632.8 \times 10^{-9} \text{ m}, \ L = 8 \text{ m} \ \text{ and } \ d = 0.15 \times 10^{-3} \text{ m}$$

(a) $d \sin\theta = m\,\lambda$ Assuming the small angle approximation $\sin\theta = \frac{y}{L}$ we get $d\,\frac{y}{L} = m\,\lambda$. (The small angle approximation will be justified if the y value is much less than L.)

The bright fringes are at integer values of m so between bright fringes we have $\Delta m = 1$.

$$y = m\,\frac{\lambda L}{d} \implies \Delta y = \Delta m\,\frac{\lambda L}{d} = 0.0337 = 3.37 \text{ cm}$$

Note that $0.0337 \text{ m} \ll 5\,m$. This justifies the small angle approximation.

(b) The distance between dark bands is the same as the distance between bright fringes.

$$\Delta\left(m + \frac{1}{2}\right) = \Delta m = 1 \implies \Delta y = 3.37 \text{ cm}$$

(c) Still assume the small angle approximation. For destructive interference (interference minimum) we get:

$$d \sin\theta = \left(m + \frac{1}{2}\right)\lambda \implies d\,\frac{y}{L} = \left(m + \frac{1}{2}\right)\lambda.$$

Note that the values of $\left(m + \frac{1}{2}\right)$ are 0.5, 1.5, 2.5, ... , so the third minimum is $\left(m + \frac{1}{2}\right) = 2.5$.

$$y = \left(m + \frac{1}{2}\right)\frac{\lambda L}{d} = 2.5\,\frac{\lambda L}{d} = 8.43 \text{ cm}$$

(d) The intensity formula is

$$I = I_0 \cos^2\left(\frac{\pi\,d \sin\theta}{\lambda}\right)$$

Using the small angle approximation we get

$$I = I_0 \cos^2\left(\frac{\pi\,d\,y}{\lambda L}\right)$$

3/4 the maximum intensity means that $\frac{I}{I_0} = 0.75$.

$$\cos^{-1}\sqrt{\frac{I}{I_0}} = \frac{\pi\, d\, y}{\lambda\, L} \implies y = 5.62\ \text{mm}$$

Make sure your calculator is in the radians mode for this calculation.

Problem L.2

700 nm light shines through a single slit of width 1.2×10^{-5} m and shines on a screen 55 cm away.
(a) What is the distance from the center to the third maximum? (Do not use the small angle approximation here?
(b) Repeat part (a) using the small angle approximation.

Solution to L.2

$$d = 1.2\times 10^{-5}\ \text{m},\ \ \lambda = 700\times 10^{-9}\ \text{m} \ \text{ and } \ L = 55\ \text{cm}$$

(a) The central maximum is at $y = 0$. For constructive interference, using the small angle approximation, we get:

$$d\ \sin\theta = m\,\lambda \ \text{ and } \ m = 3 \implies \theta = \sin^{-1}\frac{m\,\lambda}{d} = 10.079\,°$$

$$\tan\theta = \frac{y}{L} \implies y = L\tan\theta = 0.55\tan\theta = 0.0978\ \text{m}$$

(b) With the small angle approximation

$$d\ \frac{y}{L} = m\,\lambda \ \text{ and } \ m = 3 \implies y = \frac{m\,\lambda\,L}{d} = 0.963\ \text{m}$$

Problem L.3

When 632.8 nm light shines through a diffraction grating the first bright line is observed at a deflection angle of 15°. What is the spacing between the slits in the grating?

Solution to L.3

For a diffraction grating the condition for constructive interference is the same as for the single slit, $d\sin\theta = m\,\lambda$ but there is destructive interference everywhere else. The first bright fringe corresponds to $m = 1$.

$$d\sin\theta = m\,\lambda,\ \ m = 1,\ \ \theta = 15\,° \text{ and } \lambda = 632.8\times 10^{-9}\ \text{m} \implies d = 2.44\times 10^{-6}\ \text{m}$$

Problem L.4

The visible spectrum varies between $\lambda_{\text{violet}} = 400$ nm and $\lambda_{\text{red}} = 700$ nm. Consider a diffraction grating with an arbitrary slit separation d. Show that for white light the $m = 1$ maximum gives pattern that doesn't overlap the $m = 2$ maximum. Also show that the $m = 2$ and $m = 3$ patterns always overlap.

