Appendix 2

Fundamental Constants and the SI System

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The second, meter and kilogram

The SI base units we use in mechanics are seconds, meters and kilograms. Over the years these definitions evolved to allow for more precise measurements.

The original definition of a second was in terms of a day, $60 \times 60 \times 24$ seconds is one day and a day is the average time for each rotation of the earth relative to the sun. This was not appropriate for accurate measurements and the second was redefined in terms of the current most accurate way of measuring time, atomic clocks. The most common atomic clock is based on a radiation frequency of a "hyper-fine structure" transition in the common isotope of Cesium, $^{133}\mathrm{Cs}$. Since 1967, a second has been defined as exactly 9192631 cycles of this radiation frequency.

Originally, the meter was defined in terms of the dimensions of the earth; the distance from the north pole to the equator (along a meridian through Paris) was 10 000 kilometers. This was, of course, difficult to reproduce, so the standard meter was converted to the distance between two marks on a metal bar at some fixed temperature. As science progressed, and scientific precision increased, a better standard was needed. Early in the twentieth century it was redefined in terms of a fixed number of wavelengths of a specific emission line of a krypton atom. Since 1983, the modern definition is now in terms of the speed of light. We define the speed of light to be some exact value, given below, and that then defines the meter in terms of the atomic clock definition of a second.

The original definition of the gram and kilogram was in terms of lengths and properties of water; a cubic centimeter of water was one gram. For more precision, a standard kilogram was created; some artifact that defined the kilogram. This artifact changed over time but the use of some artifact as a standard kilogram persisted until 2019.

The 2019 Redefinition of the SI System

In May 2019 the SI system of units was redefined. By choosing an exact value for the speed of light we were able to define a meter. Now all SI base units are defined entirely in terms of fundamental constants whose values are chosen as exact. Planck's constant, which is now referred to officially as the Planck constant, is the fundamental constant of quantum physics. The table below shows its units involve meters, seconds and kilograms, so by choosing its value to be exact we can now define the kilogram. Although, theoretically, this could have been done when Planck first introduced his constant in 1900, as a practical matter there would have been no way to use this definition to accurate calibrate a scale to kilograms. Experimental advances late in the twentieth century made it possible to create such a scale to accurately measure kilograms; this scale is a very complicated apparatus known as a Kibble balance, named after its inventor.

Fundamental Constant Name	Symbol Constant	Exact Numerical Value of Constant	SI Units of Constant and Base Units	Defined Base Unit
HFS freq. $^{133}\mathrm{Cs}$	Δf_{Cs}	9 192 631	$\mathrm{Hz} = \mathrm{s}^{-1}$	s = second
Speed of Light	c	299 792 458	ms^{-1}	m = meter
Planck Constant	h	$6.62607015 \times 10^{-34}$	$\mathrm{J \cdot s} = \mathrm{kg \cdot m^2 s^{-1}}$	kg = kilogram
Boltzmann Constant	k_{B}	1.380649×10^{-23}	$\mathrm{J/K} = \mathrm{kg \cdot m^2 s^{-2} K^{-1}}$	K = kelvin
Avogadro Constant	N_{A}	$6.02214076 \times 10^{23}$	mol^{-1}	mol = mole
Elementary Charge	e	$1.60217634 \times 10^{-19}$	$\mathrm{C} = \mathrm{A \cdot s}$	A = ampere
Luminous Efficacy	K_{cd}	683	$\mathrm{cd/W} = \mathrm{cd \cdot kg^{-1} m^{-2} s^3}$	cd = candela

With thermodynamics we introduce two more SI base units: the kelvin, which is an absolute temperature scale, and the mole which is a very basic unit in chemistry. Now, since 2017, the kelvin is defined by choosing the Boltzmann constant to have an exact value and the mole is defined by setting the value of the Avogadro constant (the new name for Avogadro's number) to an exact value.

The other two base SI units are the ampere and the candela. The ampere is the SI unit of electric current and is central to discussions of electromagnetism; it is now defined by choosing the elementary charge e to an exact value. (Note that the electron's charge is $-e$ and the protons charge is $+e$.) The candela is unimportant for our elementary physics courses; it is a measure of the total light output of a source and its constant, the luminous efficacy, relates the total brightness to the power output of radiant energy.

Some Other Units

The atomic mass unit u is related to the Avogadro constant, $N_A\,\mathrm{u} = 1\,\mathrm{g/mol}$. Previously, before 2019, u was defined using the carbon-12 standard, where the mass of ^{12}C was defined as exactly 12 u and the Avogadro constant was then defined in terms of that. Now the Avogadro constant is exact and carbon-12 is now approximately 12 u.

The Celsius scale is now defined in terms of the Kelvin scale, where temperature differences in the two scales are the same, one celsius-degree equals one kelvin, and the zero of the scale (approximately the freezing point of water at one atmosphere) is defined as exactly 273.15 K.

Name	Constant	Numerical Value	SI Unit	Definition
Atomic Mass Unit	u	$10^{-26}/6.02214076$ (exact)	kg	u
Zero Celsius	0 °C	273.15 (exact)	K	celsius

Other 2019 Redefinitions

At some specific pressure, lower than atmospheric pressure, there is an exact temperature where all three phases of water, solid, liquid and gas, can coexist in equilibrium. This is known as the triple point of water. Previously, this triple point was defined as exactly 273.16 K = 0.01 °C, and this previously defined the kelvin. Now this 273.16 value is approximate and the kelvin is defined in terms of the Boltzmann constant.

The ampere was previously defined by choosing an electromagnetic constant that appears in the equations describing magnetism; the vacuum permeability μ_0 was simply defined as exactly $4\,\pi \times 10^{-7}\,\mathrm{N/A^2}$. Although conceptually this was a very clean way to define the ampere, it turns out that this was impractical for making very precise current measurement. Now the value of μ_0 is approximate and the ampere is defined in terms of the elementary charge. There is another electromagnetic constant ε_0, the vacuum permittivity, that can be written in terms of μ_0 and the speed of light; since μ_0 is no longer exact, neither is ε_0.

Name	Constant	Numerical Value	SI Unit
Triple Point of Water	$T_{\mathrm{tp,H_2O}}$	273.16 (approximate)	K
Vacuum Permeability	μ_0	$4\pi \times 10^{-7}$(approximate)	$\mathrm{N/A^2 = kg \cdot m^2 s^{-2}}$
Vacuum Permittivity	ε_0	$1/(\mu_0\,\mathrm{c}^2)$	–