Solution to L.4

First consider the $m = 1$ and $m = 2$ maxima.

$$d \sin\theta = m\lambda \implies \sin\theta = \frac{m\lambda}{d}$$

It is sufficient to consider the extremes only.

$$\sin\theta_{1,\text{red}} = \frac{1\,\lambda_{\text{red}}}{d} = \frac{700\text{ nm}}{d} \text{ and } \sin\theta_{2,\text{violet}} = \frac{2\,\lambda_{\text{violet}}}{d} = \frac{800\text{ nm}}{d}$$

Thus for any d we have $\sin\theta_{1,\text{red}} < \sin\theta_{2,\text{violet}}$. Since sine is an increasing function (in the first quadrant) we can conclude that

$$\theta_{1,\text{red}} < \theta_{2,\text{violet}}.$$

Now consider the $m = 2$ and $m = 3$ maxima.

$$\sin\theta_{2,\text{red}} = \frac{2\,\lambda_{\text{red}}}{d} = \frac{1400\text{ nm}}{d} \text{ and } \sin\theta_{3,\text{violet}} = \frac{3\,\lambda_{\text{violet}}}{d} = \frac{1200\text{ nm}}{d}$$

Thus for any d we have $\sin\theta_{3,\text{violet}} < \sin\theta_{2,\text{red}}$, giving $\theta_{3,\text{violet}} < \theta_{2,\text{red}}$.

Problem L.5

A diffraction grating has 300 lines/mm. What is the highest order violet (400 nm) line seen? What is the highest order red line (700 nm).

Solution to L.5

If there are 250 lines/mm, then the slit spacing is $\frac{1}{300}$ mm.

$$d = \frac{1}{300} \times 10^{-3} = 3.3333 \times 10^{-6}\text{ m} = 3333.3\text{ nm}$$

Here the small angle approximation cannot be used. The largest angle is 90° so we have $\sin\theta \le 1$. The condition for seeing the m^{th} bright fringe is:

$$d \sin\theta = m\lambda \text{ and } \sin\theta \le 1 \implies \sin\theta = \frac{m\lambda}{d} \le 1 \implies m \le \frac{d}{\lambda}$$

The highest order fringe is the largest m (an integer) that satisfies this inequality.

$$\lambda = 400\text{ nm} \implies m \le \frac{3333.3\text{ nm}}{400\text{ nm}} = 8.333 \implies m_{\max} = 8$$

(Note that since d/λ is a real number the distinction between ' ≤ ' and '<' in $m \le \frac{d}{\lambda}$ should not be important. In this problem it matters; it changes the answer. Whether the answer should be 10 or 9 is a matter of interpretation and the problem is ambiguous.)

$$\lambda = 700\text{ nm} \implies m \le \frac{3333.3\text{ nm}}{700\text{ nm}} = 4.76 \implies m_{\max} = 4$$

Problem L.6

550 nm light passes through a single vertical slit of width 0.16 mm. What is the width of the central maximum observed on a screen 1.2 m away?

Solution to L.6

$$\lambda = 550 \times 10^{-9}\ \text{m}\ ,\ L = 1.2\ \text{m}\ \text{ and }\ a = 0.16 \times 10^{-3}\ \text{m}$$

The condition for destructive interference (dark fringes) for a single slit is $a \sin\theta = m\,\lambda$, where $m = \pm 1,\ \pm 2,\ \pm 3,\ \ldots$. Note that $m = 0$ is not a dark fringe; it is the center of the central bright fringe. The the central bright fringe is from $m = -1$ to $m = +1$. Use the small angle approximation.

$$a \sin\theta = m\,\lambda \implies a\,\frac{y}{L} = m\,\lambda \implies y = \frac{m\,\lambda\,L}{a} = \frac{1 \times 550 \times 10^{-7} \times 1.2}{0.16 \times 10^{-3}} = 4.125 \times 10^{-3}$$

The width of the central maximum is $2\,y = 8.25$ mm .

Problem L.7

When a He-Ne laser beam ($\lambda = 632.8$ nm) passes through a narrow vertical slit a diffraction pattern is observed on a screen 80 cm away. If the distance from the first dark fringe to the third is 2.5 mm then what is the width of the slit?

Solution to L.7

The condition for destructive interference with the small angle approximation becomes

$$a \sin\theta = m\,\lambda \implies a\,\frac{y}{L} = m\,\lambda$$

$$y = m\,\frac{\lambda\,L}{a} \implies \Delta y = \Delta m\,\frac{\lambda\,L}{a} \implies 0.0025 = (3 - 1)\,\frac{632.8 \times 10^{-9} \times 0.80}{a} \implies a = 0.405\ \text{mm}$$

Problem L.8

When light of wavelength λ passes through a narrow vertical slit the highest order dark fringe observed (over all possible angles) is 5. (The fifth fringe can be seen and the sixth cannot.) What can one conclude about the width of the slit?

Solution to L.8

The small angle approximation cannot be used. $a \sin\theta = m\,\lambda$. To do this use the fact that $\sin\theta \le 1$ for $m = 5$ and $\sin\theta \ge 1$ for $m = 6$.

$$\sin\theta = \frac{m\,\lambda}{a} \implies \frac{5\,\lambda}{a} \le 1 \text{ and } \frac{6\,\lambda}{a} \ge 1 \implies 5\,\lambda \le a \le 6\,\lambda$$

Problem L.9

A soap film in air has a thickness of 120 nm. What visible wavelengths are strongly reflected by this film? Take the index of the soap film to be the same as for water, 1.33. The visible spectrum is between 400 nm and 700 nm.

Solution to L.9

Strongly reflected implies constructive interference. The thickness of the film is $t = 120$ nm. The soap is between air layers on both sides so $n = 1.333 > n' = 1$.

$$n > n' \text{ and const. int.} \implies 2\,t = \left(m + \frac{1}{2}\right)\frac{\lambda}{n} \implies \lambda = \frac{2\,n\,t}{\left(m+\frac{1}{2}\right)} = \frac{2\times 1.33\times 120\text{ nm}}{\left(m+\frac{1}{2}\right)} = \frac{319.23\text{ nm}}{\left(m+\frac{1}{2}\right)}$$

For values of $\left(m + \frac{1}{2}\right) = 0.5,\ 1.5,\ 2.5,\ \ldots$ we get a list of possible wavelengths.

$$\lambda = 638\text{ nm},\ 213\text{ nm},\ 128\text{ nm},\ \ldots$$

Since the visible spectrum is between 400 nm and 700 nm, the only visible wavelength in the list is 638 nm.

Problem L.10

Red light ($\lambda = 650$ nm) is normally incident on a thin oil layer ($n = 1.25$) that sits on a puddle of water.
(a) What is the smallest nonzero thickness that will strongly reflect the red light?
(b) What is the smallest nonzero thickness that will minimally reflect the red light?

Solution to L.10

$\lambda = 650$ nm. The index of the film is $n = 1.25$, which is larger than that of water $\left(n' = 1.33\right)$, so $n < n'$

(a) Strongly reflect implies constructive interference

$$2\,n\,t = m\,\lambda \implies t = m\,\frac{\lambda}{2\,n} = m\times 260\text{ nm}$$

The values of m are $0, 1, 2, \ldots$ so the smallest nonzero thickness is when $m = 1$.

$$t_{\text{min}} = 260\text{ nm}$$

(b) Minimally reflect implies destructive interference

$$2\,n\,t = \left(m + \frac{1}{2}\right)\lambda \implies t = \left(m + \frac{1}{2}\right)\frac{\lambda}{2\,n} = \left(m + \frac{1}{2}\right)\times 260\text{ nm}$$

Since the values of $m + \frac{1}{2}$ are $0.5, 1.5, 2.5, \ldots$. It follows the the minimum is when $m + \frac{1}{2} = 0.5$.

$$t_{\text{min}} = 130\text{ nm}$$

Problem L.11

You are asked to design an antiglare coating for a computer monitor. Sitting on the glass ($n = 1.50$) screen is a thin coating of magnesium fluoride with an index of refraction of 1.38. To be an antiglare coating it must satisfy two conditions. It must give destructive interference for light in the middle of the visible spectrum, at 550 nm. Secondly, there must be no constructive interference for any wavelength in the visible spectrum, from 400 nm to 700 nm. First find the different thicknesses that give the destructive interference and then use the lack of consructive interference to select the thickness uniquely.

Solution to L.11

Here we have $n = 1.38$ for the coating and the glass is $n' = 1.50$. For destructive interference at 550 nm:

$$n < n' \text{ and dest. int.} \implies 2\,n\,t = \left(m + \frac{1}{2}\right)\lambda. \implies t = \left(m + \frac{1}{2}\right)\frac{\lambda}{2\,n} = \left(m + \frac{1}{2}\right) 199.275\text{ nm}$$

$$\left(m + \frac{1}{2}\right) = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \ \dots \implies t = 99.637\text{ nm},\ 298.91\text{ nm},\ 498.19\text{ nm},\ \dots$$

Thus the minimum thickness is $t_0 = 99.637$ nm and the others are $3\,t_0,\ 5\,t_0,\ 7\,t_0,\ \dots.$

Now consider the constructive interference condition for the smallest thickness $t_0 = 99.637$ nm

$$n < n' \text{ and dest. int.} \implies 2\,n\,t_0 = m\,\lambda \implies \lambda = \frac{2\,n\,t_0}{m} = \frac{275\text{ nm}}{m}$$

$$m = 1, 2, 3, \ \dots.. \implies \lambda = 276\text{ nm},\ 138\text{ nm},\ 92\text{ nm},\ \dots.$$

The visible spectrum is between 400 nm and 700 nm, so there are no visible wavelengths that are intensified. It should also be clear that if we choose other thicknesses that give destructive interference $3\,t_0,\ 5\,t_0,\ 7\,t_0,\ \dots$, then we will multiply the above wavelength list by 3, 5, 7, 9, .. . This will always give constructive interference at some visible wavelength. The unique thickness is thus 99.6 nm

Problem L.12

Unpolarized light with an intensity of $4000\ \text{W}/\text{m}^2$ passes through three polarizing filters, the first is at a 15° angle from vertical, the second is 40° from vertical and the third is horizontal. What are the intensities between the first and second filters, between the second and third filters and after the third filter?

Solution to L.12

When polarized light with intensity I_0 passes through a polarizing filter it leaves with half the intensity $I = I_0/2$ and is polarized along the axis of the filter. It follows that the intensity between the first and second filters is:

$$I = \frac{1}{2} I_0 = \frac{1}{2} 4000 = 2000 \frac{\text{W}}{m^2},$$

and this is polarized at $\theta_1 = 15\,°$.

When polarized light with intensity I_0 passes through a filter it leaves polarized along the axis of the filter and with intensity $I = I_0 \cos^2\theta$, where θ is the angle between the polarization of the light and the axis of the filter. After the second filter at $\theta_2 = 40\,°$ the intensity is

$$I' = I\ \cos^2(\theta_2 - \theta_1) = 2000 \cos^2(40\,° - 15\,°) = 1644.8 = 1645 \frac{\text{W}}{m^2},$$

The same analysis gives the intensity after the third filter.

$$I'' = I' \cos^2(\theta_3 - \theta_2) = 1644.8 \cos^2(90\,° - 40\,°) = 679 \frac{\text{W}}{m^2},$$

Problem L.13

When the sun is what angle *above the horizon* will sunlight reflected off a still pond be totally polarized?

Solution to L.13

The polarization angle for light from air reflecting off water is θ_p.

$$\tan\theta_p = \frac{n_2}{n_1} = \frac{1.33}{1} \implies \theta_p = 53.1\,°$$

This angle is the incident angle θ_1 which is is measured from the normal. The angle above the horizon is its complement.

$$90\,° - 53.1\,° = 36.9\,°$$